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ALTERNATING CURRENT RECTIFICATION

ALTERNATING CURRENT RECTIFICATION AND ALLIED PROBLEMS

A MATHEMATICAL AND PRACTICAL TREATMENT FROM THE ENGINEERING VIEW-POINT

BY

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TO

CLIFFORD C. PATERSON

εἷς δέ τις ἀρχὸς ἀνὴρ βουληφόρος

—ILIAD, I. 144

PREFACE TO THIRD EDITION

THE chief alterations embodied in this third edition are those to be found in Part I and a new Part V, together with slight additions to the chapters on Mercury Vapour Rectifiers.

The reorganisation of Part I is consequent on the inclusion of more recent methods of Harmonic Analysis, which simplify the calculation of the Fourier coefficients. These methods have been obtained from Mr. A. Eagles' book on the subject, and should properly be studied there for fuller information.

The appearance of a book of a more physical character, dealing with the fundamental theories of rectification by Professor Gunther Schulze, translated by N. A. de Bruyne, will introduce the English reader to modern theories of rectification, which, if they are not complete, will prove an interesting and helpful speculation.

The recent developments in the capacity of solid contact rectifiers, where there is a permeable and impermeable metal-metal boundary, necessitates the segregation of this class into a separate part, and commercial rectifiers of this type are available in America. In this connection the modern theories of Schottky and Stransky, etc., have not been considered as they do not appear to be complete. For further reference on these subjects the reader is referred to the above-mentioned book by de Bruyne.

Thanks are especially due to Messrs. A. Eagles and N. A. de Bruyne for permission to publish extracts from their respective books, and to Mr. L. D. Grondahl, and the American Institute of Electrical Engineers.

L. B. W. JOLLEY.

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PREFACE TO THE SECOND EDITION

The recent improvements in the design and manufacture of rectifiers of nearly all the important types, have rendered an early revision essential; and while the general arrangement of the book remains unaltered, three new chapters have been added, viz. :—

Installation of Thermionic Rectifiers.
Radio Supplies.
Inverters.

It may be advanced that the second subject is one which should not be included in a technical work, but with the increase in the use of wireless apparatus, and the comparatively high cost of alternative supplies (such as dry batteries, etc.), it has been represented that it is worth the risk of adverse criticisms if aid can be afforded to users and manufacturers in the solution of what is, at the present juncture, a difficult problem.

In addition to the acknowledgments in the Preface of the first edition, I should wish to include the following: Acme International X-Ray Co., Mr. H. André, Prof. F. Bedell, Mr. N. A. de Bruyne, the Bureau of Standards, Mr. W. Dallenbach, Mr. A. Eagles, the "Electric Journal," the English Electric Co., for permission to reproduce extracts from their technical pamphlet on the Transverter, "Experimental Wireless," Mr. H. Giroz, Mr. W. E. Highfield, Mr. S. B. Kraut, the M.O. Valve Co., Mr. C. W. Marshall, Mr. S. W. Maxstadt, Sir Isaac Pitman & Sons Ltd., Mr. D. C. Prince, Philips Lamps Ltd., Radio Accessories Ltd., Saunders Bros. & Co. Ltd., Mr. M. G. Scroggie, and Messrs. Watson & Sons (Electro-Medical) Ltd.

Especially are thanks due to Mr. A. C. Bartlett for the contribution of the major portion of Chapter X.

In conclusion the Author would wish to thank those kindly critics of the first edition for their valuable suggestions, which in all possible cases have been embodied in this revision.

L. B. W. JOLLEY.

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PREFACE TO FIRST EDITION.

A TREATISE on the Rectification of Alternating Currents must of necessity cover a wide field, comprising voltages from 100,000 down to a small fraction of a volt. It must also include a consideration of rotating plant, special forms of arc, applications of certain electrolytic reactions, properties of certain crystals and the like; and therefore, although a number of types of apparatus must be discussed in the light of their relation to problems of rectification, restriction of space demands that questions of design and manufacture shall only receive cursory attention. In other words, a complete exposé of the various types of rotating and other plant has been deliberately omitted, except where it has been felt that the data cannot be obtained elsewhere. If it is found that the information is scattered and insufficient it is hoped that the bibliography will remedy the defect.

The use of unidirectional currents has a very large application both in the supply of power and in the laboratory, and it is the purpose of this book to attempt to describe the methods available, and to present the mathematical analysis with numerical examples where such are possible. For example, in the case of Thermionic Rectifiers, the Electron Theory, together with a full discussion of the emissivity of hot bodies, has only been cursorily examined, even though such a discussion is bound up fundamentally with the rectifying properties of thermionic valves. These and other problems have been left to the reader to probe further should he care to do so.

A moment's thought will show that if it is desired to become acquainted with the fundamental principles of rectification, the theories of electricity and magnetism generally, rotating machines and transformers, high voltage phenomena, electrolysis, crystallography, etc., would have to be thoroughly investigated, and it is obviously impossible to cope with such within the bounds of a single volume.

It has been difficult to know where to cease when discussing theories, and the indulgence of the reader is sought in any criticisms he may have as to omissions of developments which should have been included. On the other hand, an attempt has been made to make the book as complete as possible in itself: to render reference to other works as unnecessary as need be, except where more detailed information is desired.

Regarding the mathematical analyses which occur, it is preferable in considering the effect of inductance and capacity to reduce these quantities to their respective inductive and condensive reactances, as by so doing the time factor is eliminated, and it is possible to work solely in electrical degrees.

Thus if L and C are the inductance and capacity and x and x_c are their respective reactances

$$x = p\mathrm{L} \text{ and } x_c = 1/p\mathrm{C}$$

where p is the periodicity ($= 2\pi \times$ frequency).

This effects a simplification in that if the current or voltage wave obeys a sinusoidal law of the form

$$\mathrm{K} \sin pt$$

it is at once possible to write

$$\mathrm{L}\frac{di}{dt} = x\frac{di}{d\theta}$$

or

$$\frac{1}{C}\int idt = x_c\int id\theta.$$

The wave form then becomes $\mathrm{K} \sin \theta$

and $\epsilon^{\frac{r}{\mathrm{L}}t}$ becomes $\epsilon^{\frac{r}{x}\theta}$.

The impedance of a circuit containing both of these quantities is then $Z^2 = R^2 + (x - x_c)^2$ and all are in ohms.

Nothing is lost by such a conversion, as the time element can readily be introduced by taking the periodicity into consideration if so desired.

An extensive search has been made of the available literature, and the chief source of the references has been Science Abstracts; in the bibliography at the end of each section, therefore, a reference has been made to the particular Abstract concerned, as it is often found that a glance at an Abstract may render a reference to an original article in a periodical unnecessary: such periodicals, especially those of foreign origin, are often difficult of access and translations not easy to obtain.

In conclusion I should wish to tender my best thanks to the following for criticisms of the text: Messrs. W. H. Glaser, L. D. Goldsmith, G. C. Marris, J. W. Ryde, R. W. W. Sanderson, G. Granville Sharp, M. Thomson, J. Wade; and to those who have kindly given permission for the reproduction of various diagrams and tables, etc.: Dr. G. E. Bairsto, Mr. Chattock of the Birmingham City Council, the British Thomson-Houston Company, Mr. V. Bush, Mr. M. A. Codd, the Crypto Electrical Company, Mr. F. Keith Dalton, the General Electric Company, the Hewittic Electric Company, Mr. J. S. Highfield, Mr. C. H. Holbeach, Mr. A. W. Hull, Herr Jungmichl, the Lodge Cottrell Company, the North Metropolitan Electric Supply Company, the Council of the Physical Society of London, Dr. D. Owen, Power Rectifiers Ltd. and the Brown Boveri Company, Dr. R. L. Smith Rose, the late Prof. C. P. Steinmetz, Mr. G. Sutton, Dr. W. Tschudy, Prof. Miles Walker, and Prof. J. B. Whitehead, and especially to Mr. E. T. Ritchie and Mr. J. W. Ryde for most valued assistance in proof reading and corrections to the text.

L. B. W. JOLLEY.

FAIRDENE, SHEEN ROAD,
RICHMOND,
July, 1924.

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CHAPTER XVIII.

SYMBOLS EMPLOYED.

I	Maximum Current (peak value); Moment of Inertia of Beam Section. Value of Discontinuity.
i	Instantaneous Current.
$\mathcal{I}$	R.M.S. or Effective Value of the Current.
I_M	Mean Value of Current over $\frac{1}{2}$ period $\{= I_0$ in rectified circuits$\}$.
E	Maximum Voltage, and Young's Modulus.
e	Instantaneous Voltage, or Counter E.M.F., and stiffness.
$\mathcal{E}$	R.M.S. or Effective Value of the Voltage.
E_M	Mean Value of the Voltage over $\frac{1}{2}$ period $\{= E_0$ in rectified circuits$\}$.
e_g	Cut-off Value of the Voltage.
ρ	Resistivity.
τ	Temperature.
Δ	Density, and relative heating effect in Rotary Converters.
Φ	Maximum Flux.
ϕ	Instantaneous Flux.
θ	Electrical Degrees.
θ_0 θ_1, θ_a	Angular Cut-off in Electrical Degrees.
C	Capacity.
X, x	Inductive Reactance.
X_0, x_0	Condensive Reactance.
Z	Impedance.
L	Inductance.
l	Length.
R, r	Resistance.
j	$\sqrt{-1}$.
ϵ	Base to Naperian Logarithms.
$\phi\,(\,)$, $f\,(\,)$	Functions of.
$\phi'\,(\,)$, $f'\,(\,)$	First Derivatives of the Functions.
p	Periodicity.
$a_1 a_2 a_3 a_4 a_n$	Cosine Amplitudes in Fourier's Series.
$b_1 b_2 b_3 b_4 b_n$	Sine Amplitudes in Fourier's Series.
a_0	Constant Term in Fourier's Series.
W	Weight.
w	Weight per Unit Length.
κ	Damping Factor of a Reed.

Note.—Further, the practice has generally been adopted in rectifier circuits of calling the R.M.S. transformer current and voltage $\mathcal{I}'$ and $\mathcal{E}'$; and the R.M.S. rectified current and voltage $\mathcal{I}$ and $\mathcal{E}$.

ABBREVIATIONS USED IN THE BIBLIOGRAPHY.

A.I.E.E.	Journal of the American Institute of Electrical Engineers.
A.E.C.S.	Transactions of the American Electro-chemical Society.
A. der Phys.	Annalen der Physik, Germany.
A. de Phys.	Annales de Physique, France.
Akad. Amst.	Proceedings of the Royal Academy, Amsterdam (Science Section).
A. f. E.	Archiv für Elektrotechnik, Germany.
A.E.G.	A.E.G. Mitteilungen, Germany.
A.W.W.	Akademie Wissenschaften Wien.
Acc. L. Atti.	Accademie Lincei Atti, Italy.
B.B.S.	Bulletin of the Bureau of Standards, Washington.
B.B.R.	Brown Boveri Review, Switzerland.
* B.E.A.M.A.	British Electrical and Allied Manufacturers' Association's Journal.
C.E.N.	Canadian Electrical News.
Chem. Abs.	Chemical Abstracts.
C.P.S.	Transactions of the Cambridge Philosophical Society.
C.R.	Comtes Rendus, France.
C.S.E.M.K.	College of Science and Engineering, Mem, Kyoto.
D.T.	Jahrbuch der Drahtlosen Telegraphie und Telephonie, Germany.
De Ing.	De Ingenieure, Holland.
Exp. W.	Experimental Wireless.
El. World.	Electrical World, America.
E.T.Z.	Elektrotechnische Zeitschrift, Germany.
E.E.J.	Institute of Electrical Engineers, Japan.
Elect.	Electrician.
Elletro.	Elletrotechnica, Italy.
El. Times.	Electrical Times.
† El. Rev. A.	Electrical Review, America.
El. Rev.	Electrical Review, England.
El. Jnl.	Electric Journal, America.
E.T.H.	Eidgenössischen Technischen Hochschule, Zurich.
E. u. M.	Elektrotechnik und Maschinenbau, Austria.
Eng.	The Engineer.
F.I.	Journal of the Franklin Institute, America.
F.S.T.	Transactions of the Faraday Society.

* Now called "World Power."
† Now called "Industrial Engineer."

G.E.R.	General Electric Review, America.
G.S.C.I.	Gazzetta Soc. Chim. Ital.
H. de. P.	Handbuch der Physik, Winkelmann.
Helios.	Helios, Germany.
I.E.E.	Journal of the Institution of Electrical Engineers.
I.P.O.E.E.	Post Office Electrical Engineers' Journal.
Ind. E.C.	Journal of Industrial and Engineering Chemistry, America.
Jnl. de Phys.	Journal de Physique, France.
J.S.I.	Journal of Scientific Instruments.
Lum. Elect.	Lumière Électrique, France.
Met. Vick.	Metropolitan Vickers Gazette.
Mod. W.	Modern Wireless.
M.C.E.	Chemical and Metallurgical Engineering, America.
O.S.J.A.	Journal of the Optical Society of America.
P.R.S.	Proceedings of the Royal Society.
Phil. Mag.	Philosophical Magazine.
Phot. J.	Photographic Journal.
P.P.S.	Proceedings of the London Physical Society.
Phys. Zeit.	Physicalische Zeitschrift, Germany.
Phys. Rev.	Physical Review, America.
P.A.W.B.	Preuss. Akademie Wissenschaften, Berlin.
Rad. El.	Radioélectricité, France.
Rad. Rev.	Radio Review, England.
Rad. Engrs.	Proceedings of the Institute of Radio Engineers, America.
Rad. News.	Radio News, America.
Rev. Gen.	Revue Générale de l'Électricité, France.
Rev. El.	Revue Électrique, France.
R.S.	Journal of the Rontgen Society, England.
R.S.D.	Proceedings of the Royal Society, Dublin.
S.F.B.	Bulletin de la Société Francaise des Électriciens, France.
S.E.V.	Schweiz Elektrotechnische Vereins, Switzerland.
T.I.E.C.	Transactions of the International Electrical Congress, America.
W.W.	Wireless World.
W.E.	Western Electrician, America.
Wireless Age.	Wireless Age.
Z. Anorg. Chem.	Zeitschrift Anorganische Chemie, Germany.
Z. f. Phys.	Zeitschrift für Physik, Germany.
Z. f. E.	Zeitschrift für Elektrochemie, Germany.
Z. f. T.P.	Zeitschrift für Technische Physik, Germany.
Z.P.C.	Zeitschrift für Physikalische Chemie, Germany.

Note.—Unless otherwise stated, a periodical may be assumed to be of English origin.

INTRODUCTION.

Uses of Rectified Alternating Current.—However much may be said regarding the advantages of direct current, there is little doubt that at the present time, from the point of view of power generation and transmission, alternating current has considerable advantages over direct current. This fact is emphasised by the growth of the super-power station at the expense of the smaller station, where direct current may be expected to hold its own; generation at 11,000 volts, and transmission at this or higher voltages in this country; and at 150 to 200 K.V. in America, points to the absolute necessity of alternating current as the only possible system for the present and in the near future. This is beyond doubt and scarcely needs argument. But when such is admitted there will still be a demand for some form of direct current; even in the super-power station it is usual to rely on direct current for the auxiliary or control circuits, and frequently it is necessary to install a battery for this purpose alone, so that in case of a failure of supply the protective gear will still be operative. In some of the larger substations batteries are also installed for similar purposes although occasionally pilot wires may be used for the supply of the auxiliary services on alternating current circuits.

Again, even if it is presumed that such rectifying apparatus is available which would render the auxiliary supply reasonably safe, yet the failure of the main supply would involve the auxiliaries also, and hence up to the present an accumulator would appear to be an essential part of the plant.

One can only imagine the complete disappearance of direct current as a possibility in power engineering and supply when

some form of alternating current storage is developed—a state of affairs which appears to be exceedingly remote, if not actually impossible.

Thus, in Electrical Engineering problems, direct current must be accepted as a necessity even though it may be considered by some a necessary evil; but there is no doubt, that unless new rectifying apparatus is forthcoming, the application of direct current to power purposes will become less and less noticeable as time goes on, and as all of the smaller power stations become submerged in those of much bigger dimensions.

During the past two or three years the rapid improvements which have been made in rectifiers generally, have shown that this point has been appreciated by those engaged in scientific development, and the warnings which from time to time have been issued from authoritative sources appear to have had their effect. To such an extent has progress been made that many supply authorities are now considering schemes of alternating current generation and transmission, with conversion locally to direct current for distribution. This would appear to be an ideal solution, because, for many purposes, notably where variable speed motors are required, direct has distinct advantages over alternating current. It is at this point that the rectifier, whether rotary or static, will come into its own, and if it had not been believed that this would finally prove to indicate the trend of development in this country, if not elsewhere, there would have been little use for this treatise in the first instance.

At the same time, in the laboratory, test shop, and in wireless work, direct current will in any event, still be required, and in increasing quantities. So far as Electric Traction is concerned, no opinion is expressed on the possible trend of development: the problem is one which has long been debated and no concurrence of opinion is apparent.

In the laboratory, the difficulty of making small voltage and current measurements with alternating current is an ever-present problem which appears to have no immediate solution unless costly and elaborate apparatus is available; whereas in the case of direct current, meters can be obtained at reasonable

prices, reading to a few milliamperes, and moreover of considerable sensitivity. In the laboratory such instruments as the thermo-ammeter, etc., can be used for alternating current measurements with a certain degree of safety, although there is no one who would not prefer to use those of less delicate manufacture; in the test room or workshop such an instrument is almost impracticable and means are sometimes employed to convert the current to be measured into a more desirable form. Dr. Clayton Sharpe, for instance, has designed a rectifier to suit this purpose, and there is no doubt that such a device would be a welcome addition to a laboratory equipment (Chapter XXIII).

The great increase in the sale of accumulators for wireless sets has resulted in an almost impossible position regarding charging facilities. No one cares for the trouble of sending a heavy accumulator to the nearest garage for charging purposes. with the added disadvantage of inexpert attention; the cost of dry cells as an alternative is prohibitive, unless the apparatus only requires the use of small receiving valves, and it therefore becomes necessary to install some form of rectifier, especially where the apparatus is used for wireless transmission and larger power outputs are required. In the latter case, if noiseless speech reproduction is the aim (and this is surely the sole object of the present-day amateur) smoothing devices are essential in the rectified circuit, otherwise the apparatus will be a nuisance to the neighbourhood. Such devices, although not an essential part of a rectifying plant are of fundamental importance, and stress has therefore been laid on the possible use of smoothing condensers and inductances in many places in subsequent chapters. If it were realised that a considerable improvement could easily be effected in this direction by comparatively simple apparatus there would be less trouble from noisy sets than there is at present.

Coming to the case of the high voltage requirements, apparently unidirectional current will always be required in wireless operations in some form or other, i.e. with an undulatoriness which is within certain prescribed limits. High voltage,

direct current generators are anathema on account of the necessity of insulated bedplates for series operation; and the net conclusion is, that some form of rectification is required. Numerous devices have been tried, none of which are perfect, though each has its own desirable features. Consider Table I. If a high voltage converter is required, under the heading of high power rectifiers there is nothing to fill the gap except the Transverter, and in the case of low power rectifiers it is found that one of the following classes of apparatus may be used :—

(1) Spark commutators,
(2) Thermionic tubes,
(3) Vibrating flames,
(4) Point to plate discharges,
or (5) Corona.

and none of these are suitable for high power outputs.

The subdivision into high and low power rectifiers is quite arbitrary, but it is apparent that where one requires a large, high voltage, direct current output of more than one or two kilowatts the only plant available is the Transverter. So far, the practice obtains of using numbers of thermionic rectifiers in parallel, but the difficulties attending this method are not to be despised, as anyone who has had control of even a moderately sized installation will appreciate. The development of the water-cooled thermionic rectifier is to a certain extent modifying this state of affairs; but so far although the power rectified is increasing, yet the supply voltage has not progressed at the same rate. Thus comparatively large powers of the order of 200 K.W. can be rectified, but the voltage limit appears at the present to be of the order of 30 K.V. for this class of rectification—the future may tell a different story.

In no branch of the electrical industry has development been so retarded as in that of rectification. The thermionic rectifier has only been developed as a miniature half-brother to the three electrode valve and is badly in need of assistance—its short life (a few thousand hours at the most), comparative uncertainty in action, and its inefficiency, combine to render it unsuitable at the present time for high power conversion.

The most promising line for future experimentation would appear to be the commutator driven by a synchronous motor, and although no one cares to propagate such an unscientific device as a commutator there seems to be no help for it. Fundamentally, there would appear to be no reason why a commutator could not be developed which would be practically sparkless in action, although the difficulties attending its design and manufacture are not to be despised. If a commutator can be assumed as a practicable proposition, the amount of power that can be converted for a small expenditure of energy in driving the motor is considerable; a motor of two or three B.H.P. being capable of supplying the necessary torque for the conversion of high power inputs.

It may be advanced that the mercury vapour rectifier is a solution of the problem, but so far the voltage bar has been difficult to surmount and one is faced, where voltages of fifty to one hundred kilovolts are concerned, with a possibility of rows and rows of rectifiers, each unit in series and insulated from its pump and from earth in the same way as the high voltage, series, direct current system.

A strong plea is therefore put forward for serious consideration of this subject of rectification, as a separate and distinct problem. The expert in rectification problems generally must be one of considerable experience in Physics and Chemistry as well as Electrical Engineering; and he must, of necessity, be familiar with high voltage phenomena, so that all phases of the problem may be investigated. There is no doubt that unidirectional current at voltages of one hundred or even two hundred-and-fifty kilovolts will be required in the near future, and so far there is no rectification plant whatever available which will cope with the demand which does not need the installation of expensive transformers; whether these requirements will include the rectification of large power inputs or not, it is difficult to say; but is it fantastic to assume that if rectifiers could be obtained which would cope with thousands of kilowatts at hundreds of kilovolts with the ease of a static transformer then such a supply would be a serious rival of the three phase,

high voltage distribution? Certainly many present evils would be eliminated and insulations would stand up where at present they break down for the same effective voltage, and that bugbear, "power factor" would be heard of no more.

The Problem.—This introduction to the study of rectification can best be employed by discussing the problem quite generally, leaving the consideration of detail to subsequent chapters.

Rectification, by definition, should entail the conversion of a current which fluctuates symmetrically about an axis of time into one which fluctuates in any fashion whatsoever unsymmetrically about the same axis. This may be accomplished in

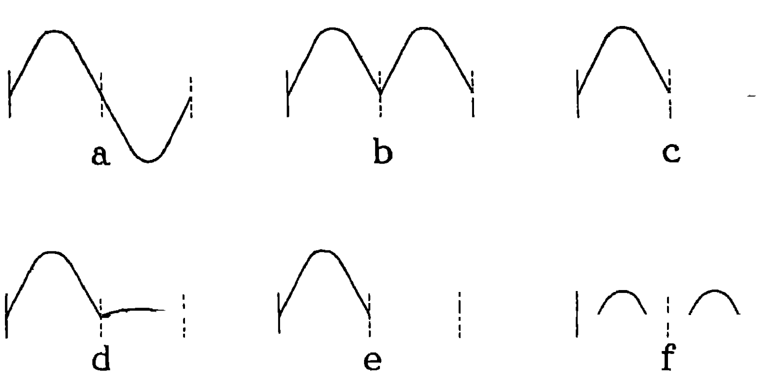

FIG. 1.—Typical wave forms of rectified current.

several ways; in Fig. 1 the alternating current wave '*a*' will be rectified in the broadest sense if it is converted into any one of the wave forms '*b*,' '*c*,' '*d*,' '*e*,' or '*f*.' The term perfect rectification is applied to such wave forms as '*b*,' '*d*,' '*e*,' and '*f*,' where the whole of the current is positive in direction; and imperfect rectification to the case of '*c*,' where a slight negative wave is present at the same time; the converse would result equally in a rectified current, viz. if the larger portion were negative and the smaller positive, although in practice this form of rectified current is rarely encountered.

Suitable apparatus can be designed to produce any of these

results. This, however, is the effect of rectification—to study more closely the cause of the change it is convenient to consider the matter from the analytical standpoint.

Assume that a piece of apparatus exists which does not obey Ohm's Law in so far as the relation of the current through the apparatus and the voltage drop is concerned, then quite generally, the following relation connecting these two quantities exists:—

$$i = \phi(e),$$

where i is the current and e the voltage, the assumption being that $\phi(e)$ is not linear. Further, assume that a small change in i, viz. δi, is accompanied by a change δe in the voltage, then

$$(i + \delta i) = \phi(e + \delta e).$$

According to Taylor's Theorem this expression may be expanded into the series

$$(i + \delta i) = \phi(e) + \delta e\phi'(e) + \frac{\delta e^2}{2!}\phi''(e) + \ldots$$

where $\phi'(e) = \dfrac{d}{de}\{\phi(e)\}$ and $\phi''(e) = \dfrac{d^2}{de^2}\{\phi(e)\}$, etc.

From this equation, substituting for i,

$$\delta i = \delta e\phi'(e) + \frac{\delta e^2}{2!}\phi''(e) + \ldots$$

Now if this small change δe consists of an alternating voltage which is impressed on the circuit, $\delta e \sin\theta$ may be substituted for δe, and

$$\delta i = \delta e \sin\theta\phi'(e) + \frac{(\delta e \sin\theta)^2}{2!}\phi''(e) + \ldots$$

which may be written in the form

$$\delta i = \frac{\delta e^2}{4}\phi''(e) + \ldots + \delta e \sin\theta\phi'(e) - \frac{\delta e^2 \cos 2\theta}{4}\phi''(e) + \ldots \quad (1)$$

If the function $\phi(e)$ is not linear in form, it will be seen that the device will allow a certain current to pass dependent on the nature of the function and its first and following derivatives, but if the relation does happen to be linear the series vanishes and i becomes

$$\phi'(e)\delta e \sin\theta = \text{constant} \times \delta e \sin\theta.$$

Hence rectification depends for its action on a volt-ampere

TABLE I.—HIGH POWER RECTIFIERS APPROXIMATELY 1/1 VOLTAGE TRANSFORMATION.

Low Voltage 0-50 volts D.C.	Medium Voltage 50-500 volts D.C.	High Voltage 500-3000 volts D.C	Extra High Input Voltage 3000-50,000 volts A.C.	Extra High Output Voltage 3000-50,000 D.C. and over.
Rotating Commutator. Rotary Converter.	Rotating Commutator. Rotary Converter. Mercury Vapour. Motor Converter.	Rotating Commutator. Mercury Vapour. Motor Converter.	Transverter.	Transverter.
HIGH POWER RECTIFIERS STEPDOWN VOLTAGE TRANSFORMATION.				
Rotating Commutator. Rotary Converter (with transformer).	Rotating Commutator. Rotary Converter (with transformer). Motor Converter.	Rotating Commutator. Rotary Converter (with transformer). Motor Converter. Mercury Vapour (with transformer).	Rotating Commutator (with transformer). Rotary Converter (with transformer). Motor Converter (with transformer). Mercury Vapour (with transformer).	Transverter.
LOW POWER RECTIFIERS.				
Rotating Commutator. Gas filled Tube. Vibrating Reed. Electrolytic. Solid Contact Rectifier.	Rotating Commutator. Gas Discharge Tube (180-500 volts). Thermionic Tube.	Rotating Commutator. Thermionic Tube.	Thermionic Tube. Lodge Rectifier.	Spark Commutator. Vibrating Flame. Thermionic Tube. Point to plate Discharge. Corona.
WIRELESS DETECTORS.				
3 electrode valve. Crystal.	3 electrode valve.	3 electrode valve.	3 electrode valve.	

characteristic which is not linear. (See footnote and also page 15.)

Fig. 2 illustrates this point graphically, where $I = \phi(E)$ is the volt-ampere characteristic of the rectifying apparatus; if PM represents the current at a given voltage OM, and an alternating E.M.F. is applied with an amplitude equal to MM' ($= MM''$), then the increase in current due to MM' is $P'T'$ whereas the reduction due to MM'' is $P''T''$; and as $P'T'$ is greater than $P''T''$ a net unidirectional current will flow through the apparatus equal to $\{\Sigma P'T' - \Sigma P''T''\}$. It will further be apparent that this value of the net current will be a maximum with a curve of the shape and form indicated in the figure, when the point P occurs at the position of maximum rate of change of slope; and also from equation (1) that if the higher derivatives are neglected the current is proportional to the square of the impressed alternating voltage.

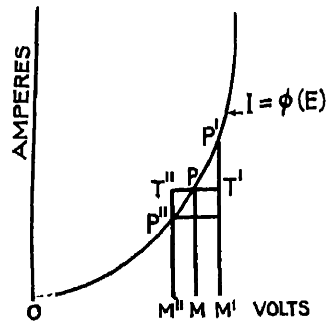

FIG. 2.—Volt-ampere characteristic of a rectifier.

Classification.—Rectifiers can be classified in various ways according to the duty they are required to perform, and Table I. will serve as a guide to the type of rectifier suited to any particular duty.

This table must not be rigidly applied, and the reader may be able to enlarge its scope considerably, but it is included so as to give a general idea of the nature of the apparatus available.

Note.—This is a generalised statement, and does not apply in each particular case; for instance, mechanical rectifiers and electrolytic cells do not depend altogether for their rectifying properties on their volt-ampere characteristic; and they therefore cannot be considered as coming within the purview of this example.

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PART I.

FOURIER'S SERIES, HARMONIC ANALYSIS, AND WAVE FORM.

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CHAPTER I.

FOURIER'S SERIES.

General Considerations.—In a perfect system of unidirectional or continuous current the wave form would consist of a straight line parallel to the axis of time; this ideal, however, is rarely realised in practice, the nearest approach being that of a homopolar form of direct current generator. In all direct current machinery employing a commutator the current consists of a ripple, often of high frequency, superimposed on the ideal horizontal straight line. This undulatoriness is due to the division of the winding and to imperfect commutation, but by suitably choosing the number of segments, the amplitude of such oscillations can be reduced to a minimum; but even with perfect commutation a smooth wave would not be obtained, the spacing factor of the winding presenting inherent difficulties. A more or less undulating wave is encountered in most cases of rectified circuits supplied from mercury vapour rectifiers, thermionic tubes and the like, with the difference that in the majority of rectifiers the periodicity of the oscillation is a multiple of the supply periodicity, and to a certain extent the undulatoriness can be controlled within well-defined limits.

It is important to be able to analyse the particular wave form generated or rectified, and although Part I. will not deal in detail with all of the various methods available, it will supply sufficient information to enable the reader to analyse most of the wave forms met with in practice.

It is not generally realised that wave form has a distinct effect on the indications of various types of instruments (page 74). One is familiar, for instance, with the fact that a moving iron

or dynamometer voltmeter or ammeter can be used on direct current circuits, but it is not generally appreciated that the readings of such instruments are identical with those of moving coil instruments only because the commutator ripple is of high frequency and low amplitude; yet on some rectified circuits this difference is noticeable and may lead to incorrect results if an instrument suited to the requirements is not employed.

There is another point on which little information is at present available. It is shown on page 22 that the general wave form of certain rectified circuits takes a certain shape which consists of a Fourier's series with a constant term and the cosines of even harmonics. If the maximum amplitude of the wave form is considered to be unity, the percentage contribution of the constant term and the harmonics is found to have the values given in Table II.

TABLE II.

Number of Phases.	Per Cent. Constant Term in Maximum Amplitude.	Per Cent. of Value of Amplitude of Harmonics in Maximum Amplitude.								
		1.	2.	3.	4.	6	8.	9.	10.	12.
1	31·8	50	21·2	—	4·25	1·8	1·0	—	0·65	0·15
2	63·6	—	42 4	—	8·50	3·60	2·0	—	1·30	0·90
3	82 5	—	—	20·7	—	4·75	—	2·1	-	1·16
4	90·4	—	—	—	12	—	2·85	—	—	1·26
6	95·5	—	—	—	—	5·5	—	—	-	1·35
12	98·9	—	—	—	—	—	—	—	—	1·40
18	99·5	—	—	—	—	—	—	—	—	—

These values are calculated from the amplitudes given on page 23, but it should be noted in passing that it is not reasonable to expect the percentages given to sum numerically to 100 per cent. seeing that in certain cases the harmonics oppose each other at the point of maximum amplitude; but the table has been included to show that on rectified circuits supplied, for instance, from a 50 cycle supply, a very considerable proportion of the amplitude may be contributed by currents of frequencies of over a 100 cycles per second. For example, in the case of the tri-phase circuit 20 per cent. of the amplitude is contributed

by a 150 cycle harmonic and thus unless the measuring instruments employed are free from frequency error, inaccurate deductions may be made from the indications of such instruments. This is well known in the case of moving iron meters, and to a lesser degree in dynamometer instruments, but there is little information as to the behaviour of moving coil meters on high frequency supplies, and it is believed that up to the present the point has not arisen.

If accurate measurements are to be made recourse must be had either to the silver or copper voltameter, or better still the electrometer.

Effect of Harmonics.—Before proceeding to discuss methods of analysis it is well for the reader to understand

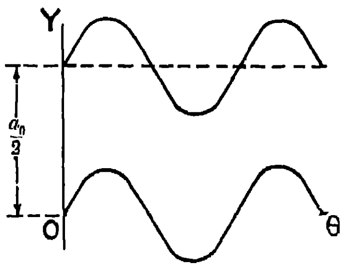

FIG. 3.—Effect of constant term on wave form.

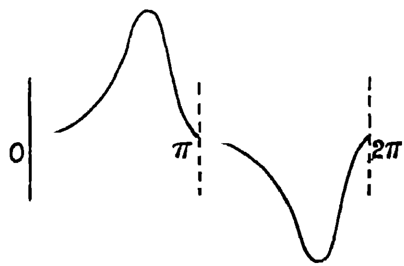

FIG. 4.—Wave form with fundamental and odd harmonics.

quite clearly the effect of various harmonics on the wave form. For instance, the inclusion of a constant term in the expression for the wave form ensures that the whole curve is moved up from the axis of θ and parallel to it, as shown in Fig. 3, and further the value of $\frac{a_0}{2}$, the value of the constant term, is seen to be the mean value of the curve between the limits of one complete cycle, and thus is a measure of the area under the curve; this is the case whatever the shape of the curve, provided that it is periodic and fulfils the requirements of Fourier's Theorem. It immediately follows as a corollary that the condition of rectification is the inclusion in the expression for the wave form of a constant term, the mean value of the rectified current equalling that term.

In most forms of alternating current work, the current or voltage wave consists of a fundamental and odd harmonics only, such as is depicted in Fig. 4, where it is seen that the loops are symmetrical about the vertical line through the point of half-cycle. If even harmonics are present the loops are not symmetrical about this line; and this is the case which more usually obtains in rectified wave forms (see page 11).

Fourier's Series.—The basis of the majority of the methods used in harmonic analysis is Fourier's Theorem which in effect states that a function $\phi(\theta)$ which is of a periodic nature may under certain conditions be represented by a series consisting of the sum of a given number of terms (in some cases any number up to infinity) of a fundamental sine and cosine wave, with the sine and cosine terms of multiple angles.

Analytically this may be expressed as follows:—

$$y = \phi(\theta) = \frac{a_0}{2} + a_1 \cos \frac{2\pi\theta}{\lambda} + a_2 \cos \frac{4\pi\theta}{\lambda} + \ldots$$
$$+ b_1 \sin \frac{2\pi\theta}{\lambda} + b_2 \sin \frac{4\pi\theta}{\lambda} + \ldots$$

where $\frac{2\pi}{\lambda}$ is the time of a complete oscillation. $\frac{a_0}{2}$ is employed for the constant term for the reason that when n is put equal to zero in the generalised expression for a_n, the constant term results and thus avoids confusion. When, however, the infinite series is employed to represent an electrical wave form, the expression

$$I_0 + I'_1 \sin \theta + I'_2 \sin 2\theta + \ldots + I'_n \sin n\theta + \ldots$$
$$+ I''_1 \cos \theta + I''_2 \cos 2\theta + \ldots + I''_n \cos n\theta + \ldots$$

is used, as being more convenient.

In the former expression, which is quite general in its application, provided that $\phi(\theta)$ is finite and continuous, and if discontinuous has only finite discontinuities, $\frac{a_0}{2}$ is a constant term, and a_1, b_1, etc., to a_n, b_n are the maximum values or amplitudes of the fundamental and the various harmonics, which it is required to ascertain.

First integrate both sides of the equation between the limits of 0 and λ whence

$$\frac{a_0}{2} = \frac{1}{\lambda}\int_0^\lambda \phi(\theta)d\theta,$$

since all the other terms vanish in accordance with the following condition :—

$$\int \cos \frac{2\pi n\theta}{\lambda} d\theta = \int_0^\lambda \sin \frac{2\pi n\theta}{\lambda} d\theta = 0.$$

It will be sufficient if the values of a_n and b_n are ascertained as the other terms can as a rule be obtained by putting $n = 1$, 2, 3, etc., in succession; and hence if both sides of the equation are multiplied by $\cos \frac{2\pi n\theta}{\lambda}$ and $\sin \frac{2\pi n\theta}{\lambda}$ respectively, and integrated between the same limits as above,

$$a_n = \frac{2}{\lambda}\int_0^\lambda \phi(\theta) \cos \frac{2\pi n\theta}{\lambda} d\theta$$

and

$$b_n = \frac{2}{\lambda}\int_0^\lambda \phi(\theta) \sin \frac{2\pi n\theta}{\lambda} d\theta,$$

because definite integrals of the form

$$\int_0^\lambda \cos p\frac{2\pi\theta}{\lambda} \sin q \frac{2\pi\theta}{\lambda} d\theta$$

equate to zero, if p and q are integers of differing numerical values.

This method can therefore be used to determine each harmonic irrespectively of its neighbours, provided that the quantities

$$\phi(\theta) \cos \frac{2\pi n\theta}{\lambda} \text{ and } \phi(\theta) \sin \frac{2\pi n\theta}{\lambda}$$

are known and their integrals can be evaluated, but a large number of products are involved in such an analysis, and unless some methods of simplification are employed the complete calculations are laborious.

Analysis using Fourier's Series.—The first application of Fourier's analysis will be to the determination of the amplitudes of the various harmonics in a single-phase wave form, as shown in Fig. 5.

If

$$y = \frac{a_0}{2} + a_1 \cos\theta + a_2 \cos 2\theta + \ldots + a_n \cos n\theta + \ldots$$
$$+ b_1 \sin\theta + b_2 \sin 2\theta + \ldots + b_n \sin n\theta + \ldots \quad (1)$$

it has been stated that

$$\frac{a_0}{2} = \frac{1}{2\pi}\int_0^{2\pi} y d\theta \text{ which in this case } = \frac{1}{2\pi}\int_0^{\pi} \sin\theta d\theta$$

and represents the mean ordinate of the curve over a whole period.

Hence $$\frac{a_0}{2} = \frac{1}{\pi}.$$

Also $$a_n = \frac{1}{\pi}\int_0^{2\pi} y \cos n\theta d\theta$$

and $$b_n = \frac{1}{\pi}\int_0^{2\pi} y \sin n\theta d\theta,$$

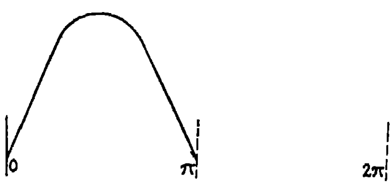

FIG. 5.—Single-phase wave form.

but in this particular case for the period 0 to π, $y = \sin\theta$ and from π to 2π, $y = 0$, and hence

$$a_n = \frac{1}{\pi}\int_0^{\pi} \sin\theta \cos n\theta d\theta$$

and $$b_n = \frac{1}{\pi}\int_0^{\pi} \sin\theta \sin n\theta d\theta.$$

Performing the integration the following expressions are arrived at:—

$$a_n = \frac{1 + \cos\pi n}{\pi(1 - n^2)}$$

$$b_n = \frac{\sin\pi n}{\pi(1 - n^2)}.$$

It is now only necessary to give n the values 1, 2 and 3, etc., to arrive at the amplitudes of the various harmonics, but it is important to note that

$$b_1 = \frac{\sin \pi}{\pi(1 - 1)} = \frac{0}{0},$$

so by applying the usual procedure and differentiating both numerator and denominator the correct value of b_1 is found to be $\frac{1}{2}$.

The final expression for this wave form is therefore

$$y = \frac{1}{\pi} + \tfrac{1}{2} \sin \theta - \frac{2}{1.3.\pi} \cos 2\theta - \frac{2}{3.5.\pi} \cos 4\theta - \ldots \quad (2)$$

$$= 0{\cdot}32 + 0{\cdot}5 \sin \theta - 0{\cdot}22 \cos 2\theta - 0{\cdot}042 \cos 4\theta - \ldots$$

To obtain the expression for the biphase wave (Fig. 6)

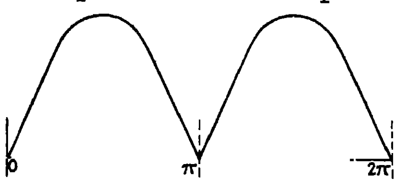

Fig. 6.—Biphase wave form.

it is necessary to replace $(\pi + \theta)$ for θ in the above equation (2), whence

$$y = \frac{1}{\pi} - \tfrac{1}{2} \sin \theta - \frac{2}{\pi}\sum_{2}^{\infty} \frac{\cos^2 \frac{n\pi}{2} \cos n\theta}{n^2 - 1} \quad \ldots \quad (3)$$

and by adding equations (2) and (3)

$$y = \frac{2}{\pi}\left[1 - 2\sum_{2}^{\infty} \frac{\cos^2 \frac{n\pi}{2} \cos n\theta}{n^2 - 1}\right] \quad \ldots \quad (4)$$

is obtained for the equation of a biphase wave. Thus it will be apparent that in this wave formation the fundamental is absent and only even harmonics are present.

Polyphase Wave Form.—It is interesting to investigate the general case of a multiphase system, and in Fig. 7 circuit

conditions are given for a hexaphase supply, the method being of general application. The wave form of such a supply is shown in Fig. 8, and if there are m phases there will be m complete loops in one cycle or 2π electrical degrees, the crest of the loop strictly following the sinusoidal curvature. The equation to the generating curve A is in this case

$$y = \cos \theta.$$

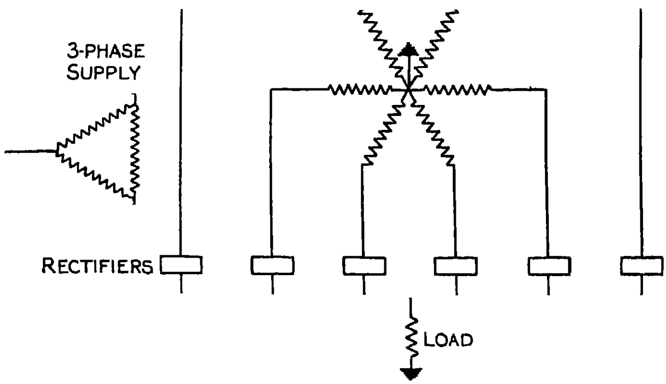

FIG. 7.—Hexaphase circuit.

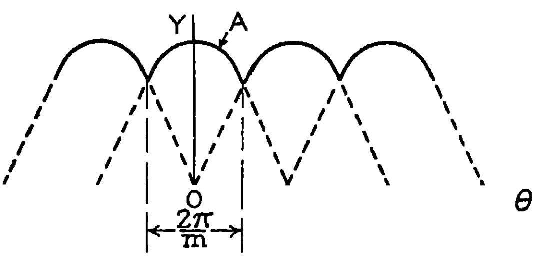

FIG. 8.—Polyphase wave form.

It is shown in treatises on Fourier's Series that the amplitudes of the nth harmonics between limits other than $\pm\,\pi$ can be obtained as follows:—

Assume the series to be

$$f(\theta) = \tfrac{1}{2}a_0 + a_1 \cos(\pi\theta/c) + a_2 \cos(2\pi\theta/c) + \ldots \infty$$
$$+ \, b_1 \sin(\pi\theta/c) + b_2 \sin(2\pi\theta/c) + \ldots \infty$$

Then

$$a_n = \frac{2}{c}\int_0^c f(\theta)\cos\left(\frac{n\pi\theta}{c}\right)d\theta$$

and

$$b_n = \frac{2}{c}\int_0^c f(\theta)\sin\left(\frac{n\pi\theta}{c}\right)d\theta$$

or

$$a_n = \frac{1}{c}\int_{-c}^{+c} f(\theta)\cos\left(\frac{n\pi\theta}{c}\right)d\theta$$

and

$$b_n = \frac{1}{c}\int_{-c}^{+c} f(\theta)\sin\left(\frac{n\pi\theta}{c}\right)d\theta.$$

In this case $c = \pi/m$ and $n\pi\theta/c = nm\theta$, whence

$$\frac{\pi a_n}{m} = \int_{-\frac{\pi}{m}}^{+\frac{\pi}{m}} \cos\theta\cos(nm\theta)\,d\theta$$

and

$$a_n = \frac{2m\sin\dfrac{\pi}{m}\cos n\pi}{\pi(1 - n^2m^2)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

Further

$$\frac{\pi b_n}{m} = \int_{-\frac{\pi}{m}}^{+\frac{\pi}{m}} \cos\theta\sin(nm\theta)\,d\theta = 0.$$

The general equation of the wave form is therefore

$$\frac{m}{\pi}\sin\frac{\pi}{m} - \frac{2m\sin\dfrac{\pi}{m}\cos m\theta}{\pi(1 - m^2)} + \frac{2m\sin\dfrac{\pi}{m}\cos 2m\theta}{\pi(1 - 2^2m^2)} + \ldots\infty \quad (6)$$

$$= \frac{m}{\pi}\sin\frac{\pi}{m}\left[1 + 2\sum_{n=1}^{\infty}\frac{\cos n\pi\cos nm\theta}{1 - n^2m^2}\right] \quad . \; . \; . \; . \; . \quad (7)$$

and all the sine terms are absent (this statement requires modification where $m = 1$, the single phase wave form considered above).

If it is desired to reproduce the wave form to a different set

of co-ordinates, as shown in Fig. 9, where the discontinuity in the curve occurs in the vertical axis, it is necessary to substitute $(\theta - \pi/m)$ for θ in equation (7) which then becomes

$$\frac{m}{\pi}\sin\frac{\pi}{m}\left[1 - \frac{2\cos m\theta}{m^2 - 1} - \frac{2\cos 2m\theta}{4m^2 - 1} - \ldots \infty\right] \quad (8)$$

$$= \frac{m}{\pi}\sin\frac{\pi}{m}\left[1 + 2\sum_{n=1}^{\infty}\frac{\cos nm\theta}{1 - n^2m^2}\right] \ldots\ldots\ldots \quad (9)$$

in which form it is more convenient. In future, and in the table below, this equation will be used.

It is interesting to apply a check to this result, and to this end an infinite series of the form

$$\frac{m}{\pi}\operatorname{cosec}\frac{\pi}{m} = 1 + 2\sum_{1}^{\infty}\frac{(-1)^n}{1 - n^2m^2}$$

is employed.

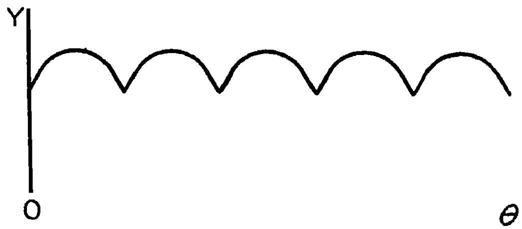

Fig. 9.—Polyphase wave form.

If the mid-point of the oscillations is considered, i.e. the point represented by $\theta = \pi/m$, the amplitudes of the harmonics all have a maximum value, and hence equation (9) equals unity, which is the peak value of the wave.

If m is given the values 2, 3, or 4, etc., to represent the numbers of the phases or loops per cycle, the following wave forms are obtained :—

$m = 2$ (Biphase) Fig. 10.

$$\frac{2}{\pi} - \frac{4\cos 2\theta}{\pi(1\,.\,3)} - \frac{4\cos 4\theta}{\pi(3\,.\,5)} - \ldots \infty$$

$m = 3$ (Triphase) Fig. 11.

$$\frac{3\sqrt{3}}{2\pi} - \frac{3\sqrt{3}\cos 3\theta}{\pi(2\,.\,4)} - \frac{3\sqrt{3}\cos 6\theta}{\pi(5\,.\,7)} - \ldots \infty$$

$m = 4$ (Quarter phase) Fig. 12.

$$\frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}\cos 4\theta}{\pi(3\,.\,5)} - \frac{4\sqrt{2}\cos 8\theta}{\pi(7\,.\,9)} - \ldots \infty$$

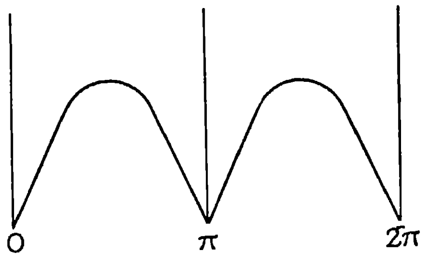

FIG. 10.

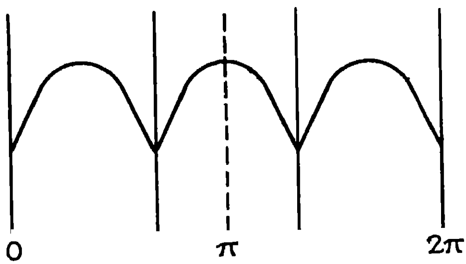

FIG. 11.

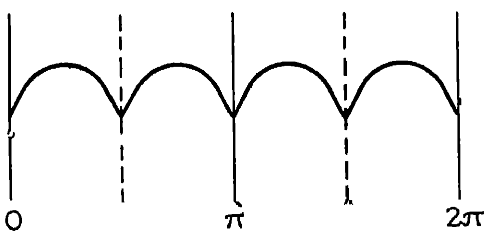

FIG. 12.

$m = 6$ (Hexaphase) Fig. 13.

$$\frac{3}{\pi} - \frac{6\cos 6\theta}{\pi(5\,.\,7)} - \frac{6\cos 12\theta}{\pi(11\,.\,13)} - \ldots \infty$$

$m = 18$ (Eighteen phase).

$$0{\cdot}999\left[1 - \frac{2\cos 18\theta}{17.19} - \frac{2\cos 36\theta}{35.37} - \ldots \infty\right].$$

With regard to the mean square value of the wave this will be equal to

$$\mathscr{I}^2 = \frac{m^2}{\pi^2}\sin^2\frac{\pi}{m}\left[1 + 2\sum_1^\infty \frac{1}{(1 - n^2m^2)^2}\right]$$

(see page 71).

To sum the series

$$\sum_1^\infty \frac{1}{(1 - n^2m^2)^2}$$

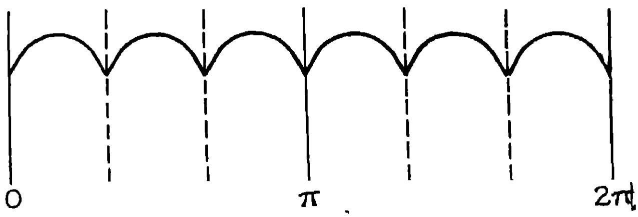

FIG. 18.

two infinite series are employed, viz.,

$$\pi \cot \pi\theta = \frac{1}{\theta} + \sum_1^\infty \frac{2\theta}{\theta^2 - n^2}$$

and

$$\pi^2 \operatorname{cosec}^2 \pi\theta = \frac{1}{\theta^2} + 2\sum_1^\infty \frac{\theta^2 + n^2}{(\theta^2 - n^2)^2}.$$

Whence putting $\theta = 1/m$

$$\sum_1^\infty \frac{1}{(1 - n^2m^2)^2} = \left(\frac{\pi}{2m}\operatorname{cosec}\frac{\pi}{m}\right)^2 + \frac{\pi}{4m}\cot\frac{\pi}{m} - \frac{1}{2}$$

and therefore

$$\mathscr{I}^2 = \frac{1}{2} + \frac{m}{4\pi}\sin\frac{2\pi}{m}.$$

This result could also have been obtained directly by evaluating

$$\frac{m}{2\pi}\int_{\frac{\pi}{2}-\frac{\pi}{m}}^{\frac{\pi}{2}+\frac{\pi}{m}} \sin^2\theta\, d\theta.$$

The actual wave forms to scale are indicated in Fig. 14, and show how the form factor is improved by increasing the number of phases.

From the above analysis, Table III can be prepared.

Special Wave Forms.—In some forms of rectifier, such, for instance, as the vibrating reed or neon tube, the voltage rises to

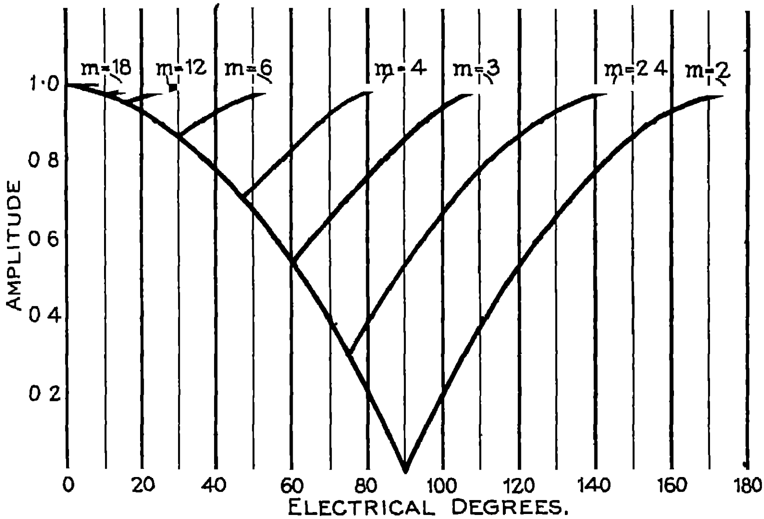

FIG. 14.—Polyphase wave forms.

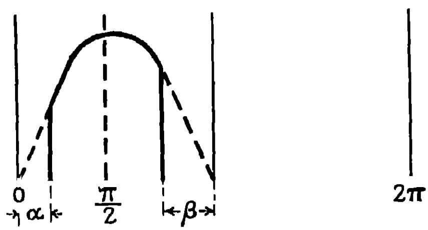

FIG. 15.—Truncated, single-phase wave form.

a certain value before the current flows, and the current is cut off before the voltage has fallen to zero. Such an oscillation will take the form of Fig. 15, where the form of the generating curve is

$$y = \sin \theta.$$

TABLE III.—POLYPHASE WAVE-FORM DATA.

No. of Phases.	R.M.S Value $\mathcal{I}$	Mean Value I_M.	Form Factor.	Amplitude of nth Harmonic.	Maximum Value ÷ R.M.S.	Maximum Value – Mean.	Minimum Ordinate.	Area under one Loop.
m	$\sqrt{\left(\frac{1}{2}+\frac{m}{4\pi}\sin\frac{2\pi}{m}\right)}$	$\frac{m}{\pi}\sin\frac{\pi}{m}$	$\frac{\mathcal{I}}{I_M}$	$\frac{2m\sin(\pi/m)}{\pi(1-n^2m^2)}$	$\frac{1}{\mathcal{I}}$	$\frac{\pi}{m}\operatorname{cosec}\frac{\pi}{m}$	$\cos\frac{\pi}{m}$	$\int_{\frac{1}{2}\pi-\pi/m}^{\frac{1}{2}\pi+\pi/m}\sin\theta d\theta$
1	$\frac{1}{2} = 0·500$	$1/\pi = 0·318$	$\frac{1}{2}\pi = 1·570$	$\frac{2}{\pi(1-4n^2)}$	2	$\pi = 3·141$	0	2
2	$1/\sqrt{2} = 0·707$	$2/\pi = 0·636$	$\frac{\pi}{2\sqrt{2}} = 1·110$	$\frac{4}{\pi(1-4n^2)}$	$\sqrt{2} = 1·414$	$\frac{1}{2}\pi = 1·570$	0	2
3	0·840	0·825	1·02	$\frac{3\sqrt{3}}{\pi(1-9n^2)}$	1·19	1·210	0·500	0·866
4	0·905	0·904	1·005	$\frac{8}{\pi\sqrt{2}(1-16n^2)}$	1·105	1·105	0·707	0·717
6	0·956	0·955	1·005	$\frac{6}{\pi(1-36n^2)}$	1·045	1·045	0·866	0·500
12	0·989	0·989	1·00	$\frac{24\sin 15°}{\pi(1-144n^2)}$	1·01	1·01	0·966	0·342
18	0·995	0·995	1·00	$\frac{36\sin 10°}{\pi(1-324n^2)}$	1·005	1·005	0·985	0·174

From Fourier's Series the amplitudes of the nth harmonics are

$$a_n = \frac{1}{\pi}\int_a^{\pi-\beta} \sin\theta \cos n\theta d\theta,$$

and

$$b_n = \frac{1}{\pi}\int_a^{\pi-\beta} \sin\theta \sin n\theta d\theta,$$

whence

$$a_n = \frac{\cos n\pi\{\cos\beta\cos n\beta + n\sin\beta\sin n\beta\} + \cos a\cos na + n\sin a\sin na}{\pi(1-n^2)}$$

and

$$b_n = \frac{\cos n\pi\{n\sin\beta\cos n\beta - \cos\beta\sin n\beta\} + \cos a\sin na - n\sin a\cos na}{\pi(1-n^2)}$$

from which

$$\tfrac{1}{2}a_0 = \frac{\cos a + \cos\beta}{2\pi}.$$

Also, by differentiating the numerator and denominator of a_n and b_n and putting $n = 1$

$$a_1 = \frac{\sin^2\beta - \sin^2 a}{2\pi}$$

and

$$b_1 = \frac{\sin a\cos a + \sin\beta\cos\beta - a - \beta + \pi}{2\pi}.$$

The complete expression for the wave form then becomes

$$y = \frac{\cos a + \cos\beta}{2\pi} + \frac{\sin a\cos a + \sin\beta\cos\beta - a - \beta + \pi}{2\pi}\sin\theta$$

$$+ \frac{\sin^2\beta - \sin^2 a}{2\pi}\cos\theta$$

$$- \sum_2^\infty \frac{\cos n\pi\{\cos\beta\cos n\beta + n\sin\beta\sin n\beta\} + \cos a\cos na + n\sin a\cos na}{\pi(n^2-1)} \times \cos n\theta$$

$$- \sum_2^\infty \frac{\cos n\pi\{n\sin\beta\cos n\beta - \cos\beta\sin n\beta\} + \cos a\sin na - n\sin a\cos na}{\pi(n^2-1)} \times \sin n\theta \quad . \qquad (10)$$

The biphase wave form, as shown in Fig. 16, is obtained by replacing $(\pi+\theta)$ for θ in equation (10), and adding the result so obtained to equation (10). Thus

$$y = \frac{\cos a + \cos \beta}{2\pi}$$
$$-\sum_{2}^{\infty} \frac{\cos\beta \cos n\beta + n \sin \beta \sin n\beta + \cos a \cos na + n \sin a \sin na}{\pi(n^2 - 1)} \times (1+\cos n\pi)\cos n\theta$$
$$-\sum_{2}^{\infty} \frac{n \sin \beta \cos n\beta - \cos \beta \sin n\beta + \cos a \sin na - n \sin a \cos na}{\pi(n^2-1)} \times (1+\cos n\pi)\sin n\theta \quad . \; . \; . \quad (11)$$

If $a = \beta$, i.e. the point of cut-off is equal to the point of cut-

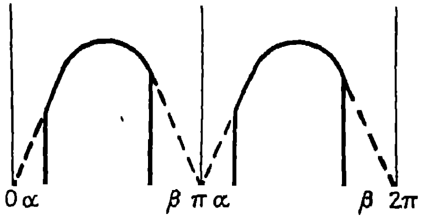

FIG. 16.—Truncated, biphase, wave form.

in, or the current wave is symmetrical, the following relationships obtain:—

Single Phase (Fig. 15).

$$y = \frac{\cos a}{\pi} + \frac{\sin 2a - 2a + \pi}{2\pi} \sin \theta$$
$$-\sum_{2}^{\infty} \frac{\cos a \cos na + n \sin a \sin na}{\pi(n^2 - 1)} (1 + \cos n\pi) \cos n\theta$$
$$-\sum_{2}^{\infty} \frac{n \sin a \cos na - \cos a \sin na}{\pi(n^2 - 1)} (\cos n\pi - 1) \sin n\theta \; . \quad (12)$$

and the fundamental cosine term disappears.

Biphase (Fig. 16).

$$y = \frac{2 \cos a}{\pi} - 2\sum_{2}^{\infty} \frac{\cos a \cos na + n \sin a \sin na}{\pi(n^2 - 1)} (1+\cos n\pi) \cos n\theta \quad (13)$$

and both fundamental terms vanish as well as all of the sine terms.

In arriving at these functions a knowledge of the summation of certain series is manifestly important, and some of the more useful ones are given at the end of this chapter.

In all forms of periodic function, the fundamental analysis will give an accurate value for the Fourier coefficients, but it is often impracticable to apply it, and other methods have to be employed. Some of these alternatives are given in Chapters II and III but the Fourier analysis has the advantage that the phase difference between the various harmonics can be taken into consideration at the same time.

Modulator Circuit.—One of the special wave forms of interest in wireless circuits is that of the distortion due to modulation; or the distortion due to the transformation of mechanical into electrical energy, as for example in the case of a telephone diaphragm.

In this instance the modulator has a steady resistance R, and on this resistance is impressed an alternating resistance which is here assumed to have the value $r \cos \theta$ due to the mechanical vibration; then the characteristic equation is

$$i = \frac{e}{R + r \cos \theta}.$$

Let $\frac{r}{R} = k$, and as r is always less than R, k is less than unity, thus

$$i = \frac{e}{R(1 + k \cos \theta)} \text{ or } y = \frac{iR}{e} = \frac{1}{1 + k \cos \theta}.$$

The Fourier coefficients are then

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{\cos n\theta d\theta}{1 + k \cos \theta}$$

and

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{\sin n\theta d\theta}{1 + k \cos \theta}$$

but the function is obviously one of the Class IV type (see page 41), and therefore

Performing the integration for a_n it is found that

$$a_n = \frac{2(-1)^n}{\sqrt{1-k^2}}\left\{\frac{1-\sqrt{1-k^2}}{k}\right\}^n$$

and

$$\frac{a_0}{2} = \frac{1}{\sqrt{1-k^2}}.$$

The series is therefore

$$\frac{iR}{e} = \frac{1}{\sqrt{1-k^2}} + 2\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{1-k^2}}\left\{\frac{1-\sqrt{1-k^2}}{k}\right\}^n \cos n\theta$$

and indicates the presence of pronounced harmonics.

This particular modulated function is of a simple type, and will not often be met in practice, but the example has been included to show the relatively large percentage amplitudes that may occur with various values of k.

Analysis of Various Wave Forms.—At this point it may be of interest to consider the analyses and equations of some of the types of wave form met with in rectifier circuits. Curves have been chosen which lend themselves to expression by a trigonometrical series as these types indicate more readily the way in which the problems can be treated.

(1) *Triangular Wave. General Case.*

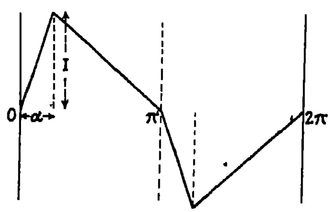

FIG. 17*a*.

The ordinary triangular wave form of the Class IV. type can be expressed generally when the peak is situated a degrees from the origin by

$$i = \frac{I}{a\pi(\pi - a)}\sum_{1}^{\infty}\frac{1-\cos n\pi}{n^2}\{\pi\cos na - \pi + 2a\}\cos n\theta$$

$$+ \frac{I}{a(\pi - a)}\sum_{1}^{\infty}\frac{1-\cos n\pi}{n^2}\sin na \sin n\theta$$

which reduces to the expression below when $a = \pi/2$ (Fig. 18).

If the equation is expanded it can be written

$$\frac{2I}{a(\pi - a)}\left[\cos(a-\theta) + \frac{1}{3^2}\cos(3a-3\theta) + \frac{1}{5^2}\cos(5a-5\theta) + \ldots\infty\right]$$
$$- \frac{2(\pi - 2a)I}{a\pi(\pi - a)}\left[\cos\theta + \frac{1}{3^2}\cos 3\theta + \frac{1}{5^2}\cos 5\theta + \ldots\infty\right]$$

whence

$$\mathscr{I} = I\sqrt{\frac{\pi^2(\pi^2 - 2a\pi + 2a^2)}{24a^2(\pi - a)^2}}.$$

The expression for the mean value introduces some interesting series, and is calculated from equation (1) page 69.

$$(I_M)_0^\pi = \frac{4I}{a\pi(\pi - a)}\sum\frac{\sin na}{n^3} - \frac{4(\pi - 2a)}{a\pi^2(\pi - a)}\sum\frac{1}{n^3}$$

n being odd.

If $a = \pi/2$ then $(I_M)_0^\pi = \frac{1}{2}I$, but if not then the term

$$\frac{4(\pi - 2a)}{a\pi^2(\pi - a)}\sum\frac{1}{n^3}$$

is not zero, and the series

$$\sum\frac{1}{n^3}$$

is not summable. Euler has found a value for it of 1·2020569 . . . and therefore

$$(I_M)_0^\pi = \frac{I}{2} - \frac{4(\pi - 2a)I}{a\pi^2(\pi - a)} \times 1{\cdot}2020569.$$

(2) *Triangular Wave* (Single Phase, Regular).

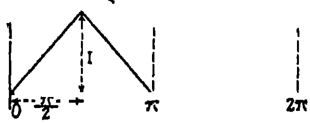

Fig. 17*b*.

$$i = \frac{I}{2} + \frac{2I}{\pi^2}\sum_2^\infty \frac{2\cos\frac{n\pi}{2} - (1 + \cos n\pi)}{n^2}\cos n\theta$$
$$+ \frac{4I}{\pi^2}\sum_2^\infty \frac{\sin\frac{n\pi}{2}}{n^2}\sin n\theta$$

$$\mathcal{I} = \frac{I}{\sqrt{6}}$$

$$I_M = \frac{I}{4}.$$

$$\text{Form Factor} = \frac{4}{\sqrt{6}}.$$

(3) *Triangular Wave* (Biphase, Regular).

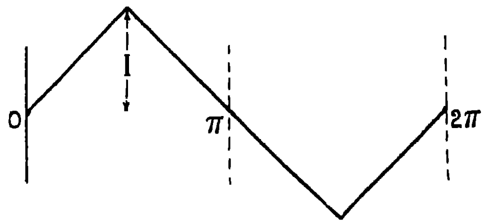

FIG. 18.

$$i = \frac{8I}{\pi^2}\left\{\sin\theta - \frac{1}{9}\sin 3\theta + \frac{1}{25}\sin 5\theta - \frac{1}{49}\sin 7\theta + \ldots\right\}$$

$$\mathcal{I} = \sqrt{\frac{I^2}{2}\cdot\frac{64}{\pi^4}\left\{1 + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\right\}} = \frac{I}{\sqrt{3}}$$

$$(I_M)_0^\pi = \frac{8}{\pi^2}\cdot\frac{2I}{\pi}\left\{1 - \frac{1}{3^3} + \frac{1}{5^3} - \ldots\right\}$$

$$= I\frac{16}{\pi^3}\cdot\frac{\pi^3}{32} = \frac{I}{2}.$$

$$\text{Form Factor} = \frac{2}{\sqrt{3}}.$$

(4) *Rectangular Wave* (Single Phase).

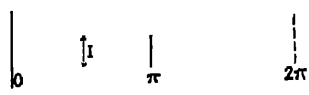

FIG. 19.

$$i = \frac{I}{2} + \frac{2I}{\pi}\left\{\sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{5}\sin 5\theta + \ldots\infty\right\}$$

$$\mathcal{I} = \frac{I}{\sqrt{2}}.$$

$$I_M = \frac{I}{2}.$$

$$\text{Form Factor} = \frac{2}{\sqrt{2}}.$$

(5) *Rectangular Wave* (Biphase).

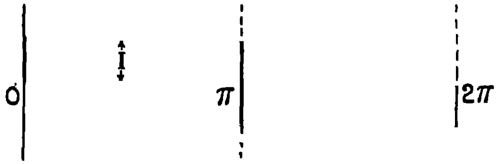

FIG. 20.

$$i = \frac{4I}{\pi}\left\{\sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{5}\sin 5\theta + \ \ldots\right\}$$

$$\mathcal{J} = I$$

$$(I_M)_0^\pi = I.$$

Form Factor = 1.

(6) *Discontinuous Rectangular Wave.*

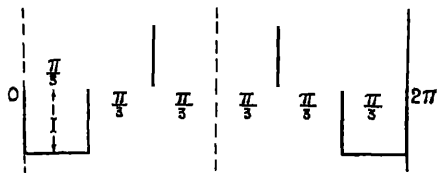

FIG. 21.

$$i = \frac{2\sqrt{3}I}{\pi}\left\{\cos\theta - \frac{1}{5}\cos 5\theta + \frac{1}{7}\cos 7\theta - \frac{1}{11}\cos 11\theta + \ \ldots\right\}$$

$$\mathcal{J} = \cdot 666I$$

$$(I_M)_0^\pi = \cdot 666I.$$

Form Factor = 1.

(7) *Trapezoid.*

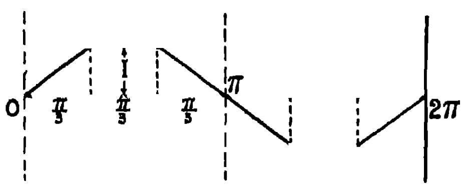

FIG. 22.

$$i = \frac{6\sqrt{3}I}{\pi^2}\left\{\sin\theta - \frac{1}{25}\sin 5\theta + \frac{1}{49}\sin 7\theta \ \ldots\right\}$$

$$\mathcal{J} = \cdot 71I$$

$$(I_M)_0^\pi = \cdot 666I.$$

Form Factor = 1·06.

(8) *Semicircular Wave.*

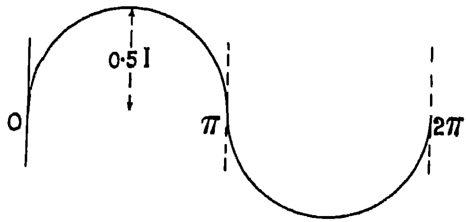

FIG. 23.

$$i = I\left\{J_1\left(\frac{\pi}{2}\right) \sin \pi\theta - \frac{1}{3}J_1\left(\frac{3\pi}{2}\right) \sin 3\pi\theta + \frac{1}{5}J_1\left(\frac{5\pi}{2}\right) \sin 5\pi\theta - \ldots\right\}$$

$$= I\{0\,566 \sin \pi\theta + 0{\cdot}094 \sin 3\pi\theta + 0{\cdot}042 \sin 5\pi\theta \ \ldots\}$$

where $J_1(\theta)$ is a Bessel function of the first order.

$$\mathscr{I} = 0{\cdot}815I$$

$$(I_M)_0^\pi = 0{\cdot}780I.$$

Form Factor = 1·04.

(9) *Single-Phase Rectified Sine Wave.*

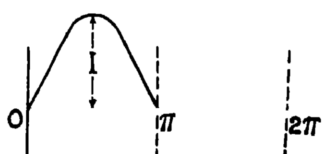

FIG. 24.

$$i = I\left\{\frac{1}{\pi} + \frac{1}{2}\sin\theta - \frac{2}{1.3.\pi}\cos 2\theta - \frac{2}{3.5.\pi}\cos 4\theta - \ldots\right\}$$

$$\mathscr{I} = \frac{I}{2}$$

$$I_M = \frac{I}{\pi}.$$

Form Factor $= \frac{\pi}{2}$.

(10) *Biphase and Polyphase Rectified Sine Waves.*

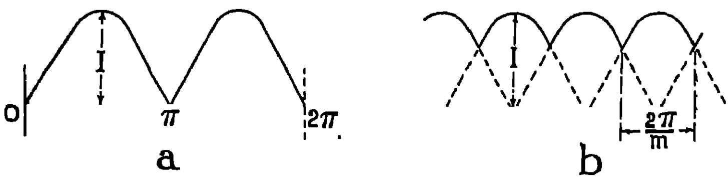

FIG. 25.

This biphase curve, as it is called (Fig. 25 *a*), can be obtained by shifting the curve in Fig. 24 through 180° and adding, whence

$$i = \frac{2I}{\pi}\left\{1 - \frac{2}{1.3}\cos 2\theta - \frac{2}{3.5}\cos 4\theta - \frac{2}{5.7}\cos 6\theta - \ldots\right\}$$

$$\mathcal{I} = \frac{I}{\sqrt{2}}$$

$$I_M = \frac{2I}{\pi}.$$

$$\text{Form Factor} = \frac{\pi}{2\sqrt{2}}.$$

This equation may be generalised in the form

$$i = I\left\{\frac{2}{\pi} - \frac{4}{\pi}\sum\frac{\cos 2n\theta}{(2n-1)(2n+1)}\right\}.$$

The polyphase wave form has been considered above, and it is quoted here merely for ease in reference.

$$i = \frac{mI}{\pi}\sin\frac{\pi}{m}\left[1 + 2\sum_{n=1}^{\infty}\frac{\cos nm\theta}{1 - n^2m^2}\right]$$

$$\mathcal{I} = \sqrt{\frac{1}{2} + \frac{m}{4\pi}\sin\frac{2\pi}{m}}$$

$$I_M = \frac{m}{\pi}\sin\frac{\pi}{m}.$$

$$\text{Form Factor} = \frac{\sqrt{\frac{1}{2} + \frac{m}{4\pi}\sin\frac{2\pi}{m}}}{\frac{m}{\pi}\sin\frac{\pi}{m}}.$$

A number of various curves are given in the generalised form in Winkelman's "Handbuch der Physik," Band 2, p. 33, and are reproduced here for reference :—

(11) *Parabola* (Single phase)

$$i = I\theta^2 = I\left\{\frac{\pi^2}{3} + 4\sum_1^\infty \frac{(-1)^m \cos m\theta}{m^2}\right\},$$

which may be written

$$i = I\theta^2 = I\left\{\frac{\pi^2}{3} - 4\cos\theta + \cos 2\theta - \frac{4}{9}\cos 3\theta + \ldots\right\}$$

$$\mathscr{I} = I\pi^2\frac{\sqrt{2}}{3}$$

$$I_M = \frac{I\pi^2}{3}.$$

Form factor $= \sqrt{2}$.

Various curves including powers of θ lend themselves to simple treatment, as for instance—

$$i = I\left\{\frac{\theta^2}{4} - \frac{\pi\theta}{2} + \frac{\pi^2}{6}\right\} = I\sum_1^\infty \frac{\cos m\theta}{m^2}$$

and

$$i = I\left\{\frac{\theta^3}{12} - \frac{\pi\theta^2}{4} + \frac{\pi^2\theta}{6}\right\} = I\sum_1^\infty \frac{\sin m\theta}{m^3}.$$

(12) *Flattened Sine Wave* (Single Phase).

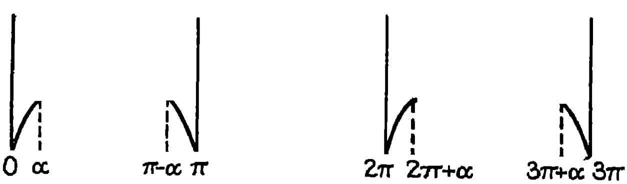

FIG. 26.

$$y = \frac{2(1-\cos a) + (\pi - 2a)\sin a}{2\pi} + \frac{\sin a \cos a + a}{\pi}\sin\theta$$

$$-\frac{1}{\pi}\sum_2^\infty \frac{1+\cos n\pi}{n^2-1}\left\{1 - \cos a \cos na - \frac{\sin a \sin na}{n}\right\}\cos n\theta$$

$$-\frac{1}{\pi}\sum_2^\infty \frac{1-\cos n\pi}{n^2-1}\left\{\frac{\sin a \cos na}{n} - \cos a \sin na\right\}\sin n\theta.$$

$$\mathcal{I} = \sqrt{\frac{1}{2\pi}\left\{a \cos 2a + \pi \sin^2 a - \frac{\sin 2a}{2}\right\}}.$$

$$I_M = \frac{1}{2\pi}\left\{(\pi - 2a) \sin a + 2(1 - \cos a)\right\}$$

$$\text{Form Factor} = \frac{\sqrt{2\pi\left\{a \cos 2a + \pi \sin^2 a - \frac{\sin 2a}{2}\right\}}}{(\pi - 2a) \sin a + 2\,(1 - \cos a)}.$$

Flattened Sine Wave (Biphase).

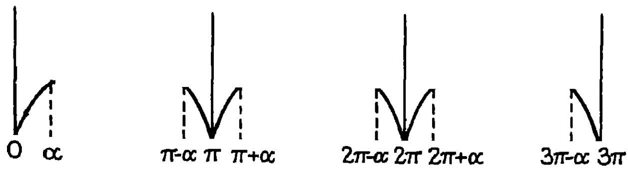

FIG. 27.

$$y = \frac{2(1 - \cos a) + (\pi - 2a) \sin a}{\pi}$$

$$- \frac{4}{\pi}\sum_{2}^{\infty}\frac{1 + \cos n\pi}{n^2 - 1}\left\{1 - \cos a \cos na - \frac{\sin a \sin na}{n}\right\} \cos n\theta.$$

$$\mathcal{I} = \sqrt{\frac{1}{\pi}\left\{a \cos 2a + \pi \sin^2 a - \frac{\sin 2a}{2}\right\}}$$

$$I_M = \frac{1}{\pi}\left\{(\pi - 2a) \sin a + 2(1 - \cos a)\right\}.$$

$$\text{Form Factor} = \frac{\sqrt{\pi\left\{a \cos 2a + \pi \sin^2 a - \frac{\sin 2a}{2}\right\}}}{(\pi - 2a) \sin a + 2(1 - \cos a)}.$$

(13) *Truncated Sine Wave Form* (Single Phase).

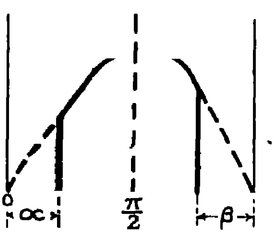

FIG. 28.

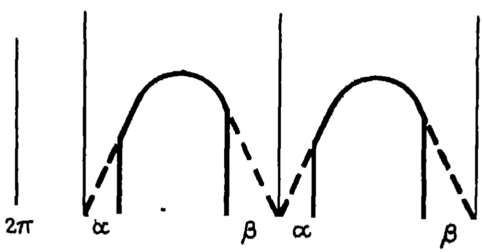

FIG. 29.

$$y = \frac{\cos a + \cos \beta}{2\pi} + \frac{\sin a \cos a + \sin \beta \cos \beta - a - \beta + \pi}{2\pi}\sin\theta - \frac{\sin^2 \beta - \sin^2 a}{2\pi}\cos\theta$$

$$- \sum_2^\infty \frac{\cos n\pi(\cos\beta\cos n\beta + n\sin\beta\sin n\beta) + \cos a\cos na + n\sin a\sin na}{\pi(n^2-1)}\cos n\theta$$

$$- \sum_2^\infty \frac{\cos n\pi(n\sin\beta\cos n\beta - \cos\beta\sin n\beta) + \cos a\sin na - n\sin a\cos na}{\pi(n^2-1)}\sin n\theta.$$

$$\mathscr{I} = \sqrt{\frac{1}{4\pi}(\pi - \beta - a + \sin\beta\cos\beta + \sin a\cos a)}$$

$$I_M = \frac{1}{2\pi}(\cos a + \cos\beta)$$

$$\text{Form factor} = \frac{\sqrt{\pi(\pi - \beta - a + \sin\beta\cos\beta + \sin a\cos a)}}{\cos a + \cos\beta}.$$

Truncated Sine Wave Form (Biphase).

$$y = \frac{\cos a + \cos\beta}{2\pi}$$

$$- \sum_2^\infty \frac{\cos\beta\cos n\beta + n\sin\beta\sin n\beta + \cos a\cos na + n\sin a\sin na}{\pi(n^2-1)}(1 + \cos n\pi)\cos n\theta$$

$$- \sum_2^\infty \frac{n\sin\beta\cos n\beta - \cos\beta\sin n\beta + \cos a\sin na - n\sin a\cos na}{\pi(n^2-1)}(1 + \cos n\pi)\sin n\theta$$

$$\mathscr{I} = \sqrt{\frac{1}{2\pi}(\pi - \beta - a + \sin\beta\cos\beta + \sin a\cos a)}$$

$$I_M = \frac{\cos a + \cos\beta}{\pi}$$

$$\text{Form factor} = \frac{\sqrt{\frac{\pi}{2}(\pi - \beta - a + \sin\beta\cos\beta + \sin a\cos a)}}{\cos a + \cos\beta}.$$

Trigonometrical Summations.—In the analyses of wave-forms the summation of various series is inevitable, and a few of the more common are given below for reference :—

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots \infty = \frac{1}{2}$$

$$\frac{1}{(1.3)^2} + \frac{1}{(3.5)^2} + \frac{1}{(5.7)^2} + \ldots \infty = \frac{\pi^2 - 8}{16}$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots \infty = \frac{\pi^2}{6}$$

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots \infty = \frac{\pi^4}{90}$$

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \infty = \frac{\pi^2}{8}$$

$$1 + \frac{1}{3^4} + \frac{1}{5^4} + \ldots \infty = \frac{\pi^4}{96}$$

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots \infty = \frac{\pi}{4}.$$

The general equation to some of these series is given by

$$S_n = \frac{2^{n+2} \, . \, n\,!}{\pi^{n+1}} \left\{ 1 + \left(-\frac{1}{3}\right)^{n+1} + \left(\frac{1}{5}\right)^{n+1} + \left(-\frac{1}{7}\right)^{n+1} \ldots \right\}$$

and $$B_m = \frac{2 \, . \, (2m)\,!}{(2\pi)^{2m}} \left\{ 1 + \frac{1}{2^{2m}} + \frac{1}{3^{2m}} + \frac{1}{4^{2m}} + \ldots \right\}$$

In the following table B_m represents the mth Bernouilli number, and the even values of S_n, Euler's number. The odd values of S_n are called "prepared Bernouillian" numbers, and are obtained as follows:—

$$S_{2n-1} = \frac{2^{2n}(2^{2n} - 1)}{2n} B_n.$$

There is a certain amount of confusion in some text-books as to which of the suffixes n, $2n$ or $2n - 1$ should be used. In this case the practice has been followed of using the suffix n only, excepting only the case above of calculating the prepared Bernouillians.

Ref.—Crystal's "Algebra," Part II., pp. 291, 342 and 368; Edward's "Calc.," p. 502.

$S_1 = 1$	$B_1 = \frac{1}{6}$
$S_2 = 1$	$B_2 = \frac{1}{30}$
$S_3 = 2$	$B_3 = \frac{1}{42}$
$S_4 = 5$	$B_4 = \frac{1}{30}$
$S_5 = 16$	$B_5 = \frac{5}{66}$
$S_6 = 61$	$B_6 = \frac{691}{2730}$
$S_7 = 272$	$B_7 = \frac{7}{6}$

$$\sum_2^\infty \frac{\cos n\pi \cos n\theta}{n^2-1} = \frac{\cos 2\theta}{1.3} - \frac{\cos 3\theta}{2.4} + \ldots \infty$$
$$= \tfrac{1}{2} - \tfrac{1}{4}\cos\theta - \tfrac{1}{2}\theta\sin\theta$$

$$\sum_2^\infty \frac{\cos n\pi(n \sin n\theta)}{n^2-1} = \frac{2\sin 2\theta}{1.3} - \frac{3\sin 3\theta}{2.4} + \ldots \infty$$
$$= \tfrac{1}{2}\theta\cos\theta + \tfrac{1}{4}\sin\theta$$

$$\sum_2^\infty \frac{\cos n\theta}{n^2-1} = \frac{\cos 2\theta}{1.3} + \frac{\cos 3\theta}{2.4} + \ldots \infty$$
$$= \tfrac{1}{2} + \tfrac{1}{4}\cos\theta - \tfrac{1}{2}\pi\sin\theta + \tfrac{1}{2}\theta\sin\theta$$

$$\sum_2^\infty \frac{n\sin n\theta}{n^2-1} = \frac{2\sin 2\theta}{1.3} + \frac{3\sin 3\theta}{2.4} + \ldots \infty$$
$$= \tfrac{1}{2}\pi\cos\theta - \tfrac{1}{4}\sin\theta - \tfrac{1}{2}\theta\cos\theta$$

$$\sum_2^\infty \frac{\cos\frac{n\pi}{2}\cos n\theta}{n^2-1} = -\frac{\cos 2\theta}{1.3} + \frac{\cos 4\theta}{3.5} - \ldots \infty$$
$$= \tfrac{1}{2} - \frac{\pi}{4}\cos\theta$$

$$\sum_2^\infty \frac{\cos\frac{n\pi}{2}(n\sin n\theta)}{n^2-1} = -\frac{2\sin 2\theta}{1.3} + \frac{4\sin 4\theta}{3.5} - \ldots \infty$$
$$= -\frac{\pi}{4}\sin\theta$$

$$\sum_2^\infty \frac{\sin\frac{n\pi}{2}\sin n\theta}{n^2-1} = -\frac{\sin 3\theta}{2.4} + \frac{\sin 5\theta}{4.6} - \ldots \infty$$
$$= \tfrac{1}{4}\sin\theta - \tfrac{1}{2}\theta\cos\theta$$

$$\sum_2^\infty \frac{\sin\frac{n\pi}{2}(n\cos n\theta)}{n^2-1} = -\frac{3\cos 3\theta}{2.4} + \frac{5\cos 5\theta}{4.6} - \ldots \infty$$
$$= -\tfrac{1}{4}\cos\theta + \tfrac{1}{2}\theta\sin\theta.$$

CHAPTER II.

HARMONIC ANALYSIS (*cont.*).

Classes of Function.—There are many methods of harmonic analysis available, some of which are accurate in the determination of the Fourier coefficients, but which are lengthy and cumbersome in operation.

As some of these methods are applicable only to particular forms of periodic function it is important to obtain a clear perception of these various types, which are bound up with a greater or less amount of symmetry of profile.

Class I. Asymmetric (Fig. 30).—The general expression for this function is

$$y=\frac{a_0}{2}+a_1\cos\theta+a_2\cos 2\theta+\ldots\infty+b_1\sin\theta+b_2\sin 2\theta+\ldots\infty.$$

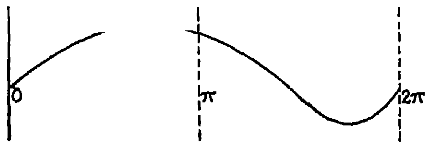

FIG. 30.—Class I. Function.

Class II. Sine Harmonic (Fig. 31).—The expression in this case is

$$y = b_1 \sin\theta + b_2 \sin 2\theta + \ldots \infty.$$

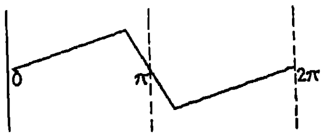

FIG. 31.—Class II. Function.

This periodic function is characterised by the fact that it is

possible to choose the origin so that $f(\theta) = -f(-\theta)$ and it is thus an odd function.

Class III. Cosine Harmonic (Fig. 32).—The expression is

$$y = \frac{a_0}{2} + a_1 \cos \theta + a_2 \cos 2\theta + \ldots \infty.$$

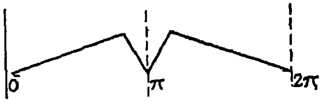

FIG. 32.—Class III. Function.

Here it is possible to choose the origin so that

$$f(\theta) = f(-\theta)$$

and it is thus an even function.

Class IV. Odd Harmonic (Fig. 33).—The expression is

$$y = a_1 \cos \theta + a_3 \cos 3\theta + \ldots \infty + b_1 \sin \theta + b_3 \sin 3\theta + \ldots \infty.$$

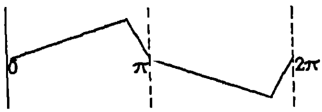

FIG. 33.—Class IV. Function.

In this case wherever the origin is situated

$$f(\pi + \theta) = -f(\theta).$$

Class V. Symmetric (Fig. 34).—The wave form is expressed by

Odd function $y = b_1 \sin \theta + b_3 \sin 3\theta + \ldots \infty.$

Even function $y = a_1 \cos \theta + a_3 \cos 3\theta + \ldots \infty.$

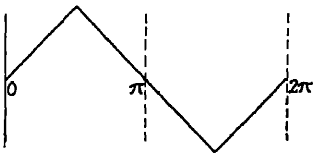

FIG. 34.—Class V. Function.

As each quarter period of this type of function is identical, it represents the highest form of symmetry possible.

It is important to recognise the differences between these five classes, as in electrical engineering practice it is rare to

encounter any forms beyond those of Classes IV. or V., and the introduction of the other three types produces some interesting problems in modern engineering.

Harmonic Analysis.—There are two types of analysis each of which has its practical application. In the first case, which is more theoretical in application, the wave form is of some known symmetrical shape, such as is shown in Figs. 31 to 34, and the first two methods of analysis result in the determination of Fourier coefficients of absolute accuracy, although it must be borne in mind that it is not always possible to analyse functions by these methods.

If the geometrical shape is not known, or is of too complicated a profile, methods of approximation have to be employed, and such cases constitute the second type of analysis.

Rigid Analysis.—The fundamental method has been employed on pages 16 to 19, but is applicable in relatively few cases, as it is rarely possible to evaluate the integral

$$\int_0^\lambda \phi(\theta) \cos \frac{2\pi n\theta}{\lambda} d\theta.$$

Russell's Method.—A useful method of analysis of waves containing odd harmonics only, i.e. Classes IV. and V., is due to Dr. A. Russell, and is described fully in his book on "Alternating Currents." It depends on the summation of two trigonometrical series, and as the method is of considerable importance, a brief description will be given.

Let C_1 and S_1 denote the two series

$$c \cos \theta - \frac{c^3}{3} \cos 3\theta + \frac{c^5}{5} \cos 5\theta - \ . \ . \ .$$

and $$c \sin \theta - \frac{c^3}{3} \sin 3\theta + \frac{c^5}{5} \sin 5\theta - \ . \ . \ .$$

respectively.

Then

$$C_1 + jS_1 = ce^{j\theta} - \frac{c^3}{3} e^{3j\theta} + \frac{c^5}{5} e^{5j\theta} - \ . \ . \ .$$

$$= \tan^{-1} ce^{j\theta} = \tan^{-1}\{c \cos \theta + cj \sin \theta\}$$

Equating real and imaginary parts by the method employed in any standard work on Trigonometry,

$$C_1 = \tfrac{1}{2} \tan^{-1} \frac{2c \cos \theta}{1 - c^2}.$$

Further let

$$C_2 = a \cos \theta + \frac{a^3}{3} \cos 3\theta + \frac{a^5}{5} \cos 5\theta + \ldots$$

and $$S_2 = a \sin \theta + \frac{a^3}{3} \sin 3\theta + \frac{a^5}{5} \sin 5\theta + \ldots$$

whence

$$jC_2 + S_2 = (ja \cos \theta + a \sin \theta) + \frac{a^3}{3} (j \cos 3\theta + \sin 3\theta) + \ldots$$

$$= j\left(a\epsilon^{-j\theta} + \frac{a^3}{3}\epsilon^{-3j\theta} + \frac{a^5}{5}\epsilon^{-5j\theta} + \ldots\right)$$

$$= \tan^{-1} ja\epsilon^{-j\theta}$$

$$= \tan^{-1} (ja \cos \theta + a \sin \theta)$$

and as before

$$S_2 = \tfrac{1}{2} \tan^{-1} \frac{2a \sin \theta}{1 - a^2}.$$

If now $c = a = 1$,

$$C_1 = \tfrac{1}{2} \tan (\pm \infty)$$

$$\text{and } S_2 = \tfrac{1}{2} \tan (\pm \infty)$$

or in other words,

$$\cos \theta - \tfrac{1}{3} \cos 3\theta + \tfrac{1}{5} \cos 5\theta - \ldots = + \frac{\pi}{4}$$

for the period $(2n - \frac{1}{2})\pi$ to $(2n + \frac{1}{2})\pi$ and $-\dfrac{\pi}{4}$ for the period $(2n + \frac{1}{2})\pi$ to $(2n + \frac{3}{2})\pi$, and also

$$\sin \theta + \tfrac{1}{3} \sin 3\theta + \tfrac{1}{5} \sin 5\theta + \ldots = + \frac{\pi}{4}$$

for the period $2n\pi$ to $(2n + 1)\pi$ and $-\dfrac{\pi}{4}$ for the period $(2n + 1)\pi$ to $(2n + 2)\pi$.

It is important also to note that the cosine series is true for all values of θ, but the sine series is only true for values of θ not equal to $n\pi$.

Now Fourier's Theorem may be applied to this result and the equation for the various amplitudes may be written

$$a_1 - \frac{a_3}{3} + \frac{a_5}{5} - \ . \ . \ . = \frac{2}{\lambda}\int_0^\lambda \phi(\theta)\left\{\cos\frac{2\pi\theta}{\lambda} - \tfrac{1}{3}\cos\frac{6\pi\theta}{\lambda} + \ . \ . \ .\right\}d\theta$$

and

$$b_1 + \frac{b_3}{3} + \frac{b_5}{5} + \ . \ . \ . = \frac{2}{\lambda}\int_0^\lambda \phi(\theta)\left\{\sin\frac{2\pi\theta}{\lambda} + \tfrac{1}{3}\sin\frac{6\pi\theta}{\lambda} + \ . \ . \ .\right\}d\theta.$$

It has been shown that the expression in the brackets may be equated to $\pm\frac{\pi}{4}$ on the imposition of various limits, which for the cosine series will be

$$+\frac{\pi}{4} \text{ when } \frac{2\pi\theta}{\lambda} = 0 \text{ to } (2n + \tfrac{1}{2})\pi$$

$$-\frac{\pi}{4} \text{ when } \frac{2\pi\theta}{\lambda} = (2n + \tfrac{1}{2})\pi \text{ to } (2n + \tfrac{3}{2})\pi$$

$$+\frac{\pi}{4} \text{ when } \frac{2\pi\theta}{\lambda} = (2n + \tfrac{3}{2})\pi \text{ to } 2\pi$$

and if n is put equal to zero

$$a - \frac{a_3}{3} + \frac{a_5}{5} - \ . \ . \ . = \frac{\pi}{2\lambda}\left\{\int_0^{\frac{\lambda}{4}}\phi(\theta)d\theta - \int_{\frac{\lambda}{4}}^{\frac{3\lambda}{4}}\phi(\theta)d\theta + \int_{\frac{3\lambda}{4}}^{\lambda}\phi(\theta)d\theta\right\}$$

and similarly

$$b_1 + \frac{b_3}{3} + \frac{b_5}{5} + \ . \ . \ . = \frac{\pi}{2\lambda}\left\{\int_0^{\frac{\lambda}{2}}\phi(\theta)d\theta - \int_{\frac{\lambda}{2}}^{\lambda}\phi(\theta)d\theta\right\}.$$

These two equations, however, do not permit of the calculation of all the amplitudes, but they may be generalised by the inclusion of a factor m which will furnish the requisite number of equations desired, by giving m the values 1, 2, 3, etc., in succession. The final equations are therefore as follows :—

$$a_m - \frac{a_{3m}}{3} + \frac{a_{5m}}{5} - \ . \ . \ .$$

$$= \frac{\pi}{2\lambda}\left\{\int_0^{\lambda/4m}\phi(\theta)d\theta - \int_{\lambda/4m}^{3\lambda/4m}\phi(\theta)d\theta + \int_{3\lambda/4m}^{5\lambda/4m}\phi(\theta)d\theta - \ . \ . \ .\right\}$$

and

$$b_m + \frac{b_{3m}}{3} + \frac{b_{5m}}{5} + \ldots$$
$$= \frac{\pi}{2\lambda}\left\{\int_0^{\lambda/2m} \phi(\theta)d\theta + \int_{\lambda/2m}^{\lambda/m} \phi(\theta)d\theta + \int_{\lambda/m}^{3\lambda/2m} \phi(\theta)d\theta + \ldots\right\}$$

Take as an example the rectangular wave in Fig. 35.

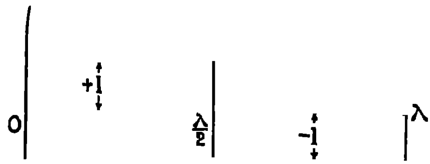

FIG. 35.—Analysis of rectangular wave form.

Firstly, $\phi(\theta) = +1$ for $\theta = 0$ to $\frac{\lambda}{2}$ and $m = 1$

and $\phi(\theta) = -1$ for $\theta = \frac{\lambda}{2}$ to λ,

whence $$a_1 - \tfrac{1}{3}a_3 + \tfrac{1}{5}a_5 - \ldots = 0,$$

and thus all the cosine terms are absent, as a_1, etc., are severally equal to zero.

Also $$b_1 + \tfrac{1}{3}b_3 + \tfrac{1}{5}b_5 + \ldots = \frac{\pi}{2}.$$

Further, putting $m = 3$

$$b_3 + \frac{b_9}{3} + \frac{b_{15}}{5} = \frac{\pi}{6}.$$

Putting $m = 5$ $$b_5 + \frac{b_{15}}{3} + \ldots = \frac{\pi}{10},$$

and so on.

Finally, the values of the amplitudes may be tabulated as follows:—

$$b_1 + \tfrac{1}{3}b_3 + \tfrac{1}{5}b_5 + \ldots\ldots = \frac{\pi}{2}$$

$$b_3 + \tfrac{1}{3}b_9 + \tfrac{1}{5}b_{15} + \ldots = \frac{\pi}{6}$$

$$b_5 + \tfrac{1}{3}b_{15} + \ldots = \frac{\pi}{10}$$

$$b_7 + \ldots = \frac{\pi}{14}$$

$$b_{15} + \ldots = \frac{\pi}{30}.$$

The results obtained from these simultaneous equations will give an approximation to the wave form; by taking only fifteen ordinates the error in b_1 is about 1 per cent., increasing to 20 per cent. for the higher harmonics.

This method is one of some accuracy compared with many others, but it involves the solution of as many simultaneous equations as there are amplitudes: a somewhat laborious process; nevertheless if accurate results are required, especially in the case of a series of infinite terms, this method is the best, if it is impossible to make use of Fourier's generalised method, an example of which will be given later.

It will be observed that the equations depend on the summation of a series of the form

$$\sin\theta + \tfrac{1}{3}\sin 3\theta + \tfrac{1}{5}\sin 5\theta + \ldots$$

and it might be thought that this method could be applied to a Fourier's series containing even harmonics. Such a series would take the form

$$\sin\theta + f(c^2)\sin 2\theta + f(c^3)\sin 3\theta + \ldots$$

where $f(c)$ is some function of the form $\frac{c}{2}$ or $\frac{c^2}{2!}$, etc. Such a series can be found in a convenient form for the summation of the sines, but not of the cosines and a different method has to be employed.

It may be remarked in passing that the form of the rectangular wave could have been predicted by the use of the above series, as follows:—

From o to π the value of the ordinate is $+1$ and hence—

$$\begin{aligned}\phi(\theta) &= +1 = \frac{\pi}{4}\times\frac{4}{\pi}\\ &= \frac{4}{\pi}\tan^{-1}(\infty)\\ &= \frac{4}{\pi}(\sin\theta + \tfrac{1}{3}\sin 3\theta + \tfrac{1}{5}\sin 5\theta + \ \ldots);\end{aligned}$$

from π to 2π the value is -1

or $$\phi(\theta) = -1 = -\frac{\pi}{4}\times\frac{4}{\pi} = \frac{4}{\pi}\tan^{-1}(\infty) \text{ as before.}$$

This is mentioned because in many cases a careful consideration of various trigonometrical series will result in a solution by inspection and much labour will be saved.

Weddle's Rule.—It will have been noted that the calculation of the constant term $\frac{a_0}{2}$ involves the calculation of the mean ordinate of the curve, and thus the evaluation of the area under the curve. This may be conveniently accomplished by a rule named after its author, although any one of a number of similar methods could be used equally well with varying degrees of accuracy.

Weddle's rule states that if y_0, y_1, y_2, etc., to y_6 are ordinates of the curve

$$y = \phi(\theta)$$

spaced equally apart, then

$$\int_0^\lambda \phi(\theta)d\theta$$

(or the area under the curve to the ordinate y_6) may be written in the form of the following equation :—

$$\text{area} \quad y_0 \text{ to } y_6 = \frac{\theta}{20}[y_0 + y_2 + y_4 + y_6 + 5(y_1 + y_5) + 6y_3].$$

The assumption underlying the theory of this rule is that the equation to the curve may be written in the form

$$y = a_0 + a_1\theta + a_2\theta^2 + a_3\theta^3 + \ . \ . \ . \ a_n\theta^n \ . \ . \ .$$

and it can be proved that complete accuracy is obtained when n is not greater than 5; and if n is as great as 10 the inaccuracy will not exceed 1 per cent.

Thus if a sine curve is to be integrated the accuracy with which it can be represented by the above equation determines the accuracy of the resulting calculations.

Certain precautions should, however, be taken if the curve is discontinuous, such, for instance, as the case of a triangular wave, where the curve should be divided into two portions at the apex, and be considered as two distinct curves, and the area calculated independently for each.

Method of Discontinuities.—A later development of Fourier's analysis by the use of the Method of Discontinuities

has been described by Eagle, and will prove of great benefit to those engaged in the solution of rectifier problems. For a complete description of the theory of this method, reference should be made to the work published on the subject; but it is pertinent to include a brief description.

Assume a wave form as shown in Fig. 36, which possesses discontinuities at points whose abscissæ are a, b, c, and d. It is apparent that if the wave form of the curve is

$$y = f(\theta)$$

then at the point where $\theta = a$, there is a discontinuity in the value of $f(\theta)$, the magnitude of which (viz. the length A_1A_2) can be written symbolically

$$f(a + o) - f(a - o)$$

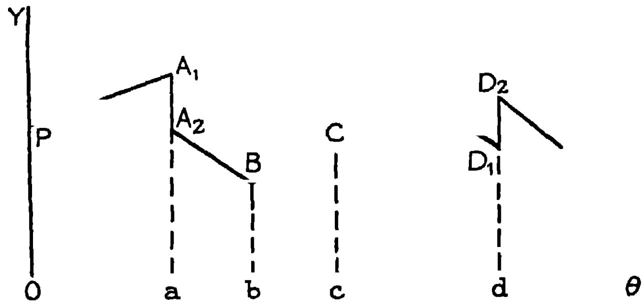

FIG. 36.—Generalised wave form.

where $f(a + o)$ and $f(a - o)$ represent the ordinates of the portion of the curve on the right and left of the point a respectively.

Similarly at the point where $\theta = b$ there is no discontinuity in the function, but one is found in the first differential; whence its value can be written

$$f'(b + o) - f'(b - o).$$

At point C there is only a discontinuity in the second derivative, wherefore it may be expressed as

$$f''(c + o) - f''(c - o)$$

and at D it is

$$f(d + o) - f(d - o) + f''(d + o) - f''(d - o).$$

To enable the various amplitudes to be calculated the value

of each discontinuity at the points a, b, c, and d is separately multiplied by either sin na, cos na, sin nb, cos nb, etc., respectively. If, therefore,

$$\begin{aligned} I_a &= f(a + o) - f(a - o) \\ I'_a &= f'(a + o) - f'(a - o) \\ I'_d &= f'(d + o) - f'(d - o), \text{ etc.,} \end{aligned}$$

then it can be proved that

$$\pi a_n = -\frac{1}{n}\Sigma I_a \sin na - \frac{1}{n^2}\Sigma I'_a \cos na + \frac{1}{n^3}\Sigma I''_a \sin na + \frac{1}{n^4}\Sigma I'''_a \cos na - \dots \infty$$

and

$$\pi b_n = \frac{1}{n}\Sigma I_a \cos na - \frac{1}{n^2}\Sigma I'_a \sin na - \frac{1}{n^3}\Sigma I''_a \cos na + \frac{1}{n^4}\Sigma I'''_a \sin na + \dots \infty.$$

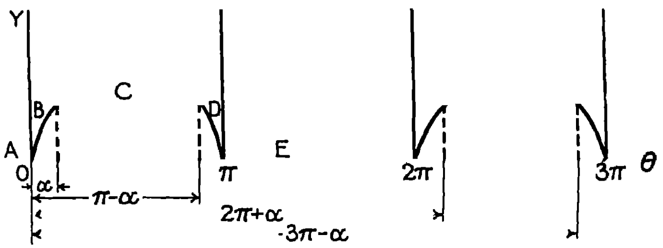

Fig. 37.—Flattened sine wave.

As an example of the use of the method consider the flattened sine wave form in Fig. 37. This curve takes the form $y = \sin\theta$ from 0 to a, $y = \sin a$ from a to $\pi - a$, and $y = \sin\theta$ from $\pi - a$ to π.

The first point of note is that in this case there is no discontinuity in the function itself, and therefore

$$I_a = 0.$$

Considering the first derivative, the summation consists of that of the discontinuities represented by

$$[f'(B) - f'(A)]_{\theta=0} \cos 0$$
$$[f'(C) - f'(B)]_{\theta=a} \cos na$$
$$[f'(D) - f'(C)]_{\theta=\pi-a} \cos n(\pi - a)$$
$$[f'(E) - f'(D)]_{\theta=\pi} \cos n\pi$$

which may be written

$$\begin{aligned}
&[\cos\theta - 0]_{\theta=0} &&= 1\\
&[0 - \cos\theta]_{\theta=a} \cos na &&= -\cos a \cos na\\
&[\cos\theta - 0]_{\theta=\pi-a} \cos n(\pi - a) &&= -\cos a \cos(n\pi - na)\\
&[0 - \cos\theta]_{\theta=\pi} \cos n\pi &&= \cos n\pi.
\end{aligned}$$

Hence

$$\Sigma I'_a \cos na = (1 + \cos n\pi)(1 - \cos a \cos na).$$

Similarly,

$$\Sigma I''_a \sin na = \sin a \sin na(1 + \cos n\pi)$$

and

$$\Sigma I'''_a \cos na = -(1 + \cos n\pi)(1 - \cos a \cos na), \text{ etc.}$$

Therefore

$$\pi a_n = +(1 + \cos n\pi)\left[(1 - \cos a \cos na)\left\{-\frac{1}{n^2} - \frac{1}{n^4} \ldots \infty\right\} + \left\{\frac{1}{n^3} + \frac{1}{n^5} + \frac{1}{n^7} + \ldots \infty\right\} \sin a \sin na\right]$$

$$= \frac{1 + \cos n\pi}{1 - n^2}\left\{1 - \cos a \cos na - \frac{\sin a \sin na}{n}\right\}.$$

Similarly it may be shown that

$$\pi b_n = \frac{1 - \cos n\pi}{1 - n^2}\left\{\frac{\sin a \cos na}{n} - \cos a \sin na\right\}.$$

Since putting $n = 1$ results in infinite values for a_n and b_n, it is necessary to differentiate the numerator and denominator with respect to n from which it is found that $a_1 = 0$,* and

$$b_1 = \frac{\sin a \cos a + a}{\pi}$$

while

$$\frac{a_0}{2} = \frac{2\int_0^a \sin\theta d\theta + (\pi - 2a) \sin a}{2\pi}$$

$$= \frac{2(1 - \cos a) + (\pi - 2a) \sin a}{2\pi}.$$

* *Note.*—In this connection it is particularly important that the function be reduced to the indeterminate form $\frac{0}{0}$ before differentiating.

The complete expression for the wave form is therefore

$$y = \frac{2(1 - \cos a) + (\pi - 2a)\sin a}{2\pi} + \frac{\sin a \cos a + a}{\pi}\sin\theta$$

$$- \frac{1}{\pi}\sum_{2}^{\infty}\frac{1 + \cos n\pi}{n^2 - 1}\left\{1 - \cos a \cos na - \frac{\sin a \sin na}{n}\right\}\cos n\theta$$

$$- \frac{1}{\pi}\sum_{2}^{\infty}\frac{1 - \cos n\pi}{n^2 - 1}\left\{\frac{\sin a \cos na}{n} - \cos a \sin na\right\}\sin n\theta.$$

This wave form is an approximation to that obtained with a thermionic rectifier working with a saturated filament, and if a is given the value of 30°

$$y = 0{\cdot}209 - 0{\cdot}120\cos 2\theta - 0{\cdot}061\cos 4\theta - 0{\cdot}034\cos 6\theta - \ldots \infty$$
$$+ 0{\cdot}307\sin\theta + 0{\cdot}069\sin 3\theta + 0{\cdot}007\sin 5\theta + \ldots \infty$$

showing that in the practical case of a saturated rectifier the wave form consists of a sine fundamental, even cosine and odd sine harmonics.

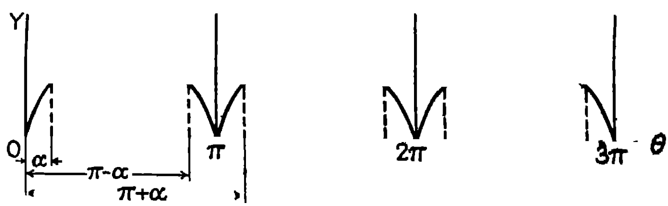

FIG. 38.—Flattened sine wave, biphase.

In the biphase case indicated in Fig. 38 by putting $\theta = \pi + \theta$ and adding the two curves

$$y = \frac{2(1 - \cos a) + (\pi - 2a)\sin a}{\pi}$$

$$- \frac{4}{\pi}\sum_{2}^{\infty}\frac{1 + \cos n\pi}{n^2 - 1}\left\{1 - \cos a \cos na - \frac{\sin a \sin na}{n}\right\}\cos n\theta$$

and it will be noted that all sine terms have vanished.

This method of discontinuities has been shown to be of considerable utility in solving certain Fourier's series which could not otherwise be obtained. The problem of evaluating the integral of such a form as

Single Phase Wave Form $y = \sin\theta$

$$\pi a_n = -\frac{1}{n}\Sigma I_\alpha \sin n\alpha - \frac{1}{n^2}\Sigma I'_\alpha \cos n\alpha + \frac{1}{n^3}\Sigma I''_\alpha \sin n\alpha + \frac{1}{n^4}\Sigma I'''_\alpha \cos n\alpha - \dots \infty$$

	$\alpha = 0$			$\beta = \pi$			$\gamma = 2\pi$			$\delta = 3\pi$				
I	—	—	0	—	—	0	—	—	0	—	—	0	$-\frac{1}{n}\Sigma I_\alpha \sin n\alpha$	—
I'	$\cos\theta$	$\cos 0$	1	$-\cos\theta$	1	$\cos n\pi$	—	—	0	—	—	0	$-\frac{1}{n^2}\Sigma I'_\alpha \cos n\alpha$	$-\frac{1}{n^2}(1+\cos n\pi)$
I''	$-\sin\theta$	0	0	—	—	0	—	—	0	—	—	0	$+\frac{1}{n^3}\Sigma I''_\alpha \sin n\alpha$	—
I'''	$-\cos\theta$	$-\cos 0$	1	$\cos\theta$	$\cos\pi$	$\cos n\pi$	—	—	0	—	—	0	$+\frac{1}{n^4}\Sigma I'''_\alpha \cos n\alpha$	$-\frac{1}{n^4}(1+\cos n\pi)$
I^{iv}	—	—	0	—	—	0	—	—	0	—	—	0	$-\frac{1}{n^5}\Sigma I^{iv}_\alpha \sin n\alpha$	—

$$\pi b_n = \frac{1}{n}\Sigma I_\alpha \cos n\alpha - \frac{1}{n^2}\Sigma I'_\alpha \sin n\alpha - \frac{1}{n^3}\Sigma I''_\alpha \cos n\alpha + \frac{1}{n^4}\Sigma I'''_\alpha \sin n\alpha - \dots \infty$$

$$\pi a_n = -(1+\cos n\pi)\left\{\frac{1}{n^2} + \frac{1}{n^4} + \dots \infty\right\} = -\sum_{2}^{\infty} \frac{1+\cos n\pi}{n^2-1} \text{ and } a_1 = 0$$

	$\alpha = 0$			$\beta = \pi$			$\gamma = 2\pi$			$\delta = 3\pi$				
I	—	—	0	—	—	0	—	—	0	—	—	0	$\frac{1}{n}\Sigma I_\alpha \cos n\alpha$	—
I'	$\cos\theta$	$\cos 0$	0	$-\cos\theta$	$\cos\pi$	$\sin n\pi$	—	—	—	—	—	—	$-\frac{1}{n^2}\Sigma I'_\alpha \sin n\alpha$	$-\frac{1}{n^2}\sin n\pi$
I''	—	—	0	—	—	0	—	—	0	—	—	0	$-\frac{1}{n^3}\Sigma I''_\alpha \cos n\alpha$	—
I'''	$-\cos\theta$	$\cos 0$	0	$\cos\theta$	$\cos\pi$	$-\sin n\pi$	—	—	0	—	—	0	$+\frac{1}{n^4}\Sigma I'''_\alpha \sin n\alpha$	$-\frac{1}{n^4}\sin n\pi$
I^{iv}	—	—	0	—	—	0	—	—	0	—	—	0	$+\frac{1}{n^5}\Sigma I^{iv}_\alpha \cos n\alpha$	—

$$\pi b_n = -\frac{\sin n\pi}{n^2-1} = 0 \text{ except when } n = 1 \text{ when } b_1 = \tfrac{1}{2}$$

$$y = \frac{1}{\pi} + \frac{1}{2}\sin\theta - 2\sum_{2}^{\infty} \frac{1+\cos n\pi}{n^2-1}\cos n\theta$$

FIG. 39.—Schedule illustrating use of method of discontinuities.

$$\int_0^{2\pi} f(\theta)\sin n\theta \, d\theta$$

is reduced to that of the summation of a series, which can usually be more easily ascertained. In the case of an aperiodic

function, such for instance, as that containing a short power series, the method presents no difficulty and may in most cases be employed in place of the older one.

To facilitate calculation it is convenient to prepare a schedule similar to that in Fig. 39, and the sole work then involved is that of filling in the requisite squares in order of sequence. To illustrate the use of the schedule, the analysis has been prepared for the single phase sine wave of Fig. 24.

Approximate Analysis.—Considerable ingenuity has been expended on the several methods of approximate solutions to wave form analysis. Generally, it will be found worth the time spent if consideration is given to the particular class in which the periodic function falls, as it is usually the case that the analysis becomes progressively simpler from Class I. to V.

Consider the general case of a Class I. function first.

Wave Analysis for all Harmonics, First Ordinate Method.—It is believed that this method was first evolved by the late Prof. S. P. Thompson, and the fundamental theory which can be easily proved is (to quote his own words): "the fact that if a series of $2n$ ordinates is measured at intervals apart of $\frac{\pi}{n}$ where n is the numeric representing the order of the harmonic, and if their values, taken alternatively, positively and negatively are averaged over a whole period, the mean so obtained is either simply the amplitude of that harmonic or else is the sum of the amplitudes, and of certain higher harmonics, viz. those the ordinal numeric of which is an odd multiple of n. For cosine components the series of $2n$ ordinates must begin (or end) at the beginning (or end) of the period. For sine components the series must begin at $\frac{\pi}{2n}$ from the beginning of the period."

It should be understood that this is quite a general method which could be equally applied to the case given above, but it has the disadvantage of less accuracy where a series contains a large number of terms, and, moreover, becomes very laborious as the number of terms increases. The analysis of a wave form which includes harmonics down to the 11th or 15th will be accurate if suitable corrections are applied.

From the above, assuming Fourier's series

$$y = \frac{a_0}{2} + a_1 \cos\frac{2\pi\theta}{\lambda} + a_2 \cos\frac{4\pi\theta}{\lambda} + a_3 \cos\frac{6\pi\theta}{\lambda} + \ldots$$
$$+ b_1 \sin\frac{2\pi\theta}{\lambda} + b_2 \sin\frac{4\pi\theta}{\lambda} + b_3 \sin\frac{6\pi\theta}{\lambda} + \ldots$$

the values of the amplitudes a, b, etc., may be written down at once, and are here given for the constant term and for odd and even harmonics up to the seventh: the subscripts represent the angle in degrees at which the ordinate y is measured:—

$$a_7 = \tfrac{1}{14}[y_0 - y_{25\cdot71} + y_{51\cdot43} - y_{77\cdot14} + y_{102\cdot85} - y_{128\cdot56} + y_{154\cdot28} - y_{180}$$
$$+ y_{205\cdot71} - y_{231\cdot43} + y_{257\cdot14} - y_{282\cdot85} + y_{308\cdot56} - y_{334\cdot28}].$$
$$b_7 = \tfrac{1}{14}[y_{12\cdot84} - y_{38\cdot56} + y_{64\cdot28} - y_{90} + y_{115\cdot71} - y_{141\cdot43} + y_{167\cdot14}$$
$$- y_{192\cdot85} + y_{218\cdot56} - y_{244\cdot28} + y_{270} - y_{295\cdot71} + y_{321\cdot43} - y_{347\cdot14}].$$
$$a_6 = \tfrac{1}{12}[y_0 - y_{30} + y_{60} - y_{90} + y_{120} - y_{150} + y_{180} - y_{210} + y_{240}$$
$$- y_{270} + y_{300} - y_{330}].$$
$$b_6 = \tfrac{1}{12}[y_{15} - y_{45} + y_{75} - y_{105} + y_{135} - y_{165} + y_{195} - y_{225} + y_{255}$$
$$- y_{285} + y_{315} - y_{345}].$$
$$a_5 = \tfrac{1}{10}[y_0 - y_{36} + y_{72} - y_{108} + y_{144} - y_{180} + y_{216} - y_{252} + y_{288} - y_{324}].$$
$$b_5 = \tfrac{1}{10}[y_{18} - y_{54} + y_{90} - y_{126} + y_{162} - y_{198} + y_{234} - y_{270} + y_{306} - y_{342}].$$
$$a_4 = \tfrac{1}{8}[y_0 - y_{45} + y_{90} - y_{135} + y_{180} - y_{225} + y_{270} - y_{315}].$$
$$b_4 = \tfrac{1}{8}[y_{22\cdot5} - y_{67\cdot5} + y_{112\cdot5} - y_{157\cdot5} + y_{202\cdot5} - y_{247\cdot5} + y_{292\cdot5} - y_{337\cdot5}].$$
$$a_3 = \tfrac{1}{6}[y_0 - y_{60} + y_{120} - y_{180} + y_{240} - y_{300}].$$
$$b_3 = \tfrac{1}{6}[y_{30} - y_{90} + y_{150} - y_{210} + y_{270} - y_{330}].$$
$$a_2 = \tfrac{1}{4}[y_0 - y_{90} + y_{180} - y_{270}] - a_6.$$
$$b_2 = \tfrac{1}{4}[y_{45} - y_{135} + y_{225} - y_{315}] + b_6.$$
$$a_1 = \tfrac{1}{2}[y_0 - y_{180}] - a_3 - a_5 - a_7.$$
$$b_1 = \tfrac{1}{2}[y_{90} - y_{270}] + b_3 - b_5 + b_7.$$
$$\frac{a_0}{2} = y_0 - a_1 - a_2 - a_3 - a_4 - a_5 - a_6 - a_7.$$

An example will make the use of the method clear. Take the case of a single-phase rectified circuit as shown in Fig. 40, where the wave is of sinusoidal shape for the period 0 to π, and no current flows from π to 2π. This case is then considered up to the 6th harmonic.

FIG. 40.—Single-phase rectified wave form.

Filling in the values for y all the odd harmonics are absent except b_1 and

$$a_6 = \tfrac{1}{12}(0 - 0{\cdot}5 + 0{\cdot}86 - 1{\cdot}0 + 0{\cdot}86 - 0{\cdot}5 + 0) = -0{\cdot}02.$$
$$a_4 = \tfrac{1}{8}(0 - 0{\cdot}707 + 1{\cdot}0 - 0{\cdot}707 + 0) = -0{\cdot}052.$$
$$a_2 = \tfrac{1}{4}(-1) + 0{\cdot}02 = -0{\cdot}23.$$
$$a_1 = 0.$$
$$b_6,\ b_4,\ \text{and}\ b_2 = 0.$$
$$b_1 = 0{\cdot}5.$$

and $\dfrac{a_0}{2} = 0 - (-0{\cdot}23 - 0{\cdot}052 - 0{\cdot}02) = 0{\cdot}302,$

and the wave form is represented by the equation

$$y = 0{\cdot}302 + 0{\cdot}5\sin\theta - 0{\cdot}23\cos 2\theta - 0{\cdot}052\cos 4\theta \quad [-0{\cdot}02\cos 6\theta\ \ldots].$$

If this result is compared with the rigid expression obtained on page 19, viz.

$$y = \frac{1}{\pi} + \frac{1}{2}\sin\theta - \frac{2}{\pi\,.\,1\,.\,3}\cos 2\theta - \frac{2}{\pi\,.\,3\,.\,5}\cos 4\theta - \ldots \infty$$
$$= 0{\cdot}32 + 0{\cdot}5\sin\theta - 0{\cdot}22\cos 2\theta - 0{\cdot}042\cos 4\theta - \ldots$$

it indicates at once that the ordinate method has no very great accuracy, where an infinite series is concerned, and where only six terms are considered; a greater accuracy could be obtained by taking a larger number of terms, but the correction factor would become complicated and the method cumbersome.

Second Ordinate Method.—In this method, which is described in some detail, it is assumed that it is possible accurately to measure in one complete period the length of $2p$ equally spaced ordinates $Y_1\ Y_2\ Y_3\ Y_4\ \ldots\ Y_{2p}$, which are $\dfrac{\pi}{p}$ degrees apart.

Let the equation to the periodic function be

$$f(\theta) = y = \frac{a_0}{2} + a_1\cos\theta + \ldots + a_{p-1}\cos(p-1)\theta + \frac{a_p}{2}\cos p\theta$$
$$+ \beta_1\sin\theta + \ldots + \beta_{p-1}\sin(p-1)\theta$$

since $\sin p\theta = \sin n\pi = 0$.

Put $\theta = \dfrac{\pi}{p}, \dfrac{2\pi}{p}\ \ldots\ 2\pi$ in succession in this equation, and add

the results so obtained. Then it is known that

$$\sum_{q=1}^{p} \cos \frac{2\pi nq}{p} = \sum_{q=1}^{p} \sin \frac{2\pi nq}{p} = 0,$$

wherefore

$$2p\frac{a_0}{2} = \Sigma Y_q,$$

or

$$\frac{a_0}{2} = \frac{1}{2p}\Sigma Y_q.$$

Further put $\theta = \frac{\pi}{p}, \frac{2\pi}{p}$, etc., but also multiply by $\cos \frac{n\pi}{p}$, $\cos \frac{2n\pi}{p} \ldots \cos \frac{2\pi nq}{p}$ and remember that

$$\left.\begin{aligned} \sum_{q=1}^{p} \cos \frac{2\pi mq}{p} \cos \frac{2\pi nq}{p} &= 0 \\ \sum_{q=1}^{p} \sin \frac{2\pi mq}{p} \sin \frac{2\pi nq}{p} &= 0 \\ \sum_{q=1}^{p} \sin \frac{2\pi mq}{p} \cos \frac{2\pi nq}{p} &= 0 \end{aligned}\right\} m \neq n$$

whence

$$a_n = \sum_{q=1}^{2p} \cos^2 \frac{\pi nq}{p} = \sum_{q=1}^{2p} Y_q \cos \frac{\pi nq}{p}$$

but

$$\sum_{q=1}^{p} \cos^2 \frac{\pi nq}{p} = p$$

and therefore

$$a_n = \frac{1}{p}\sum_{q=1}^{2p} Y_q \cos \frac{\pi nq}{p} \quad . \quad . \quad . \quad (1)$$

In the same way it can be shown that

$$\beta_n = \frac{1}{p}\sum_{q=1}^{2p} Y_q \sin \frac{\pi nq}{p} \quad . \quad . \quad . \quad (2)$$

This result, however, does not lead very far, unless it can be shown that a_n and β_n bear some relation to the Fourier co-

efficients. If the distance $\frac{\pi}{p}$ between the successive ordinates is decreased until it becomes in the limit $d\theta$, then

$$\cos \frac{\pi n q}{p} = \cos n\theta$$

and
$$Y_q = f(\theta)$$

whence

$$\underset{p=\infty}{Lt} \frac{1}{p} \sum_{q=1}^{2p} Y_q \cos \frac{\pi n q}{p} = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta \, d\theta$$

and
$$\underset{p=\infty}{Lt} \, a_n = a_n.$$

Equations (1) and (2) are eminently suited to the construction of simple schedules, and to illustrate their use, the single phase triangular wave form of Fig. 17*b* is analysed.

If the maximum ordinate is assumed to be 10 units, and 20 ordinates are chosen per period, then $p = 10$, and

$$Y_0 = 0,\ Y_1 = 2,\ Y_2 = 4,\ Y_3 = 6,\ Y_4 = 8,\ Y_5 = 10,\ Y_6 = 8,$$
$$Y_7 = 6,\ Y_8 = 4,\ Y_9 = 2,\ Y_{10} = 0, \text{ and } Y_{11} \text{ to } Y_{20} = 0.$$

It is necessary to choose a value for p which is equal to the number of the last harmonic required; and in this case, therefore, it will be possible to calculate the tenth harmonic.

Next ascertain the values of

$$\frac{1}{p} \sin \frac{\pi q}{p} \quad \text{and} \quad \frac{1}{p} \cos \frac{\pi q}{p}$$

for values of q of 1 to 20, when they can be set out in the form of Tables IV. and V.

Consider the first harmonic—the ordinates Y_1 to Y_9 are set in column $n = 1$ opposite the appropriate value for

$$\frac{1}{p} \sin \frac{\pi q}{p} \quad \text{and} \quad \frac{1}{p} \cos \frac{\pi q}{p}$$

and are multiplied by that value.

In the case of the second harmonic, as the summation is now of the form

$$\sum Y_q \sin \frac{2\pi q}{p},$$

TABLE IV.—ANALYSIS OF A TRIANGULAR WAVE FORM (FIG. 17) BY MEANS OF 10 ORDINATES PER HALF CYCLE ($p = 10$) OR

$Y_0 = 0$ $Y_1 = 2$ $Y_2 = 4$ $Y_3 = 6$ $Y_4 = 8$
$Y_5 = 10$ $Y_6 = 8$ $Y_7 = 6$ $Y_8 = 4$ $Y_9 = 2$
$Y_{10} = 0$ Y_{10} to $Y_{20} = 0$

Sine-Harmonics.

q.	$\frac{1}{p}\sin\frac{\pi q}{p}$ ($p=10$).	Harmonic. 1.	2.	3	4.	5.	6.	7.	8.	9.	10.
0	0	0									
1	0·0309	0·0618		0·1854				0·1854		0·0618	
2	0·0588	0·2352	0·1176					0·4704			
3	0·0809	0·4854		0·1618				0·1618		0·4854	
4	0·0951	0·7608	0·3804	0·3804							
5	0·1000	1·0000				0·2000 0·2000 1·0000				1·0000	
6	0·0951	0·7608	0·5706	0·3804							
7	0·0809	0·4854		0·1618				0·1618		0·4854	
8	0·0588	0·2352	0·4704					0·4704			
9	0·0309	0·0618		0·1854				0·1854		0·0618	
10	0	0	0			0					
11	− 0·0309										
12	− 0·0588		− 0·4704	− 0·4704						− 0·2352	
13	− 0·0809										
14	− 0·0951		− 0·3804					− 0·3804		− 0·7608	
15	− 0·1000			− 1·000		− 0·6000 − 0·6000		− 1·0000			
16	− 0·0951		− 0·5706					− 0·3804		− 0·7608	
17	− 0·0809										
18	− 0·0588		− 0·1178	− 0·4704						− 0·2352	
19	− 0·0309										
20	0		0								
	Σ	4·0664 $= \beta_1$	0	− 0·4856 $= \beta_3$	0	0·2000 $= \beta_5$	0	− 0·1266 $= \beta_7$	0	0·1024 $= \beta_9$	0

TABLE V.—COSINE-HARMONICS.

$$\frac{a_0}{2}=\frac{1}{20}(2+4+6+8+10+8+6+4+2)=2{\cdot}5.$$

q.	$\frac{1}{p}\cos\frac{\pi q}{p}$ ($p=10$)	Harmonics. 1.	2.	3.	4.	5.	6.	7.	8	9.	10
0	0·1000	0									
1	0·0951	0·1902		0·5706							
2	0·0809	0·3236	0·1618				0·4854				
3	0·0588	0·3528		0·1176							
4	0·0809	0·2472	0·1236	0·1236	0·0618 0·2472		0·2472		0·1854 0·1236		
5	0	0									
6	– 0·0309	– 0·2472	– 0·1854	– 0·1236			– 0·0618				
7	– 0·0588	– 0·3528		– 0·1176							
8	– 0·0809	– 0·3236	– 0·6472		– 0·3236 – 0·4854		– 0·3236		– 0·1618 – 0·6472		
9	– 0·0951	– 0·1902		– 0·5706							
10	– 0·1000	0	– 1·0000				– 1·0000				– 0·2000 – 0·6000 – 1·0000 – 0·6000 – 0·2000
11	– 0·0951										
12	– 0·0809		– 0·6472	– 0·6472	– 0·4854 – 0 3236		– 0·3236		– 0·6472 – 0·1618		
13	– 0·0588										
14	– 0·0309		– 0·1854				– 0·0618				
15	0			0							
16	0·0809		0·1236		0·2472 0·0618		0·2472		0·1236 0 1854		
17	0·0588										
18	0·0809		0·1618	0·6472			0·4854				
19	0·0951										
20	0·1000				1·0000				1·0000		0·4000 0·4000 0·8000 0·8000
	Σ	0	– 2·0944 $= a_2$	0	0	0	– 0·3056 $= a_6$	0	0	0	– 0·2000 $= a_{10}$

Y_q must be set opposite a value of

$$\frac{1}{p}\sin\frac{2\pi q}{p}$$

and multiplied by it, that is in line two, four, and so forth, due regard being paid to the correct sign. In this way the Tables IV. and V. are constructed for all the harmonics.

The wave form is finally found to be

$$\begin{aligned}2{\cdot}5 &- 2{\cdot}0944\cos 2\theta - 0{\cdot}3056\cos 6\theta - 0{\cdot}2000\cos 10\theta + \ldots \\ &+ 4{\cdot}0864\sin\theta - 0{\cdot}4856\sin 3\theta + 0{\cdot}2000\sin 5\theta \\ &- 0{\cdot}1266\sin 7\theta + 0{\cdot}1204\sin 9\theta - \ldots\end{aligned}$$

If the approximate Fourier coefficients so calculated are compared with those obtained by the rigid method considerable inaccuracies will be noted especially in the higher harmonics, due to the fact that p is not infinitely great. To investigate this point a schedule has been prepared for several types of wave form and a remarkable fact materialises.

In Table VI. six examples have been taken, and the coefficients ascertained by the rigid method, and finally measured by the second ordinate method. In five out of the six cases the error in the calculation of the coefficient is extraordinarily constant, even though the wave form differs widely.

The constancy of the error,* which is shown in Fig. 41 as a percentage variation, does not appear to vary with the class of periodic function so much as on the decrease of amplitude as the harmonic becomes of greater frequency.

* Since the preparation of this section, Eagle has shown that with a given grade of discontinuity, viz. I', I'', etc. (page 50), the error curve is represented by a simple series which sums to a trigonometrical function. Thus in the first five of the examples in Table VI., the discontinuity occurs in the first derivative, viz. I', and the error curve is given by the expression

$$\left[\frac{\frac{\pi x}{2}}{\sin\frac{\pi x}{2}}\right]^2$$

where $x = \frac{q}{p} = \frac{q}{10}$ in Fig. 41.

Fig. 41 was originally drawn from the data in Table VI., but this new expression fits it exactly.

This intriguing fact is being further investigated, especially with regard to the exception to the rule, but the result is not available for this edition.

It will readily be understood that if harmonics are present of only small amplitudes, the accuracy of the measurement of the ordinates Y_0 Y_1 Y_2, etc., will largely affect their value. This is emphasised in Table VI., the last two lines, which clearly indicate that the error curve applies in such instances; or more truly applies only when the accuracy of measurement is sufficiently great. On the whole, however, its application will

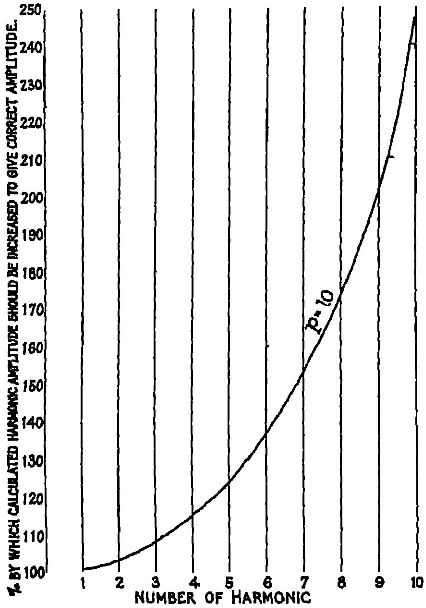

FIG. 41.—Error curve.

result in a reduction of the error due to the difference between the α's and β's and the Fourier coefficients.

Simplified Schedules.—For all but the Class I. function it is possible to simplify the procedure outlined in Tables IV. and V. considerably, due to a certain amount of symmetry present in each of the Classes II. to V.

Class II.—In this class let the ordinates over the period be

$$0 \quad Y_1'' \; Y_2'' \; . \; . \; . \; Y_{p-1}'' \quad 0 \quad -Y_{p-1}'' \; . \; . \; . \; -Y_3'' \quad -Y_2'' \quad -Y_1'' \quad 0,$$

then it can be shown that

$$\beta_n = \frac{2}{p}\Big[(Y_1'' + (-1)^{n-1}Y_{p-1}'')\sin\frac{n\pi}{p}$$

$$(Y_2'' + (-1)^{n-1}\ Y_{p-2}'')\sin\frac{2n\pi}{p} + \ . \ . \ . \ \Big] \ldots(3)$$

all of the Y's being counted once and once only.

Thus for $p = 10$ Table VII. represents the method of obtaining the values of β_1 to β_9.

Class III.—In this case the ordinates will be

$$Y_0''' \quad Y_1''' \ . \ . \ . \ Y_{p-1}''' \quad Y_p''' \quad Y_{p-1}''' \ . \ . \ . \ Y_1''' \quad Y_0'''$$

and

$$a_n = \frac{2}{p}\Big[\frac{Y_0''' + (-1)^n Y_p'''}{2} + \{Y_3''' + (-1)^n Y_{p-1}'''\}\cos\frac{n\pi}{p}$$

$$+ \{Y_2''' + (-1)^n Y_{p-2}'''\}\cos\frac{2n\pi}{p} + \ . \ . \ . \ \Big] \ldots(4)$$

and the schedule is as given in Table VIII., where $p = 10$.

Class IV.—The ordinates are

$$0 \quad Y_1'''' \quad Y_2'''' \ . \ . \ . \ Y_{p-1}'''' \quad 0 \qquad -Y_3'''' \quad -Y_2'''' \ . \ . \ . \ -Y_{p-1}'''' \quad 0$$

and

$$a_n = \frac{2}{p}\Big[(Y_2'''' - Y_{p-1}'''')\cos\frac{n\pi}{p} + (Y_2'''' - Y_{p-2}'''')\cos\frac{2n\pi}{p} + \ . \ . \ . \ \Big]$$

$$\beta_n = \frac{2}{p}\Big[(Y_1'''' + Y_{p-1}'''')\sin\frac{n\pi}{p} + (Y_2'''' + Y_{p-2}'''')\sin\frac{2n\pi}{p} + \ . \ . \ . \ \Big] \ldots(5)$$

and the schedules are given in Table IX., for $p = 10$.

The method of construction of these schedules has not been fully described, although a careful consideration of equations (3) to (5) and the setting out of the U's and V's in the form of the full schedule in the tables will indicate how they are constructed. It should be remembered that they can be applied as they stand, and that the calculations of the a_n and β_n can be accomplished with rapidity; further, the conversion of the a's and β's to the true Fourier coefficients by means of the error curve of Fig. 41 will, in the case of Classes I. to IV., give a reasonably

TABLE VII.—SCHEDULE FOR A CLASS II. FUNCTION ($p = 10$).

$$y = \beta_1 \sin x + \beta_2 \sin 2x + \ . \ . \ . \ + \beta_9 \sin 9x.$$

Ordinates	0	Y''_1	Y''_2	Y''_3	Y''_4	Y''_5
	0	Y''_9	Y''_8	Y''_7	Y''_6	
Sums	0	U_1	U_2	U_3	U_4	U_5
Differences	0	V_1	V_2	V_3	V_4	V_5

q.	$\frac{2}{p}\sin\frac{\pi q}{p} = K$.	Harmonics. 1 and 9.		3 and 7.		5.	2 and 8		4 and 6.	
0	0									
1	0·0618	KU_1		KU_3						
2	0·1176		KU_2		$-KU_4$		KV_1	KV_4	$-KV_3$	KV_2
3	0·1618	KU_3		KU_1						
4	0 1902		KU_4		KU_2		KV_3	KV_2	KV_1	$-KV_4$
5	0·2000	KU_5		$-KU_5$		KU_1 $-KU_3$ KU_5				
	Σ	P_1	Q_1	P_3	Q_3	P_5	P_2	Q_2	P_4	Q_4
		$\beta_1 = P_1 + Q_1$ $\beta_9 = P_1 - Q_1$		$\beta_3 = P_3 + Q_3$ $\beta_7 = P_3 - Q_3$		$\beta_5 = P_5$	$\beta_2 = P_2 + Q_2$ $\beta_8 = P_2 - Q_2$		$\beta_4 = P_4 + Q_4$ $\beta_6 = P_4 - Q_4$	

TABLE VIII.—SCHEDULE FOR A CLASS III. FUNCTION ($p = 10$).

$$y = \frac{a_0}{2} + a_1 \cos x + \ldots + a_{10} \cos 10x.$$

Ordinates	Y'''_0	Y'''_1	Y'''_2	Y'''_3	Y'''_4	Y'''_5
	Y'''_{10}	Y'''_9	Y'''_8	Y'''_7	Y'''_6	
Sums	U_0	U_1	U_2	U_3	U_4	U_5
Differences	V_0	V_1	V_2	V_3	V_4	

q	$\frac{2}{p}\cos\frac{\pi q}{p} = L.$	Harmonics.										
		0 and 10		2 and 8.		4 and 6.		1 and 9.		3 and 7.		5.
0	0·2000	$\frac{LU_0}{2}$ LU_2 LU_4	LU_1 LU_3 LU_5	$\frac{LU_0}{2}$	$-LU_5$	$\frac{LU_0}{2}$	LU_5	$\frac{LV_0}{2}$		$\frac{LV_0}{2}$		$\frac{LV_0}{2}$ $-LV_2$ LV_4
1	0·1902								LV_1		$-LV_3$	
2	0·1618			$-LU_4$	LU_1	$-LU_2$	$-LU_3$	LV_2		$-LV_4$		
3	0·1176								LV_3		LV_2	
4	0·0618			LU_2	$-LU_3$	LU_4	LU_1	LV_4		$-LV_2$		
5	0											
	Σ	P_0	Q_0	P_2	Q_2	P_4	Q_4	P_1	Q_1	P_3	Q_3	P_5
		$\frac{a_0}{2} = \frac{P_0 + Q_0}{2}$ $\frac{a_{10}}{2} = \frac{P_0 - Q_0}{2}$		$a_2 = P_2 + Q_2$ $a_8 = P_2 - Q_2$		$a_4 = P_4 + Q_4$ $a_6 = P_4 - Q_4$		$a_1 = P_1 + Q_1$ $a_9 = P_1 - Q_1$		$a_3 = P_3 + Q_3$ $a_7 = P_3 - Q_3$		$a_5 = P_5$

TABLE IX.—SCHEDULE FOR A CLASS IV. FUNCTION ($p = 10$).

$$y = \alpha_1 \cos x + \alpha_3 \cos 3x + \ldots \alpha_9 \cos 9x + \alpha_{10} \cos 10x + \beta_1 \sin x + \beta_2 \sin 2x + \ldots + \beta_9 \sin 9x.$$

Ordinates	Y_0''''	Y_1''''	Y_2''''	Y_3''''	Y_4''''	Y_5''''
		Y_9''''	Y_8''''	Y_7''''	Y_6''''	
Sums	0	U_1	U_2	U_3	U_4	U_5
Differences	0	V_1	V_2	V_3	V_4	

q	$\frac{2}{p}\sin\frac{\pi q}{p}$ $= M.$	$\frac{2}{p}\cos\frac{\pi q}{p}$ $= N$	Harmonics. 1 and 9.				3 and 7				5.	
0	0	0·2000										$-NV_2$ NV_4
1	0·0618	0·1902	MU_1			NV_1	MU_3			$-NV_3$		
2	0·1176	0·1618		NV_2	MU_2			$-NV_4$	$-MU_4$			
3	0·1618	0·1176	MU_3			NV_3	MU_1			NV_1		
4	0·1902	0·0618		NV_4	MU_4			$-NV_2$	MU_2			
5	0·2000	0	MU_5				$-MU_5$				MU_1 $-MU_3$ MU_5	
		ΣU_n ΣV_n	P_1	R_1	Q_1	S_1	P_3	R_3	Q_3	S_3	P_5	R_5
			$\beta_1 = P_1 + Q_1$ $\beta_9 = P_1 - Q_1$ $\alpha_1 = R_1 + S_1$ $\alpha_9 = R_1 - S_1$				$\beta_3 = P_3 + Q_3$ $\beta_7 = P_3 - Q_3$ $\alpha_3 = R_3 + S_3$ $\alpha_7 = R_3 - S_3$				$\beta_5 = P_5$ $\alpha_5 = R_5$	

accurate result. Care must be taken with all Class V. functions where the wave form is approximately sinusoidal.*

With regard to the analysis of a Class IV. function, this can readily be accomplished by means of Table VII. for a Class II. function omitting all the even harmonics. This is done automatically as the differences V_n are zero.

* See note on page 61.

CHAPTER III.

WAVE FORM AND ITS MEASUREMENT.

HAVING considered in Chapters I. and II. the fundamentals of Fourier's series, and theoretical and practical analysis, the final requirement is that of the effect of harmonics of the readings of instruments. The possible errors which may occur have been foreshadowed in the introductory remarks in Chapter I., but in this chapter the problem is analysed in greater detail.

On rectified circuits, as both alternating and direct current instruments will provide an indication, it is often found that a moving coil ammeter and a moving iron voltmeter are employed together, and if by chance a direct and alternating current ammeter are both used the widely divergent results will cause confusion.

Consider, therefore, first the theoretical reading which may be expected in each case.

Calculation of Mean Values of Complex Waves.—In considering the value of the mean ordinate of the wave it is often difficult if not impossible to state which is the zero value from which the integration should commence, on account of the number of positive and negative loops in the wave, due to the presence of higher harmonics. It is therefore convenient to begin the integration from the zero value of the fundamental, and in what follows this has been done. Further, if the oscillation consists of odd harmonics only the mean value of a complete period is zero, as the positive half of the loop equals in area the negative half, but in rectified circuits it frequently happens that the wave has a form as shown in Fig. 42, where the positive wave persists for a greater time than the negative

portion; and in such requirements as battery charging, the battery is being charged during the positive half, and discharged during the smaller negative portion. Thus the mean value in this case will be the area of the curve from 0 to θ less the area from θ to 2π.

The general expression for a current wave containing harmonics, and when no constant term is present is

$$i = I'_1 \sin\theta + I'_2 \sin 2\theta + I'_3 \sin 3\theta + \ldots + I'_n \sin n\theta + \\ + I''_1 \cos\theta + I''_2 \cos 2\theta + I''_3 \cos 3\theta + \ldots + I''_n \cos n\theta +$$

which may be written

$$i = I_1 \sin(\theta - \theta_1) + I_2 \sin(2\theta - \theta_2) + I_3 \sin(3\theta - \theta_3) + \ldots \\ \ldots + I_n \sin(n\theta - \theta_n) + \ldots$$

and the average value of which is

$$I_M = \frac{1}{\pi}\int_0^\pi i d\theta = \frac{1}{\pi}\sum\int_0^\pi I_n \sin(n\theta - \theta_n) d\theta = \frac{2}{\pi}\sum \frac{I_n \cos\theta_n}{n} \quad (1)$$

when n is odd, or zero when n is even.

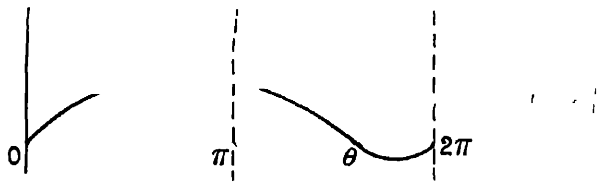

FIG. 42.—Type of imperfectly rectified wave form.

Further, when $\theta_n = 0$, i.e. when the harmonics are all in phase with the fundamental (or when the series consists of sine terms only)

$$I_M = \frac{2}{\pi}\sum\frac{I_n}{n} \quad . \quad . \quad . \quad . \quad (2)$$

when n is odd, or zero when n is even as before.

In the general expression for Fourier's series it has been stated that when a constant term I_0 is present the average ordinate equals that term; in other cases the average value of the curve is obtained by the formula

$$\left(I_M\right)_0^\pi = \frac{2}{\pi}\sum\frac{I_n \cos\theta_n}{n} \text{ or } \frac{2}{\pi}\sum\frac{I_n}{n} \text{ or zero}$$

according as to whether cosine terms are present or not, and whether n is odd or even.

These results may be summarised more explicitly :—

(a) $i = I_0$ plus other terms

$$I_M = I_0.$$

(b) $i = I'_1 \sin \theta + I'_2 \sin 2\theta \ldots + I'_n \sin n\theta \ldots$

$$+ I''_1 \cos \theta + I''_2 \cos 2\theta \ldots + I''_n \cos n\theta \ldots$$

$$\left(I_M\right)_0^\pi = \frac{2}{\pi}\sum \frac{I_n \cos \theta_n}{n} \quad (n \text{ is odd only})$$

where
$$I_n = \sqrt{I'^2_n + I''^2_n}$$

and
$$\tan \theta_n = -\frac{I''_n}{I'_n}$$

(c) $i = I_1 \sin \theta + I_2 \sin 2\theta \ldots + I_n \sin n\theta \ldots$

$$\left(I_M\right)_0^\pi = \frac{2}{\pi}\sum \frac{I_n}{n} \quad (n \text{ is odd only}).$$

These three results (a), (b) and (c) refer to a wave form only between the limits 0 and π; between the limits 0 and 2π (b) and (c) reduce to zero, n being odd or even, and the net unidirectional current is I_0 ($= I_M$). Thus the condition of rectification is that the wave form shall contain a constant term.

This point has been elaborated to distinguish between I_0 and I_M—where the wave form is symmetrical, i.e. where I_0 is zero, I_M is zero over the complete period, and also over the half period except where n is odd.

Calculation of the Effective Value of Complex Waves.—The effective or R.M.S. value of the current wave presents no such difficulty as that introduced by the phase displacements of the various harmonics. The general equation of the wave can be written

$$i = I_0 + I_1 \sin (\theta - \theta_1) + I_2 \sin (2\theta - \theta_2) + \ldots$$
$$\ldots + I_n \sin (n\theta - \theta_n) + \ldots$$

The square of the instantaneous value of the current is

$$i^2 = I_0^2 + I_1^2 \sin^2 (\theta - \theta_1) + I_2^2 \sin^2 (2\theta - \theta_2) + \ldots$$
$$\ldots + 2I_1I_n \sin (\theta - \theta_1) \sin (n\theta - \theta_n) + \ldots$$
$$\ldots + 2I_2I_n \sin (2\theta - \theta_2) \sin (n\theta - \theta_n) + \ldots$$
$$\ldots + I_0I_1 \sin (\theta - \theta_1) + \ldots$$

and the effective value of this equation will be given by

$$\mathscr{I}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta.$$

Now definite integrals of the form

$$\int_0^{2\pi} \sin p(\theta - \theta_1) \cos q(\theta - \theta_2) d\theta$$

or

$$\int_0^{2\pi} \sin p(\theta - \theta_1) \sin q(\theta - \theta_2) d\theta$$

equate to zero when p and q are not equal, and therefore components containing the squares of the sines of the angles only remain. Each of these reduces to π on evaluating the definite integrals, and therefore

$$\mathscr{I}^2 = I_0^2 + \tfrac{1}{2}(I_1^2 + I_2^2 + I_3^2 + I_4^2 + \ldots + I_n^2 +) \quad \ldots . \quad (3)$$

Thus the effective value of the current is seen to bear no relation whatever to phase displacement, and depends on the constant term, and the maximum value of each of the amplitudes of the harmonics only.

Calculation of Power.—The average power in the circuit can also be calculated if the analysis of the current and voltage waves is available. In a perfectly general form

$$i = I_0 + I_1 \sin(\theta - \theta_1) + I_2 \sin(2\theta - \theta_2) + \ldots$$
$$e = E_0 + E_1 \sin(\theta - \gamma_1) + E_2 \sin(2\theta - \gamma_2) + \ldots$$

are the instantaneous values of the current and voltage.

Then the instantaneous power

$$\begin{aligned} ei = E_0 I_0 &+ E_1 I_1 \sin(\theta - \theta_1) \sin(\theta - \gamma_1) \\ &+ E_2 I_1 \sin(2\theta - \gamma_2) \sin(\theta - \theta_1) \\ &+ E_n I_n \sin(n\theta - \gamma_n) \sin(n\theta - \theta_n) + \ldots \end{aligned}$$

The average power is equal to

$$\frac{1}{2\pi}\int_0^{2\pi} ei\, d\theta$$

and in performing the integration, the terms involving differing values of n are as before equal to zero, and all the remaining integrals take the form

$$\int_0^{2\pi} \sin (n\theta - \theta_n) \sin (n\theta - \gamma_n) d\theta$$
$$= \tfrac{1}{2}\int_0^{2\pi} [\cos (\theta_n - \gamma_n) - \cos \{2n\theta - (\theta_n + \gamma_n)\}] d\theta$$
$$= \tfrac{1}{2}\int_0^{2\pi} \cos (\theta_n - \gamma_n) d\theta$$

since
$$\int_0^{2\pi} \cos \{2n\theta - (\theta_n + \gamma_n)\} d\theta = 0.$$

Thus P (the average power)

$$= E_0 I_0 + \tfrac{1}{2}[E_1 I_1 \cos (\theta_1 - \gamma_1) + E_2 I_2 \cos (\theta_2 - \gamma_2) + \ldots$$
$$\ldots E_n I_n \cos (\theta_n - \gamma_n) + \ldots]$$
$$= E_0 I_0 + \tfrac{1}{2}[E_1 I_1 \cos \phi_1 + E_2 I_2 \cos \phi_2 + \ldots$$
$$\ldots + E_n I_n \cos \phi_n + \ldots] \quad . \quad (4)$$

where ϕ_n is the phase displacement between the current and the voltage of the nth harmonic.

This point is one of considerable importance as it demonstrates the fact that the real value of the power is made up of the power contributed by the fundamental and each harmonic; and also because it throws light on the argument as to what the actual power is in some forms of rectified circuits (page 77).

Effective and Mean Values in a Rectified Circuit.—It is instructive at this stage to take a typical case and calculate the R.M.S. and mean values of a rectified wave, and for this purpose the example given in Fig. 24 of the single phase sine wave will be used. The equation to this curve has been found to be

$$i = I\left\{\frac{1}{\pi} + \frac{1}{2}\sin\theta - \frac{2}{3\pi}\cos 2\theta - \frac{2}{15\pi}\cos 4\theta \ldots\right\}$$

and the mean value of which is $\frac{I}{\pi}$. For the R.M.S. value, from equation (3)

$$\mathscr{I}^2 = I^2\left[\frac{1}{\pi^2} + \frac{1}{2}\left\{\frac{1}{4} + \frac{4}{\pi^2(3)^2} + \frac{4}{\pi^2(3.5)^2} + \ldots\right\}\right]$$
$$= I^2\left[\frac{1}{\pi^2} + \frac{1}{2}\left\{\frac{1}{4} + \frac{4}{\pi^2}\left(\frac{1}{(1.3)^2} + \frac{1}{(3.5)^2} + \ldots\right)\right\}\right].$$

The series

$$\left(\frac{1}{1.3}\right)^2 + \left(\frac{1}{3.5}\right)^2 + \left(\frac{1}{5.7}\right)^2 + \ldots$$

may by partial fractions be split up into a series whose nth term is

$$\frac{1}{4(2n-1)^2} + \frac{1}{4(2n+1)^2} + \frac{1}{4(2n+1)} - \frac{1}{4(2n-1)}$$

the first two terms of which are of the form

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots \infty$$

and the sum of which is $\pi^2/8$. The second two terms can be shown to sum to $-\frac{1}{4}$ when an infinite number of terms is taken. Thus

$$\left(\frac{1}{1.3}\right)^2 + \left(\frac{1}{3.5}\right)^2 + \left(\frac{1}{5.7}\right)^2 + \ldots \infty = \frac{\pi^2 - 8}{16} \text{ (see also page 39).}$$

Filling in this value into the expression for $\mathscr{I}$

$$\mathscr{I}^2 = \frac{I^2}{4}$$

or

$$\mathscr{I} = \frac{I}{2}$$

as the effective ordinate. In this particular case, as the curve takes the sinusoidal form, this result might have been arrived at by the simpler method

$$\mathscr{I}^2 = \frac{I^2}{2\pi}\int_0^\pi \sin^2\theta d\theta = \frac{I^2}{4}$$

or

$$\mathscr{I} = \frac{I}{2},$$

It has been shown that where a constant term is present the average ordinate equals that term, but consider a case where there is no such term, as for example the rectangular wave. The equation for such a curve is

$$i = I\left\{\frac{4}{\pi}(\sin\theta + \tfrac{1}{3}\sin 3\theta + \tfrac{1}{5}\sin 5\theta \ldots)\right\}$$

$$(I_M)_0^\pi = I\frac{2}{\pi}\sum\frac{I_n}{m}$$
$$= I\frac{2}{\pi}\frac{4}{\pi}\left\{1 + \frac{1}{3^2} + \frac{1}{5^2} + \ . \ . \ .\right\}$$
$$= I\frac{8}{\pi^2}\frac{\pi^2}{8} = I.$$

Average Power in a Rectified Circuit.—The equation for the mean power has been shown to be

$$P = E_0I_0 + \tfrac{1}{2}[E_1I_1 \cos \phi_1 + E_2I_2 \cos \phi_2 + \ . \ . \ . \ ]$$

and if the biphase rectified current and E.M.F. waves of Fig. 25 *a* are evaluated

$$e = E\left[\frac{2}{\pi} - \frac{4}{1.3.\pi}\cos 2\theta - \frac{4}{3.5.\pi}\cos 4\theta \ . \ . \ .\right]$$

and $$i = I\left[\frac{2}{\pi} - \frac{4}{1.3.\pi}\cos 2\theta - \frac{4}{3.5.\pi}\cos 4\theta \ . \ . \ .\right]$$

where all of the harmonics of current are in phase with the E.M.F.

The average power will then be

$$P = \frac{4}{\pi^2}EI + \tfrac{1}{2}\left[\frac{16EI}{\pi^2(1.3)^2} + \frac{16EI}{\pi^2(3.5)^2} + \ . \ . \ .\right]$$
$$= \frac{4}{\pi^2}EI + \frac{8EI}{\pi^2}\left(\frac{\pi^2 - 8}{16}\right)$$
$$= \frac{EI}{2} = \mathscr{E}\mathscr{I}$$

where $\mathscr{E}$ and $\mathscr{I}$ are the effective values of the current and the E.M.F.

Type of Instruments to use on a Rectified Circuit.—The type of instrument which is used on a rectified circuit has to be carefully chosen, and a brief description of the more common types may be of value in deciding which to employ.

D.C. Moving Coil.—In this type of instrument a coil, through which a definite proportion of the main current flows, rotates between the poles of a permanent magnet, which thus provides

a constant flux. The torque on the coil is consequently proportional to the current in the coil; therefore in the case of a varying current the meter will read the average value of the current I_0, and if connected to an alternating current circuit will give no permanent deflection, as the average value of i (0 to 2π) is zero. If, however, the instrument is provided with delicate moving parts of small inertia the needle will be observed to vibrate with the same period as the frequency of supply, and if the inertia is sufficiently small the needle will register the amplitude of the supply current (see page 80).

On a rectified circuit, viz. one where a constant term is always present, the meter will read a value corresponding to

$$I_M = I_0 = \frac{1}{2\pi}\int_0^{2\pi} i d\theta,$$

and this integral equals that term. Further, it may be noted that this reading is that which registers the charging current of a battery connected in circuit, and therefore on all electrolytic work a moving coil instrument should be used, or more generally —one which indicates the mean value of the current.

Hot Wire, Dynamometer, and Moving Iron Instruments.— All of these instruments fall into the category of "square law" instruments, viz. those in which the deflection is proportional to the mean value of the current squared. The hot wire ammeter depends on the heating and consequent expansion of a thin wire for its deflection; and as the heating effect equals the I^2R losses the deflection varies as the square of the current. This instrument has another valuable property, viz. that the indications are independent of the frequency of supply, a feature which is not shared by any of the other types mentioned.

The dynamometer contains two coils, one stationary and the other capable of rotation. The current passes through both of these coils connected in series and the torque is thus proportional to the square of the current. Lastly the moving iron meter depends on the induction in a small piece of soft iron rotating in a variable field, and the indications are again proportional to the square of the current flowing round the coil.

All of these meters will read accurately on a direct current circuit (with the qualifications below), but this must not be accepted as evidence that they will give the same result as a moving coil instrument on a rectified circuit.

It is convenient to look upon a moving coil meter as reading the electrolytic value of the current or the equivalent of a certain quantity of silver deposited in a certain time in a silver voltameter, whereas the square law instruments register the dynamic or heating value of the current. This distinction is a real one, and although rarely met with except in cases of rectification, yet it is important that the difference should be clearly understood.

It has been stated that both types of instrument will register correctly on alternating and direct current provided that the above distinction is borne in mind. Suppose that a dynamometer and a moving coil ammeter are connected in series on a circuit which consists of a small alternating current $i \sin \theta$ superposed on a nonpulsating current of value I; the reading on a moving coil meter will be I but on the dynamometer it will be

$$\sqrt{I^2 + \tfrac{1}{2}i^2}$$

according to equation (3), and the ratio of the two readings will be

$$\sqrt{1 + \tfrac{1}{2}\left(\frac{i}{I}\right)^2}$$

$$= 1 + \tfrac{1}{4}\left(\frac{i}{I}\right)^2 - \tfrac{1}{16}\left(\frac{i}{I}\right)^4 + \ldots$$

The difference between the two indications is thus dependent on the ratio of the superposed current to the mean current and if this is small the readings will be approximately the same. Further, if the frequency of the oscillations is high the inertia of the moving parts may be such as to render the vibrations unnoticeable.

Wattmeters and Power Measurements.—The wattmeter is a dynamometer in which the stationary coil is connected in

shunt or through a current transformer, so as to measure the current flowing, and the moving coil is placed across the supply to measure the voltage; the deflection of the needle is proportional to the product of the currents in the two coils, and is therefore a measure of the mean value of the power.

It has been shown in equation (4) that the average power $P = E_0I_0 + \frac{1}{2}[E_1I_1 \cos\phi_1 + E_2I_2 \cos\phi_2 + E_nI_n \cos\phi_n + \ldots]$ and that this is the general expression for the power in any circuit.

Now the effective values of the current and E.M.F. are

$$\mathscr{I} = \sqrt{I_0^2 + \tfrac{1}{2}[I_1^2 + I_2^2 + \ldots]}$$

and

$$\mathscr{E} = \sqrt{E_0^2 + \tfrac{1}{2}[E_1^2 + E_2^2 + \ldots]}$$

and hence if the power in the circuit is to be measurable by the product of the effective values of the current and the E.M.F. multiplied by an equivalent power factor

$$\sqrt{[I_0^2 + \tfrac{1}{2}\{I_1^2 + I_2^2 + \ldots\}][E_0^2 + \tfrac{1}{2}\{E_1^2 + E_2^2 + \ldots\}]}\cos\Phi$$

must equal

$$E_0I_0 + \tfrac{1}{2}[E_1I_1 \cos\phi_1 + E_2I_2 \cos\phi_2 + \ldots].$$

If the current and E.M.F. waves are similar in shape and only differ by a constant factor such that generally

$$E_1 = k^2I_1, \text{ and } E_n = k^2I_n, \text{ etc.,}$$

then $\mathscr{E}\mathscr{I} \cos\Phi$ reduces to

$$k^2[I_0^2 + \tfrac{1}{2}(I_1^2 + I_2^2 + \ldots)] \cos\Phi$$
$$= \{E_0I_0 + \tfrac{1}{2}(E_1I_1 + E_2I_2 + \ldots)\} \cos\Phi.$$

If now it is inferred from this relation that

$$\cos\phi_1 = \cos\phi_2 \ldots = \cos\Phi = 1,$$

i.e. $$\phi_1 = \phi_2 = \Phi = 0,$$

then $$P = \mathscr{E}\mathscr{I},$$

and under no other circumstances will this be true.

It is interesting to note in passing that the product of the current and voltage as read on moving coil meters, viz. E_0I_0, will

only accurately measure the power when the amplitudes of all harmonics are zero, i.e.

either $\quad E_1 = E_2 = \ldots 0, \quad$ or $\quad I_1 = I_2 = \ldots 0,$

or in other words, when the wave form of current or voltage is a straight line parallel to the axis of time and when there is no frequency error. This is only approximately the case even in three and six phase circuits and hence "square law" instruments are likely to give a truer indication of power than those of the moving coil type, again, provided that the frequency error is not too great.

To show the order of the error involved if the frequency error is negligible, assume a single phase sinusoidal wave form with unity power factor, in which case effective amperes multiplied by effective volts will give true power.

Then $$P_1 = \mathscr{E}\mathscr{I} = \frac{EI}{4}.$$

The power measured on moving coil meters is

$$P_2 = E_0 I_0 = \frac{EI}{\pi^2} = \frac{EI}{10} \text{ approximately}$$

and $$\frac{P_1}{P_2} = \frac{5}{2}.$$

In the case of a biphase sinusoidal wave form

$$\frac{P_1}{P_2} = \frac{5}{4}$$

and thus the difference between the readings is reduced, and the error reduced from 150 per cent. to 20 per cent.

In the case therefore of a rectified circuit where either (1) the current is out of phase with the E.M.F. or (2) the current wave is not similar to the E.M.F. wave, the *only* way to ascertain the true power is to use an electrostatic wattmeter which of course inherently performs the integration

$$\frac{1}{2\pi}\int_0^{2\pi} ei\,d\theta$$

and is not susceptible to frequency errors. If a dynamometer ammeter and voltmeter are used in conjunction with a wattmeter the equivalent power factor may under certain circumstances be obtained and will be

$$\cos\Phi = \frac{P}{\mathcal{E}\mathcal{I}}$$

$$= \frac{E_0 I_0 + \frac{1}{2}[E_1 I_1 \cos\phi_1 + E_2 I_2 \cos\phi_2 + \ . \ . \ .]}{\sqrt{[E_0^2 + \frac{1}{2}\{E_1^2 + E_2^2 + \ . \ . \ .\}][I_0^2 + \frac{1}{2}\{I_1^2 + I_2^2 + \ . \ . \ .\}]}}$$

but in the case of a wave form with harmonics present the value of Φ is meaningless excepting that of the above expression, and further, this is only true provided that the error introduced by harmonics present of frequencies outside the range of 25 to 150 cycles per second, is less than the inherent error of the instrument.

It cannot be too carefully emphasised that it is rarely true in rectified circuits that the two conditions above mentioned apply; and large errors may be introduced if any instrument but a wattmeter, which is not susceptible to frequency error, is used to measure power output. In fact it is not too much to say that if true power is required either some form of calorimeter is necessary or alternatively an electrometer (see also page 14).

It may be advanced that the insertion of a power factor meter may enable correct readings to be obtained, but such an instrument is susceptible to frequency errors, and these errors will mount up with the higher harmonics which are always present in a rectifier circuit. Although a power factor meter will reduce the correction necessary it may not give an absolutely correct result.

Lastly, it will be found that the difference in the indications of moving coil and square law instruments becomes less as the number of phases in the rectifier is increased, which is equivalent to a decrease in the amplitudes of the higher harmonics: when this is the case either the moving coil or square law instruments will give an approximate reading of the power, and the power factor will approximate to unity.

Form Factor.—The form factor of an oscillation is the ratio

$$\frac{\text{Effective value}}{\text{Mean value}}$$

and can therefore only be unity in the case of a rectangular wave, and in all other cases must be greater than unity.

The form factor is not of great practical utility, but it does give an indication of the peakiness or otherwise of the wave.

The form factor can be at once calculated from the readings given by a moving coil and dynamometer instrument in series, and if this ratio conforms to a given factor the general wave form can be approximately foretold.

Oscillographs.—Little has been said so far of the deleterious effects of certain types of wave, but there is little doubt that in some cases, such as, for instance, the inclusion in the circuit of a transformer, it is advisable to keep the wave form as nearly sinusoidal as possible, as otherwise at the crest value the iron may reach a high degree of saturation. The form factor may assist in giving an approximate idea of the peakiness of the wave, but actual demonstration of the shape of the curve by means of an oscillograph is preferable.

There are several forms of oscillograph, one of which should be found in every laboratory and test shop; the best known is that named after Duddell, and consists of a permanent magnet system of high magnetic induction, and two fine filaments to which is attached a mirror, and which suffer angular distortion when a current is passed through them. As the inertia of the moving parts is kept as small as possible, the mirror will more or less accurately follow the form of the oscillation, and if a spot of light be suitably reflected from it on to a sensitised moving film a record will be obtained of the actual wave form. It is not necessary to describe in detail the various designs and modifications which are embodied in the typical oscillographs obtainable on the market, as such particulars can be found in text-books and catalogues.

Two other types of oscillograph are briefly described which are easy to manipulate and which lend themselves to use in the test shop.

Gas Tube.—A glass tube about 3 inches long and 1 inch in diameter is filled with gas, preferably argon, to a predetermined pressure and is provided with two electrodes LL (Fig. 43) which are in linear juxtaposition with a small gap between them. The bulb is held between two contacts CC screwed to a wooden disc A about 18 inches in diameter, which is rotated by a silk thread from a motor M. The current is led to the contacts by two brushes bearing on slip rings (not shown) and a balance weight W is provided to counteract out-of-balance forces. It has been found that the negative glow (see Chapter VII.) will creep along the electrodes from the gap to the cap, a distance which is proportional to the current flowing through the tube; and the length of the glow is therefore proportional to the current flowing.

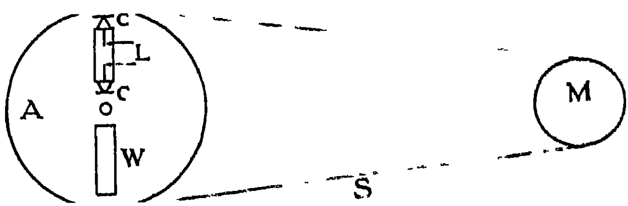

FIG. 43.—Gas tube oscillograph.

The speed of the motor determines the length of the axis of the oscillogram, and as the speed is increased the geometrical patterns of the base line change from square to pentagon, hexagon, etc., according as the speed is a multiple of the periodicity in the ratio of 4/1, 5/1, etc. The patterns change gradually from one design to another and the oscillogram appears as a continuous glow, as shown in Fig. 44. One advantage of the apparatus lies in the fact that it can be used directly on a high tension circuit without the interposition of any transformer, the motor being insulated by the silk thread.

Cathode Ray Oscillograph.—This form of oscillograph is also very convenient and has been described recently in several articles in the technical press. Cathode rays are projected from the filament F which is mounted in a somewhat unusual type

of bulb, as shown in Fig. 45. The rays pass between a metallic constriction A which is the anode, and thence between two plates P_2 parallel to one another and to the plane of the paper, and also between two similar plates P_1 at right angles to the plane of the paper; the anode, one of the P_1 plates and one of the P_2 plates are connected to a source of high potential.

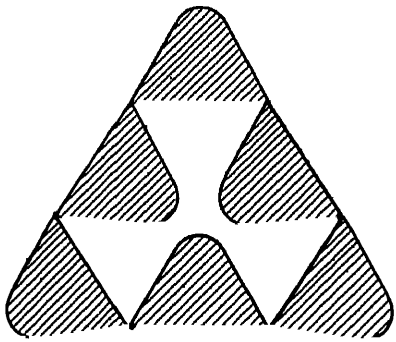

FIG. 44.—Typical oscillogram from a gas tube oscillograph.

Cathode rays are subject to deflection by an electrostatic and also by an electromagnetic field, and hence if plates P_1 are connected to an alternating supply and plates P_2 to a generator of oscillations of a known frequency the rays will be deflected so as to record the current wave on a fluorescent screen S at the end of the bulb. A further and interesting development is obtained by connecting plates P_1 in shunt with a choke coil and approaching the iron of the choke to the plates P_2. The plates P_1 will cause the rays to be deflected proportionately to the current flowing in the choke and hence to the magnetising force H, and the leakage flux from the iron will cause them to be deflected in a direction at right angles, and of an amplitude proportional to the magnetic induction and thus the B-H cyclic curve of magnetisation will become visible on the screen. This ocular demonstration is of value when considering the effect of rectified currents on the cyclic curve of a transformer iron circuit.

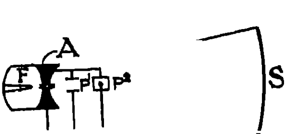

FIG. 45.—Cathode ray oscillograph.

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PART II.

MECHANICAL RECTIFIERS.

PART II.

MECHANICAL RECTIFIERS.

CHAPTER IV.

ROTATING MACHINERY.

FROM the standpoint of alternating current rectification, many of the existing types of machinery could be classed under the heading of rectifiers. For example, the process of commutation in a direct current generator, involving as it does the conversion of an inherent alternating E.M.F. in the windings into an unidirectional voltage at the brush terminals, is a rectifier.

Of the large machines which perform the function of rectification, in that a direct current output is obtained from an alternating current input, the four following types may be mentioned :—

(1) Motor Generator
(2) Rotary Converter
(3) Motor Converter
and (4) Transverter.

The motor generator set driven by an alternating current motor connected mechanically to a direct current generator is most commonly found in spite of its comparative inefficiency, but the theory of operation can be considered from the standpoint of a separate motor and generator, and the discussion of such combined plant is not included; but the reason for the prevalent use of motor generator sets is difficult to appreciate The only sound cause for the installation of such an arrangement would be the case where an earthed supply is available

and the rectified current has to be unearthed, and conversely; or where special voltage regulation is essential.

Of the remaining types of machine, viz. (2), (3) and (4), the Transverter will be considered in some detail, as little information is available in the current text-books, but as this chapter is devoted to machines whose windings rotate with their commutating devices, the Transverter will be discussed in the next chapter.

Rotary Converters—General.—Item (2), the Rotary Converter, is also only briefly considered, as the general theory of operation is well understood. But it has been thought advisable to develop the analysis of the voltage relationships on the A.C. and D.C. sides, as well as the calculation of the heating effect of the combined currents in the rotor, as both these problems show a certain similarity to allied phenomena in other forms of rectifier. The rotary converter, therefore, will only be considered from these two standpoints in what follows.

The rotating part of this machine consists of a drum type of armature revolving between two fixed pole pieces and provided with a commutator, as in the case of a direct current generator; but on the opposite end of the armature to this commutator, slip rings are mounted on the shaft and connected as indicated diagrammatically in the Fig. 46.

Thus the current in the armature consists of a supplied alternating component, and a generated continuous current.

Rotary Converter — Voltage Relationships.—Take the case depicted in Fig. 46—the two connections to the slip rings are made to opposite ends of the windings, and the brushes D′D″ are shown bearing on the commutator segments. Assuming an uniform winding, the alternating component of the E.M.F. will vary according to a sinusoidal law, viz. the instantaneous value

$$e = E \sin \theta$$

where E is its maximum amplitude. At the instant when $\theta = \frac{\pi}{2}$, $e = E$, and this is also the value of the continuous

voltage across the brushes. The effective value of the alternating E.M.F. is $\frac{E}{\sqrt{2}}$, and therefore the R.M.S. of the alternating voltage equals the direct E.M.F. divided by $\sqrt{2}$.

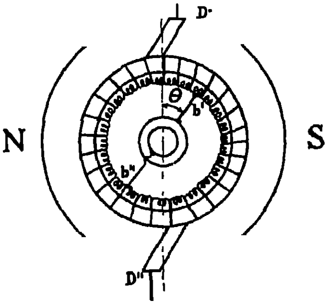

FIG. 46.—Diagrammatic representation of a rotary converter.

In the case of a three-phase supply the connections from the armature coils are taken out at three equally spaced positions to three collector rings, as shown in Fig. 47, and the

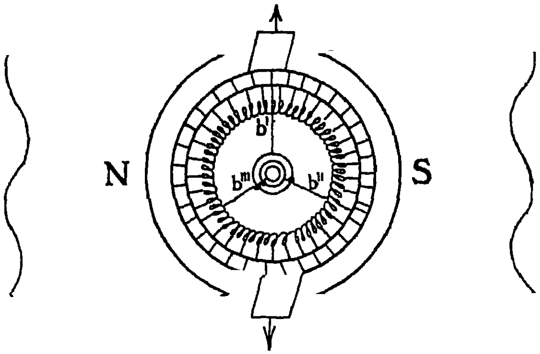

FIG. 47.—Diagram of three-phase rotary converter.

E.M.F. between the slip rings is given by the following equation:—

$$\mathscr{E} = \frac{E_M\sqrt{3}}{2\sqrt{2}} = 0{\cdot}612E_M$$

where E_M is the direct voltage, and $\mathscr{E}$ is the effective alternating E.M.F. of supply. And generally it can be shown that if m is the number of phases

$$E_M = \frac{\sqrt{2}\,\mathscr{E}}{\sin\frac{\pi}{m}}.$$

Thus the two E.M.Fs. have a perfectly definite relation depending only on the number of phases; and theoretically no variation of this relation is possible, but in practice this statement is not strictly true.

The actual relation between the two voltages is given in Table X. for the varying number of phases.

TABLE X.

Continuous Voltage.	R.M.S Voltage between Slip Rings. Number of Phases.					
	1.	2.	3.	6.	12.	m.
1	0·707	0·707	0·612	0·354	0·183	$\frac{1}{\sqrt{2}}\sin\frac{\pi}{m}$

Rotary Converter—Rating of Machine.—The current produced in the armature winding consists of a combination of the alternating and direct components. In Fig. 48, D represents the

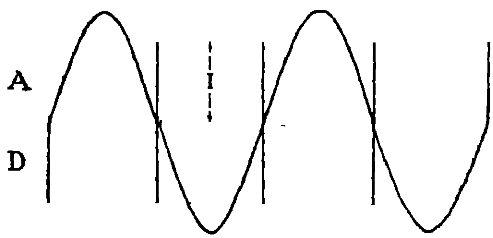

FIG. 48.—A.C. and D.C. current waves in a rotary converter.

direct current in the coil varying in value from $+I$ to $-I$, and A the alternating component which varies according to the equation

$$i = I\sin\theta.$$

The net current in the armature coil for a single-phase machine will therefore be obtained by an algebraical superposition of these two curves, and takes the form of the wave of Fig. 49. The mean square ordinate of this curve represents the heating of the armature, and therefore the armature loss.

To calculate how this armature heating varies for differing numbers of phases, it is first necessary to calculate the relation of the currents; and for simplicity assume for the moment that the efficiency and power factor are both 100 per cent. whence

$$E_M I_M = m\,\mathscr{E}\mathscr{I}$$

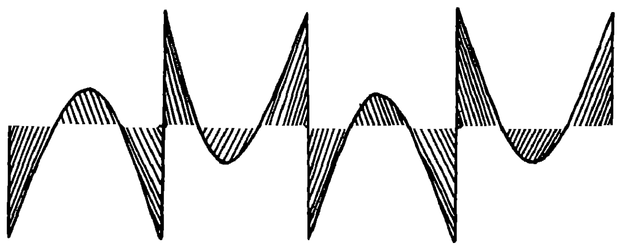

FIG. 49.—Net current in the armature of a rotary converter.

where I_M is the D.C. output current and $\mathscr{I}$ is the effective current input per phase.

As
$$E_M = \frac{\sqrt{2}\mathscr{E}}{\sin\frac{\pi}{m}}$$

it follows that
$$I_M = \frac{m\,\mathscr{I}\sin\frac{\pi}{m}}{\sqrt{2}}.$$

Take the case of a coil in the armature which is situated at an angle a from the midway position; the direct component is $\frac{I_M}{2}$ and the alternating component

$$I\sin(\theta - a)$$
$$= \sqrt{2}\mathscr{I}\sin(\theta - a).$$

The actual current in the coil is therefore

$$i = \sqrt{2}\mathscr{I} \sin (\theta - a) - \frac{I_M}{2}$$

$$= \frac{I_M}{2}\left\{\frac{4 \sin (\theta - a)}{m \sin \frac{\pi}{m}} - 1\right\}.$$

The mean square of this current is

$$\frac{1}{\pi}\int_0^{\pi} i^2 d\theta = \frac{I_M^2}{4}\left\{\frac{8}{m^2 \sin^2 \frac{\pi}{m}} + 1 - \frac{16 \cos a}{m\pi \sin \frac{\pi}{m}}\right\}.$$

The ratio of the direct component squared to this mean effective current is therefore

$$\delta = \frac{8}{m^2 \sin^2 \frac{\pi}{m}} + 1 - \frac{16 \cos a}{m\pi \sin \frac{\pi}{m}}$$

where δ is a factor which represents the ratio of the loss due to the armature heating to that of the same output of direct current generator.

This ratio will have maximum and minimum values at

$$a = \frac{\pi}{m} \text{ and } 0$$

and the mean value is therefore

$$\Delta = \frac{m}{\pi}\int_0^{\frac{\pi}{m}} \delta da = \frac{8}{m^2 \sin^2 \frac{\pi}{m}} + 1 - \frac{16}{\pi^2}.$$

This ratio Δ can be expressed in another way; if it is desired to obtain the same heating effect in the rotary converter its output can be increased in the ratio

$$\frac{1}{\sqrt{\Delta}}$$

over that of the same sized direct current generator.

Values for Δ and $\frac{1}{\sqrt{\Delta}}$ are given in Table XI. for varying numbers of phases :—

TABLE XI.

		D C. Machine.	Number of Phases. 2.	3.	4.	6.	12.	∞.
Rating taking continuous current machine as unity . . .	$\frac{1}{\sqrt{\Delta}}$	1 00	0·85	1·34	1·64	1·96	2·24	2·31
	Δ	1 00	1·37	0 55	0·37	0·26	0·20	0·18

and the curves are plotted in Fig. 50.

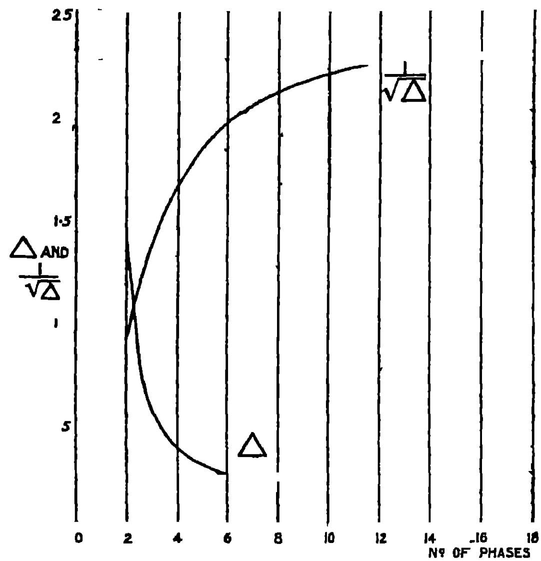

FIG. 50.—Effect of number of phases on rating.

In what has been said it has been assumed that the direct and the alternating currents are in phase, as is shown in Fig. 48; if, however, the circuit has a lagging or a leading power factor, the alternating current is displaced from the rectangular wave an amount equivalent to the power factor of the circuit, and the ratio δ will no longer have maximum and minimum values at $a = \pi/m$ and 0 respectively. The calculation of the heating current in this, the general case, can be effected as follows :—

If $\cos\phi$ is the power factor, the ratio between the wattless and the power currents will be $\tan\phi$, and as η is the overall efficiency of the machine, the input current will have to be increased by $1/\eta$ to make up for the deficit due to the losses; thus both the wattless and the power currents will be increased to this extent.

The alternating component of the armature current will therefore be

$$\sqrt{2}\,\mathscr{I}\left\{\frac{1}{\eta}\sin(\theta - a) + \frac{\tan\phi}{\eta}\cos(\theta - a)\right\}$$

and the current in the coil represented by the angle a will be

$$i = \sqrt{2}\,\mathscr{I}\left\{\frac{1}{\eta}\sin(\theta - a) + \frac{\tan\phi}{\eta}\cos(\theta - a)\right\} - \frac{I_M}{2}$$

$$= \frac{I_M}{2}\left[\frac{4}{m\sin\frac{\pi}{m}}\left\{\frac{1}{\eta}\sin(\theta - a) + \frac{\tan\phi}{\eta.}\cos(\theta - a)\right\} - 1\right]$$

$$= r\sin(\theta - a) + s\cos(\theta - a) - t$$

where

$$r = \frac{2I_M}{m\eta\sin\frac{\pi}{m}} \qquad s = \frac{2I_M\tan\phi}{m\eta\sin\frac{\pi}{m}} \qquad \text{and} \qquad t = \frac{I_M}{2}.$$

The mean square of the current is

$$\frac{1}{\pi}\int_0^{\pi} i^2 d\theta = \frac{r^2 + s^2}{2} - \frac{4rt\cos a}{\pi} - \frac{4st\sin a}{\pi} + t^2$$
$$= \delta'.$$

This is the heating current in any one coil displaced an angle a from the neutral point, and hence the average current is

$$\frac{m}{2\pi}\int_{-\frac{\pi}{m}}^{+\frac{\pi}{m}}\delta'\,da$$

$$= \frac{m}{2\pi}\int_{-\frac{\pi}{m}}^{+\frac{\pi}{m}}\left\{\frac{r^2 + s^2}{2} - \frac{4rt\cos a}{\pi} - \frac{4st\sin a}{\pi} + t^2\right\}da$$

$$= \frac{r^2 + s^2}{2} + t^2 - \frac{4rt}{\pi^2}\sin\frac{\pi}{m}$$

whence filling in the values for r, s and t

$$\Delta' = \left\{ \frac{8}{m^2\eta^2 \cos^2\phi \sin^2 \frac{\pi}{m}} + 1 - \frac{16}{\eta\pi^2} \right\}.$$

It will be noted that as there is no definite information as to the position of the maximum amplitude the integration has to be performed over one cycle, and that with a reduction in efficiency or power factor the ratio $\frac{1}{\sqrt{\Delta'}}$ decreases rapidly, and thus showing a falling off in the favourable comparison with a direct current generator.

The effect of power factor on the armature loss is illustrated in Fig. 51.

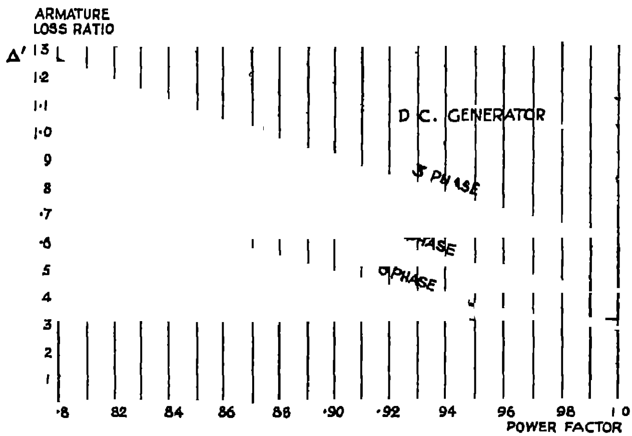

Fig. 51.—Rotary converter—power factor *v.* armature loss.

A single-phase rotary converter is not as efficient as the equivalent direct current machine, but the six-phase converter is seen to be nearly twice as efficient. The gain by increasing to twelve phases is not proportionately so great considering the increased complication, and therefore in practice it is usual to find the six-phase rotary is the one employed.

Rotary Converter—Inverted Running.—It may be gathered from what has been previously said that a rotary converter can be run under inverted conditions, that is, it is electrically

reversible, and can be driven from a direct current main to supply alternating current.

If the converter is driven from a direct current supply it functions in the same way as a D.C. shunt motor, as the excitation is connected across the brushes of the machine. Thus a weakening of the field produces an increase of speed and conversely. But the direct and alternating E.M.Fs. have a fixed relation, and hence a variation in speed under inverted conditions will theoretically only affect the frequency of supply on the A.C. side, and not the A.C. voltage. This is not strictly true as the armature reaction has some effect on the A.C. voltage, but to all intents and purposes it will function in the same way as does an A.C. generator.

A rotary converter is a valuable piece of laboratory apparatus as a source of variable frequency, but it is advisable to guard against an accidental weakening of the field and consequent high rotor speeds. In practice on large machines an automatic cut-out is often mounted on the shaft which operates on the main circuit breaker and controls the supply in cases of excessive speeds.

Rotary Converter—Direct Running.—When a rotary is operated from the A.C. side a totally different set of conditions appertains: the machine is now functioning as a synchronous motor and has all the characteristics of such a motor. Thus it will only continue to run satisfactorily at a synchronous speed, and if the load is increased beyond a point it will fall out of step, and the high currents in the stator will inevitably cause damage unless the supply is controlled.

A rotary converter, however, possesses the valuable property of the synchronous motor with regard to power factor improvement, which can be effected without difficulty. On the other hand, this asset is to a certain extent nullified by a corresponding increase in the rating of the machine, which can be imagined by considering the curve A in Fig. 48 being moved to the right or left relatively to the curve D. The additive effect of the two currents I_A and I_D will result in an increased armature loss which is clearly indicated in Fig. 51 where the copper

FIG. 53.—Substation with two G.E.C. rotary converters.

[*To face page* 97.

loss in the armature is plotted against the power factor. For a fuller description as to the way by which this is effected the reader is referred to the works mentioned in the bibliography, but it may not be out of place to include the characteristic curves or "V" curves of a synchronous motor. These characteristics (Fig. 52) apply equally to a rotary converter and show that with constant load and varying excitation the current input attains high values for low excitation which decrease to a minimum value and increase with increased excitation. The curves are almost symmetrical about a line curving to the right-hand side of the figure; and this line of minimum current input is

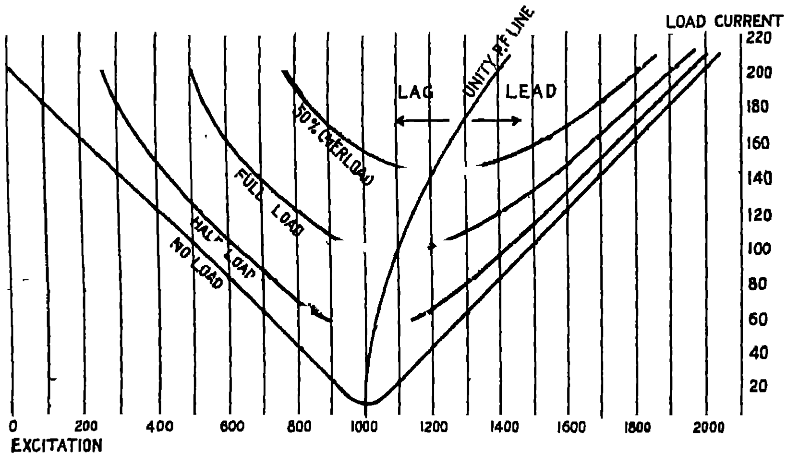

FIG. 52.—"V" curves of a synchronous motor.

also the line of unity power factor. For any one particular loading, excitation to the left of the line will produce a lagging, and to the right of the line a leading power factor. Thus a variation of excitation during direct running will not produce a voltage ratio variation, but the power factor only will be affected, and by these means correction for other inductive loads on the system may be obtained.

Starting of Rotary Converters.—As a rotary converter has the characteristics of a synchronous motor, it follows that if the machine is operating from the A.C. side it can only be started if

special means are adopted to preclude heavy currents being taken from the mains. In some cases a separate D.C. motor is mounted on the shaft, but this is only practicable when a D.C. supply is available. An alternative is to run up to synchronous speed using a small induction motor mounted in a similar fashion; but the simplest and best way is to specify a machine which is self-starting as an induction motor and which can be synchronised without difficulty. Such rotaries are quite common and result in a considerable saving in capital expenditure. If a large D.C. supply is available the obvious course to adopt is to start up from the D.C. side, using an ordinary starting resistance and rheostat.

Motor Converter.—A specialised form of rotary converter which is in some ways preferable to the latter is the la Cour type of motor converter.

This machine is in principle a rotary converter running in concatination with an induction motor rotor on the same shaft. The induction motor operates at one-half synchronous speed, and the stator only is connected to the supply. The rotor which is connected mechanically with the armature of the rotary converter is usually divided into twelve phases, which may be considered as the supply to the rotary in the usual way.

Fig. 54 indicates the arrangement. One-half of the input energy to the motor is converted into electrical energy in the armature, and the other half into mechanical energy which is absorbed in rotating the D.C. armature

The advantages of this type of converter are considerable: it will start up as an induction motor without any difficulty by the insertion of a three-phase resistance in the rotor. The only slip rings are those required for this resistance, and they may be short-circuited during actual running. As the leads from the rotor to the armature rotate with the rotor, the advantage of the effect of the greater number of phases on the increased efficiency can be employed. Further, as the driving unit is an induction motor which is quite separate from the converter the supply voltage can be varied within wide limits without effect on the D.C. generated voltage. The complete unit can be

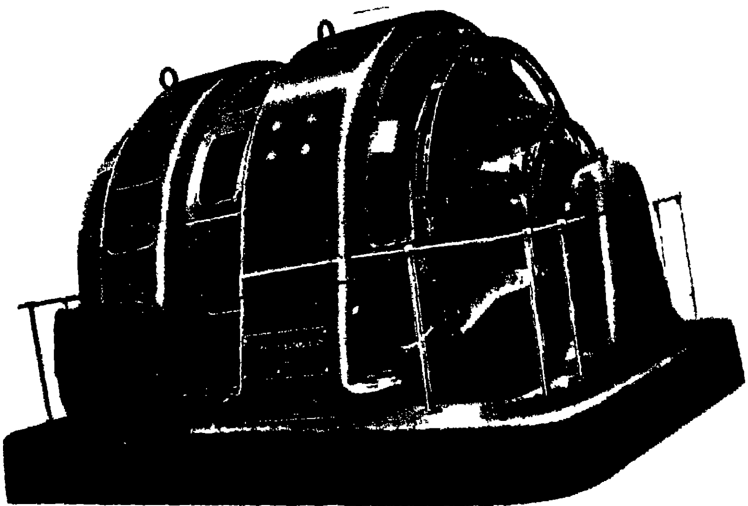

FIG. 55.—1000 K.W. motor converter.

[To face page 99.

operated direct from a 6600 volt supply and generate at 110 volts without any transformation plant.

It also lends itself to the supply of a three-wire system as shown in the illustration. By means of the triple-pole change-over switch, and after the slip rings have been short-circuited, the middle wire is connected to all three rings, when automatic

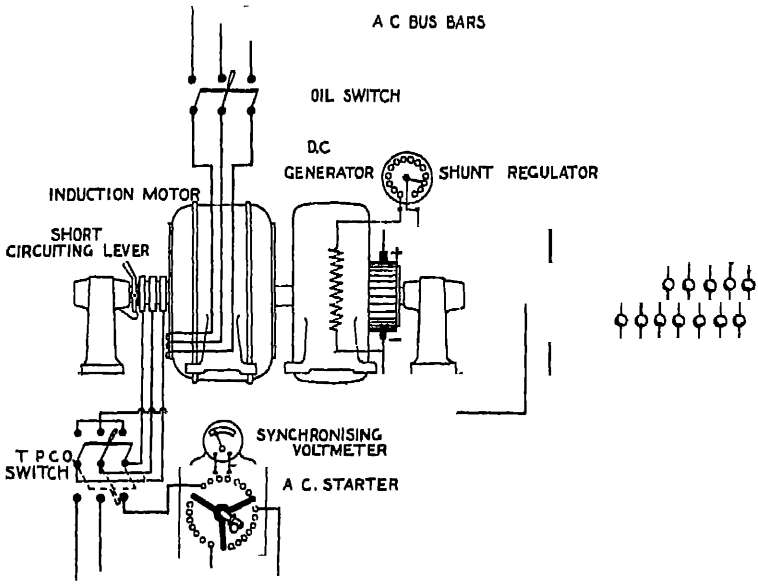

FIG. 54.—Diagrammatic sketch of motor converter.

balancing is obtained. There is little fear of a flash-over on short-circuit or heavy loads, but no power-factor control is possible with this type of machine.

A 1000 K.W. motor converter manufactured by the General Electric Co., supplying direct current at 480-540 volts at 500 R.P.M., is illustrated in Fig. 55.

CHAPTER V.

MECHANICAL RECTIFIERS (*cont.*).

SYNCHRONOUS COMMUTATOR RECTIFIERS. FUNDAMENTAL CONSIDERATIONS.

General.—The synchronous commutator rectifier has so far not had an extended use, in part due, no doubt, to the difficulties attending satisfactory commutation, which in all probability have been over-emphasised in this particular design. The evils attending this essential part of direct current machinery have been mentioned, and these difficulties are incapable theoretically of real solution unless some extraneous methods, such as the use of interpoles, are employed; the cause of the trouble is in the inductance which the coil under commutation possesses; hence it might be considered that if this inductance could be eliminated, perfect commutation would result. This is in part true, but in the case of the commutator rectifier there is no inductive coil to be short-circuited during the period of rectification, excepting that of the supply transformer, if such there be. The assumption, therefore, that commutation should be sparkless, is theoretically true with the reservation that this can only be achieved at one definite current loading.

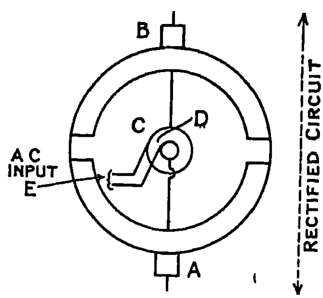

FIG. 56.—Two-part commutator.

Consider the case of a two-part commutator as shown in Fig. 56, where the supply E, alternating in character, is connected to a commutator through two slip rings and thence

to brushes C and D. If the disc rotates at synchronous speed, brushes A and B will supply a unidirectional current to the load, and the rectified current and voltage wave will, under these circumstances, be similar to the curves in Fig. 57. But if the load is inductive or if a separate inductance is inserted in the rectified circuit, the current wave will take the form of that in Fig. 58. The supply current wave to the rectifier must be so constituted that on reversal it maintains a

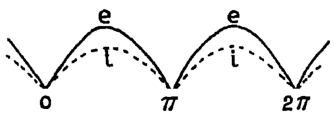

FIG. 57.—Rectified current and voltage with no inductance.

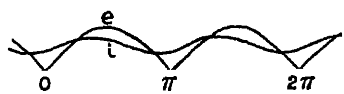

FIG. 58.—Rectified current with smoothing inductance.

wave form similar to it, but reversed at a predetermined point. It must therefore take the shape depicted in Fig. 59, and must of necessity be discontinuous in form in this particular case, as it has to jump instantaneously from a positive to a negative value. It has been postulated that when inductance is present, an abrupt current change is impossible, and the current wave will therefore assume a more or less smooth shape, as shown in full line in Fig. 59. It follows that the current in the rectified

FIG. 59.—Supply current wave to rectifier with inductance.

FIG. 60.—Inductive current in commutator.

circuit and in the A.C. circuit cannot be the same, the difference being commensurate with the curve in Fig. 60. This current exists as a short-circuit arc on the commutator unless special means are provided to absorb it in some other way. Such means are available, and in some cases resistances are inserted in the A.C. and rectified circuits, so calculated as to absorb this excess current adequately. It is apparent that a resistance can only absorb the current correctly at one particular load, and moreover that the efficiency will be impaired by the

rheostatic losses The fact, however, must not be lost sight of, that perfect commutation is possible in a commutator rectifier, even in the case of an inductive load, and if arrangements can be made to vary the short-circuit resistance for any particular load, the synchronous commutator will have advantages which will render it a very serious competitor of the more usual types of converter.

Open and Short-circuit Types.—There are two forms of synchronous rectifier which can be designed, and which operate on different principles :—

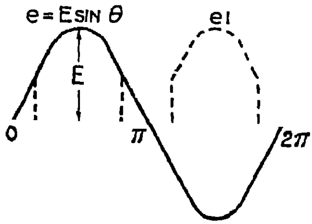

FIG. 61.—Supply and rectified voltage in short-circuit rectifier.

(i) the Short-circuit type, in which the brushes are wide enough to connect together any two adjacent commutator segments, and

(ii) the Open-circuit type, where the segments are never interconnected.

The late Prof C. P. Steinmetz has investigated the wave forms resulting in each case, and certain of his investigations are given.

Short-circuit Rectifier.—The voltage of supply $e = E \sin \theta$ and the rectified voltage e_1 are shown in Fig. 61 where the dotted lines represent the E.M.F. of the unidirectional current.

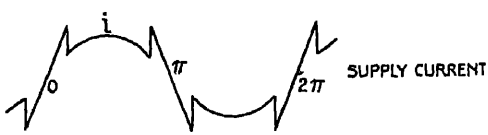

FIG. 62.—Supply current to short-circuit rectifier.

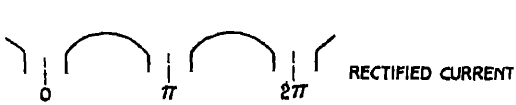

FIG. 63.—Rectified current with non-inductive load.

The supply and rectified currents on a non-inductive load are indicated in Figs. 62 and 63.

If the load is inductive the current wave is distorted as shown in Figs. 64 and 65.

It will be noticed from these curves that inductance in the D.C. circuit causes the current to die away slowly—as would be expected, but further that it results in a smoothing of the wave form rather than in any movement of the current wave relatively to the voltage wave, as in ordinary A.C. circuits. Further, in this case at the moment of reversal the alternating

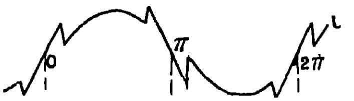

Fig. 64.—Supply current with inductive rectified load; short-circuit type.

Fig. 65.—Rectified current with inductive load; short-circuit type.

supply current has reached a higher value than the direct current, and it is this excess current which it is necessary to absorb if commutation is to be sparkless.

In the case of Fig. 66, where the inductance has the correct value, it has been possible to design a resistance to absorb the arcing current completely, and stable conditions ensue. i represents the supply current and i_1 the rectified current.

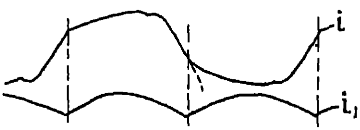

Fig. 66.—Short-circuit rectifier; supply and direct current with stable conditions.

Open-circuit Rectifier.—With this class of commutator the current and voltage curves are similar to those of the short-circuit type on a non-inductive load; but where inductance is present the current wave takes the form of Fig. 67. The cause of the arcing in this case, the position and amount of which is shown by the shaded area, is due to the interruption of the current whilst still considerable in value, and also because of the inductive circuit. In this case there is no correct cure; but a compromise is possible—similar in effect to

the use of interpoles in a D.C. generator—viz. a shifting of the brushes in the correct direction will result in the current being broken at a time when the voltage has reversed sufficiently to

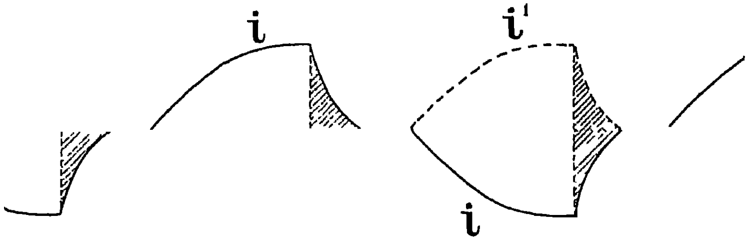

FIG. 67.—Open-circuit rectifier; inductive load.

cause a small reverse current to flow which is nearly enough to neutralise the decreasing supply current.

From what has been seen of the two cases it will be observed that difficulties attend the design of all commutator rectifiers; but the experience of the open-circuit type would lead one to believe that the shifting of the brushes would result in improved conditions in the short-circuit rectifier; and in practice it is found that a brush lead, together with a variation in the duration of the short-circuit tends to beneficial results.

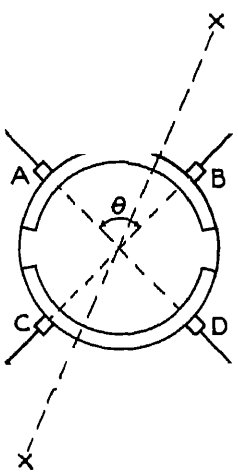

FIG. 68.—Commutator with brush shifting gear.

The way in which the duration of the short-circuit is effected is ingenious: two or, in some cases, three brushes are used, and are so connected electrically that they can be shifted independently of each other. In Fig. 68 the brushes A and B are shorted, as are also C and D, and the effective angular brush width is equal to the angle θ.

To give effect to the angular shift, assume that the zero line

is XX relatively to the synchronous driving motor, then the following will represent the cycle of events (Fig. 69):—

0 to θ_1 period of short circuit or rectification.

θ_1 to $\pi - \theta_2$ current flowing from A.C. to D.C. circuit.

$\pi - \theta_2$ to $\pi + \theta_1$ period of rectification.

A model made of cardboard and similar to the arrangement of Fig. 69 will make the connections, during the revolution of the commutator, clear.

In Fig. 68 the brushes are shown directly connected, and during certain portions of the cycle, heavy local currents will flow. In the ensuing analysis the direct connection will be substituted by a series of resistances which will effectively curtail these currents.

Short-circuit Rectifier with Control Resistances.—In Fig. 69 assume that the six brushes A, B, C, D, E, and F are so disposed that A, B, and C form one pole and D, E, and F the other; and that between A and B, and C and B resistances r' and r'' are connected, and r''' and r'''' similarly between D and E, and F and E. The A.C. supply is then connected, by means of slip rings G and H, to the metal segments.

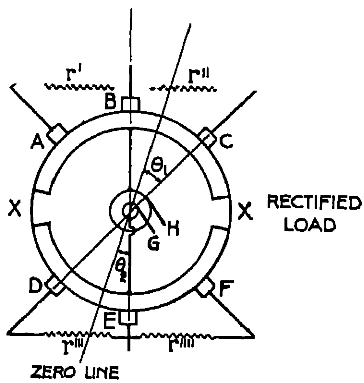

FIG. 69.—Commutator with brush gear and shorting resistances.

In the position shown, i.e. when the D.C. and A.C. sides are connected to one another, the brushes B and E will pick up the greater part of the current, and that supplied from the others can be neglected.

But in the period when B and E are midway between the gaps in the segments, viz. at XX, the A.C. supply is short-circuited by a resistance

$$r_1 = \frac{1}{\frac{1}{r' + r''} + \frac{1}{r''' + r''''}}$$

and the rectifier circuit by a resistance

$$r_2 = \frac{1}{\frac{1}{r' + r'''} + \frac{1}{r'' + r''''}}.$$

Hence, by suitably choosing r', r'', r''' and r'''', r_1 and r_2 may be made to assume any value to suit the conditions of sparkless commutation.

Theoretical Analysis.—Assume that r_0, x_0, and r and x are the resistance and inductance of the A.C. and D.C. circuits respectively; further, that E is the maximum value of the A.C. supply voltage, and e is the counter E.M.F. in the D.C. circuit. Then the following equations apply, where i_1 is the through current in the rectified circuit, and i_2 and i_3 the currents in the rectified and A.C. circuits during periods of short-circuit respectively:—

Period when A.C. and D.C. Circuits are Interconnected.

$$E \sin \theta - e - i_1(r + r_0) - (x + x_0)\frac{di_1}{d\theta} = 0. \qquad (1)$$

A solution of equation (1) is of the form

$$i_1 = a + \beta\epsilon^{-a\theta} + \gamma \sin (\theta - \lambda)$$

and when the constants a, β, γ, a and λ are eliminated equation (1) becomes

$$i_1 = -\frac{e}{r + r^0} + A_1\epsilon^{-\frac{r+r_0}{x+x_0}\theta} + \frac{E\{(r + r_0)\sin\theta - (x + x_0)\cos\theta\}}{(r + r_0)^2 + (x + x_0)^2} \qquad (2)$$

This, then, is the state of affairs during the period θ_1 to $\pi - \theta_2$.

Period when the A.C. and D.C. Circuits are Short-circuited. D.C. Circuit. Period $\pi - \theta_2$ to $\pi + \theta_1$.

$$-e - i_2(r + r_2) - x\frac{di_2}{d\theta} = 0 \qquad (3)$$

and the solution of (3) is found to be

$$i_2 = -\frac{e}{r + r_2} + A_2\epsilon^{-\frac{r+r_2}{x}\theta} \qquad (4)$$

A.C. Circuit.

$$E \sin\theta - i_3(r_0 + r_1) - x_0\frac{di_3}{d\theta} = 0 \qquad . \qquad . \qquad (5)$$

Integrating this in the same way as equations (1) and (4) the solution is

$$i_3 = A_3\epsilon^{-\frac{r_0+r_1}{x_0}\theta} + \frac{E\{(r_0 + r_1)\sin\theta - x_0\cos\theta\}}{(r_0 + r_1)^2 + x_0^{\,2}} \qquad . \qquad (6)$$

The equations (2), (4), and (6) then represent the instantaneous currents in the two circuits under the two sets of conditions. There are other conditions, however, which represent a state of sparkless commutation: at the position represented by $\pi - \theta_2$ the currents must be equal, and at $\pi + \theta_1$, $i_2 = i_1$ at θ_1, and is equal and opposite to i_3 at $\pi + \theta_1$.

These conditions may be written in the form

$$i_1(\theta = \pi - \theta_2) = i_2(\theta = \pi - \theta_2) = i_3(\theta = \pi - \theta_2)$$

and
$$i_1(\theta = \theta_1) = i_2(\theta = \pi + \theta_1) = -\, i_3(\theta = \pi + \theta_1).$$

The four equations connecting the circuit constants are thus obtained, viz. :—

$$-\frac{e}{r+r_0} + A_1\epsilon^{-\frac{r+r_0}{x+x_0}(\pi-\theta_2)} + \frac{E\{(r + r_0)\sin\theta_2 + (x + x_0)\cos\theta_2\}}{(r + r_0)^2 + (x + x_0)^2}$$

$$= -\frac{e}{r + r_2} + A_2\epsilon^{-\frac{r+r_2}{x}(\pi-\theta_2)} = A_3\epsilon^{-\frac{r_0+r_1}{x_0}(\pi-\theta_2)}$$

$$+ \frac{E\{(r_0+r_1)\sin\theta_2 + x_0\cos\theta_2\}}{(r_0 + r_1)^2 + x_0^{\,2}} \qquad (7)$$

and

$$-\frac{e}{r + r_0} + A_1\epsilon^{-\frac{r+r_0}{x+x_0}\theta_1} + \frac{E\{(r + r_0)\sin\theta_1 - (x + x_0)\cos\theta_1\}}{(r + r_0)^2 + (x + x_0)^2}$$

$$= -\frac{e}{r + r_2} + A_2\epsilon^{-\frac{r+r_2}{x}(\pi+\theta_1)} = -\,A_3\epsilon^{-\frac{r_0+r_1}{x_0}(\pi+\theta_1)}$$

$$+ \frac{E\{(r_0 + r_1)\sin\theta_1 - x_0\cos\theta_1\}}{(r_0 + r_1)^2 + x_0^{\,2}} \qquad (8)$$

The following quantities have to be obtained from these four equations (7) and (8), viz. :—

$$e,\ E,\ r,\ r_0,\ x,\ x_0,\ r_1,\ r_2,\ \theta_1,\ \theta_2,\ \mathrm{A}_1,\ \mathrm{A}_2,\ \text{and}\ \mathrm{A}_3,$$

and it is therefore necessary to assume values for nine of them before the equations can be solved.

If the counter E.M.F. consists of a battery under charge there is a further equation which may be evolved to provide a greater latitude in the choice of constants. For any given size of battery the best charging current is a fixed quantity. Call this value I_M, then the condition is that the mean value of i shall be equal to I_M.

From equation (2)

$$i_1 = -\frac{e}{r + r_0} + A_1 \epsilon^{-\frac{r + r_0}{x + x_0}\theta} + \frac{E\{(r + r_0)\sin\theta - (x + x_0)\cos\theta\}}{(r + r_0)^2 + (x + x_0)^2}$$

and the condition is that the mean value of i_1 during the period θ_1 to $\pi - \theta_2$ less the mean value of i_2 during the period $\pi - \theta_2$ to $\pi + \theta_1$ (since during this latter period the battery is being discharged) equals I_M, and hence

$$I_M = \frac{1}{\pi - \theta_1 - \theta_2}\int_{\theta_1}^{\pi - \theta_2} i_1 d\theta - \frac{1}{\theta_1 + \theta_2}\int_{\pi - \theta_2}^{\pi + \theta_1} i_2 d\theta \quad . \quad . \quad (9)$$

These five equations, (7), (8), and (9), are the characteristic equations for a mechanical rectifier of this description, and from these general equations the particular cases of the various circuits can be calculated. The first case to be considered is that of accumulator charging by synchronous rectifiers.

Theoretical Analysis—Battery Charging.—If an ordinary alternating current supply from a transformer be used for this purpose with no inductance in circuit, the voltage and current may for all practical purposes be taken to be in phase, and the counter E.M.F. constant (this latter condition is not strictly true, as the battery E.M.F. increases as the charge proceeds). These conditions result in zero values for r_0 and x_0.

Further, it will be shown that if an inductance is inserted in the secondary side, that is, if x has any appreciable value, the current equations generally become impossible of solution. This may be readily demonstrated—assume that the circuit constants have the following values :—

$$e = 110, \; r = 5, \; x = 10 \text{ and } r_2 = 55,$$

then equating the current equations as has been done above

$$-\frac{110}{5} + A_1\epsilon^{-0\cdot5(\pi-\theta_2)} + \frac{E\{5\sin\theta_2 + 10\cos\theta_2\}}{125}$$

$$= -\frac{110}{60} + A_2\epsilon^{-\frac{60}{10}(\pi-\theta_2)} = \frac{E\sin\theta_2}{r_1} \quad . \quad . \quad (10)$$

for the conditions $i_1 = i_2 = i_3$ when $\theta = \pi - \theta_2$.

When

$$i_1\,(\theta = \theta_1) = i_2\,(\theta = \pi + \theta_1) = -\,i_3\,(\theta = \pi + \theta_1)$$

then

$$-\frac{110}{50} + A_1\epsilon^{-0\cdot5\theta_1} + \frac{E\{5\sin\theta_1 - 10\cos\theta_1\}}{125}$$

$$= -\frac{110}{60} + A_2\epsilon^{-\frac{60}{10}(\pi+\theta_1)} = \frac{E\sin\theta_1}{r_1} \quad . \quad . \quad (11)$$

and from the second equation of (10) and (11) it is obvious that θ_1 and θ_2 cannot have identical values. Thus the point of cut-off of the battery cannot be the same as the point of cut-in and sparking will ensue. If the equations are solved with the values assigned it will in general be found to be impossible to arrive at a positive value for both the resistances r_1 and r_2 in every case. It can therefore be assumed that the equations are meaningless if an inductance is present in either the secondary side of the transformer or in the rectifier circuit. The practical view of this result is that sparkless commutation is impossible in battery charging where any inductance is present, and it is therefore necessary to reduce it to a minimum value. This is also true in the general case where any constant counter E.M.F. is in the load circuit.

In the ensuing calculations, therefore, x is assumed to have a zero value and the equations contract to the following:—

$$i_1 = -\frac{e}{r} + \frac{E\sin\theta}{r}$$

$$i_2 = -\frac{e}{r + r_2}$$

$$i_3 = \frac{E\sin\theta}{r_1}.$$

Inserting the terminal conditions for the points of cut-in and cut-off

$$-\frac{e}{r}+\frac{E\sin\theta_2}{r}=-\frac{e}{r+r_2}=\frac{E\sin\theta_2}{r_1}$$

when $\qquad i_1 = i_2 = i_3 \quad \text{for} \quad \theta = \pi - \theta_2$

and

$$-\frac{e}{r}+\frac{E\sin\theta_1}{r}=-\frac{e}{r+r_2}=\frac{E\sin\theta_1}{r_1}$$

when $\quad i_1\,(\theta = \theta_1) = i_2\,(\theta = \pi + \theta_1) = -\,i_3\,(\theta = \pi + \theta_1).$

It follows that $\theta_1 = \theta_2$ and the four equations become

$$-\frac{e}{r}+\frac{E\sin\theta_1}{r}=-\frac{e}{r+r_2}=\frac{E\sin\theta_1}{r_1} \quad . \quad . \quad (12)$$

The condition that the charging current must be within certain limits is given by

$$I_M = -\frac{e}{r}+\frac{2E\cos\theta_1}{r(\pi-2\theta_1)}.$$

Eliminating E from these equations

$$E\sin\theta_1 = er\left(\frac{1}{r}-\frac{1}{r+r_2}\right)=\frac{er_2}{r+r_2}$$

and $$E\cos\theta_1 = \frac{(I_M r+e)(\pi-2\theta_1)}{2}$$

whence

$$\tan\theta_1 = \frac{2er_2}{(r+r_2)(I_M r+e)(\pi-2\theta_1)} \quad . \quad (13)$$

If this equation is solved for θ_1 it will be found that for θ_1 to have a value between 20° and 30°

$$\frac{2er_2}{(r+r_2)(I_M r+e)} \text{ must equal unity,}$$

which results in a quadratic for r,
and finally

$$r = \frac{\sqrt{I_M^2 r_2^2 + e^2 + 6I_M r_2 e} - (I_M r_2 + e)}{2I_M} \quad . \quad . \quad (14)$$

Thus (assuming $\theta_1 = 20°$ to 30°) the resistance of the secondary circuit is completely dependent on three factors, viz.

(i) the number of cells (that is on e), (ii) on the charging current (that is on I_M), and on the shunt resistance r_2. Now the charging current I_M will to a certain extent determine the size of the battery, and on this factor will the amount of current which can be allowed to flow through the short-circuiting resistance depend.

As an example let one-ninth the charging current flow through the short-circuiting resistance r_2, and

$$\frac{I_M}{9} = \frac{e}{r_2}.$$

Equation (14) then reduces to

$$r = 0{\cdot}0925 r_2.$$

Filling in this value in equation (12)

$$E \sin \theta_1 = \frac{e}{1{\cdot}0925}.$$

It has been stipulated that

$$\tan \theta_1 = \frac{1}{\pi - 2\theta_1} \text{ which gives}$$

$$\theta_1 = \text{approximately } 23^\circ$$

and hence $$E = 2{\cdot}35e.$$

It is now possible to ascertain the current curves for any particular case, and accordingly the following size of battery will be considered :—

The counter E.M.F. $e = 110$ (i.e. 55 cells) with a charging current of 18 amperes. Then

$$E = 2{\cdot}35 \times e = 258 \text{ volts (maximum)}$$

$$= 183 \text{ volts effective}$$

and $$r_2 = \frac{9e}{I_M} = 55 \text{ ohms}$$

and $$r = 0{\cdot}0925, \quad r_2 = 5{\cdot}1 \text{ ohms.}$$

Thus $$i_1 = \frac{(258 \sin \theta - 110)}{5{\cdot}1}$$

$$= 50{\cdot}7 \sin \theta - 21{\cdot}6$$

$$i_2 = -\frac{110}{5 + 55} = -1{\cdot}83 \text{ amperes.}$$

These equations may be tabulated as in Table XII. and the current curve will be as shown in the accompanying Fig. 70.

TABLE XII.

θ.	0.	23.	30.	60.	90.	120.	150.	157.	180.	203.
i_1	—	−1·7	8·8	43·9	50·7	43·9	8·8	−1·7	—	—
i_2	−1·8	—	—	—	—	—	—	−1·8	−1·8	−1·8

The average value of the current is seen to be 18·3 amperes which agrees with the figure allowed.

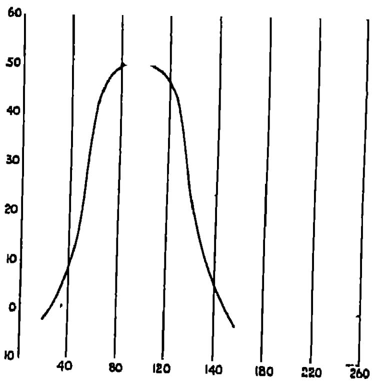

FIG. 70 —Battery charging current wave.

If no short-circuiting resistance r_2 is required then

$$\frac{2er_2}{(r + r_2)(I_M r + e)} = 1$$

where $r_2 = \infty$

and hence $2e = I_M r + e$, or $e = I_M r$.

Also $e = E \sin \theta_1$

and $$i_1 = \frac{I_M E \sin \theta}{e} - I_M$$

and in the above case

$$E = 281 \text{ volts}, \quad r = 6{\cdot}12 \text{ ohms}, \quad \text{and } i_2 = 0.$$

As before $\theta_1 = 23°$

and $i_1 = 46 \sin \theta - 18.$

Rectifier supplying an Inductive Circuit.—When the counter E.M.F. is zero and the load consists of an inductive circuit, the case is somewhat different as both r and x must be taken into consideration. As before $r_0 = x_0 = 0$.

The equations for the current wave then become

$$i_1 = A_1\epsilon^{-\frac{r}{x}\theta} + \frac{E(r \sin \theta - x \cos \theta)}{r^2 + x^2}$$

$$i_2 = A_2\epsilon^{-\frac{r+r_2}{x}\theta}$$

and $$i_3 = \frac{E \sin \theta}{r_1}.$$

Cut-in and cut-off conditions give rise to the equations

$$A_1\epsilon^{-\frac{r}{x}(\pi - \theta_2)} + \frac{E(r \sin \theta_2 + x \cos \theta_2)}{r^2 + x^2} = A_2\epsilon^{-\frac{r+r_2}{x}(\pi + \theta_2)}$$
$$= E \sin \theta_2/r_1$$

for the period when $\theta = \pi - \theta_2$, and $i_1 = i_2 = i_3$.

And further

$$A_1\epsilon^{-\frac{r}{x}\theta_1} + \frac{E(r \sin \theta_1 - x \cos \theta_1)}{r^2 + x^2} = A_2\epsilon^{-\frac{r+r_2}{x}(\pi \ \theta_1)}$$
$$= E \sin \theta_1/r_1$$

when $i_1(\theta = \theta_1) = i_2(\theta = \pi + \theta_1) = -i_3(\theta = \pi + \theta_1)$.

In these equations there are nine unknown quantities, and as four equations are available, five of these unknowns must be chosen.

To take a numerical example, choose the following values for the five constants

$E = 141$ volts, $\theta_2 = \frac{\pi}{6}$, $r = 10$ ohms, $x = 20$ and $r_2 = 5$ ohms.

Filling in the values in the equations above

$$0\,270\, A_1 + 6{\cdot}30 = 0{\cdot}141\, A_2 = \frac{70{\cdot}05}{r_1}$$

and $$A_1\epsilon^{-0\cdot5\theta_1} + 2\cdot82 \sin\theta_1 - 5\cdot64 \cos\theta_1 = A_2\epsilon^{-0\cdot75(\pi+\theta_1)} = \frac{141 \sin\theta_1}{r_1}.$$

Eliminating A_1 and A_2

$$e^{-0\cdot75(\pi+\theta_1)} = 0\cdot283 \sin\theta_1$$

whence by trial θ_1 is found to be 16°.

From this

$$A_2 = 66\cdot7, \quad A_1 = 11\cdot5, \quad \text{and } r_1 = 7\cdot46 \text{ ohms.}$$

The equations for the instantaneous values of the currents then become

$$i_1 = 11\cdot5\epsilon^{-0\cdot5\theta} + 2\cdot82 \sin\theta - 5\cdot64 \cos\theta$$
$$i_2 = 66\cdot7\epsilon^{-0\cdot75\theta}$$
$$i_3 = 18\cdot9 \sin\theta$$

which may be set out in tabular form

TABLE XIII.

$\theta°$	16.	30.	60.	90.	120.	150.	180.	196.
i_1	5·36	5·38	6·48	8·07	9·29	9·39		
i_2						9·42	6·33	5·06
i_3						9·45	0	− 9·45
i_1'	3·90	7·05	12·2	14·1	12·2	7·05	0	

i'_1 is the equivalent sine wave with no inductance corresponding to a condition of $x = 0$ whence

$$i'_1 = \frac{E \sin\theta}{r} = 14\cdot1 \sin\theta.$$

These values are plotted in Fig. 71 and it will be seen, comparing i_1 and i_2 with i'_1, that the effect of the inductance is to tend to smooth out the inequalities of the wave rather than to shift the current wave relatively to the space and time scale, as is usually the case in A.C. circuits.

The mean value of i_1 and i_2 is approximately 7·5 amperes as against the effective value of 7·63 amperes—giving a form factor of 1·04, which compares with the sinusoidal factor of 1·11.

It should be explained that as i_2 is the current in the D.C. side during the period of short-circuit, and the conditions stated

that at the time represented by $\pi - \theta_2$, $i_1 = i_2$, the continuity of the two curves forms a check on the arithmetic of the calculations.

Constant Current System.—The above cases apply only to a condition of constant potential. If the supply is at a constant current, different conditions prevail and new equations must be evolved.

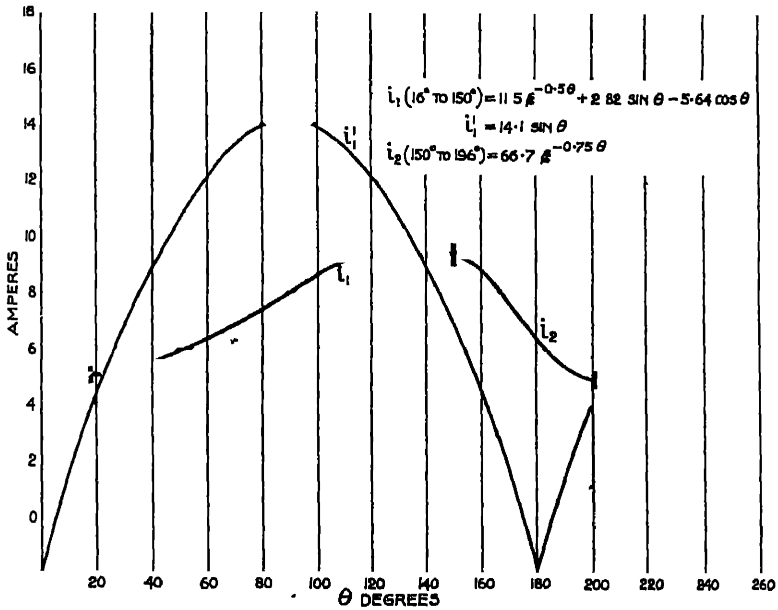

FIG. 71.—Constant potential rectifier with inductance in D.C. circuit.

As before a resistance r_2 short-circuits the rectified circuit during the period of rectification, but this resistance is not disconnected when the A.C. and D.C. sides are electrically joined to one another.

Let i_1 be the instantaneous current in the rectifier and i_2 the current in the resistance; and further suppose the supply current is of the form

$$i_3 = I \sin \theta.$$

Then the differential equations connecting the various quantities are

$$i_1 + i_2 = I \sin \theta$$

and
$$i_2 r_2 = i_1 r + x \frac{di_1}{d\theta},$$

and from these two equations the instantaneous current in the rectifier circuit may be obtained.

Eliminating i_2

$$x \frac{di_1}{d\theta} + i_1 (r + r_2) - Ir_2 \sin \theta = 0.$$

The solution of this equation is of the form

$$i_1 = A_1 \epsilon^{-\frac{r+r_2}{x}\theta} + \frac{r_2 I\{(r + r_2) \sin \theta - x \cos \theta\}}{(r + r_2)^2 + x^2}$$

In the period of short-circuit

$$i_2 r + x\frac{di_2}{d\theta} = 0.$$

Whence

$$i_2 = A_2 \epsilon^{-\frac{r}{x}\theta}.$$

At the time instant represented by $\theta = \pi - \theta_2$

$$i_1 = i_2 = I \sin \theta$$

and also

$$i_1(\theta = \theta_1) = i_2(\theta = \pi + \theta_1) = I \sin \theta \; (\theta = \theta_1)$$

from which the four characteristic equations may be obtained. These are

$$A_1 \epsilon^{-\frac{r + r_2}{x}(\pi - \theta_2)} + \frac{Ir_2\{(r + r_2) \sin \theta_2 + x \cos \theta_2\}}{(r + r_2)^2 + x^2}$$

$$= A_2 \epsilon^{-\frac{r}{x}(\pi - \theta_2)} = I \sin \theta_2 \quad . \quad . \quad . \quad (15)$$

and

$$A_1 \epsilon^{-\frac{r + r_2}{x}\theta_1} + \frac{Ir_2\{(r + r_2) \sin \theta_1 - x \cos \theta_1\}}{(r + r_2)^2 + x^2}$$

$$= A_2 \epsilon^{-\frac{r}{x}(\pi + \theta_1)} = I \sin \theta_1 \quad . \quad . \quad . \quad (16)$$

It is interesting to take an actual case, and the following values may therefore be assigned:—

$$I = 50 \text{ amperes}, \theta_2 = \frac{\pi}{6}, r = 5 \text{ ohms and } x = 10$$

from which

$$92{\cdot}5\epsilon^{-0{\cdot}5(\pi+\theta_1)} = 50 \sin \theta_1$$

and by drawing the curves for

$$92{\cdot}5\epsilon^{-0{\cdot}5(\pi+\theta_1)} \text{ and } 50 \sin \theta_1$$

the point of intersection is found to exist at $\theta_1 = 19°$.

Filling in this value, the equation for r_2 will be

$$\epsilon^{-1{\cdot}144-0{\cdot}220r_2} = \frac{10{\cdot}15 - r_2}{6{\cdot}60 + 1{\cdot}80r_2},$$

and by again drawing two curves for various values of r_2, 9·25 ohms will equate the two sides.

Eliminating θ_1 and r_2 $\qquad A_1 = 37{\cdot}0$.

The equations are then solved and the three currents are represented by

$$i_1 = 37\epsilon^{-1{\cdot}425\theta} + 21{\cdot}65 \sin \theta - 15{\cdot}4 \cos \theta$$
$$i_2 = 92{\cdot}5\epsilon^{-0{\cdot}5\theta}$$
$$i_3 = 50 \sin \theta.$$

Tabulated for various values of θ the results given in the table are obtained.

TABLE XIV.

$\theta°$.	19.	30.	60.	90.	120.	150	180	199.
i_1	15·6	15·0	19·4	25·5	28·36	25 05	—	—
i_2	—	—	—	—	—	25 00	19·8	16·3
i_3	—	—	—	—	—	25·00	0	− 16·3

Fig. 72 illustrates the current wave in this case.

As before a check on the accuracy of the arithmetical working is given by the result, first of all at the angle $\pi - \theta_2$ (in this case 150°) and at this point $i_1 = i_2 = i_3$; then again at $\pi + \theta_1$ (= 199°), i_2 should equal i_1 at θ_1 (= 19°) and i_2 at $(\pi + \theta_1) = - i_3$ at $(\pi + \theta_1)$.

It will be seen from the above table that this is not strictly true, but as in all of these examples at some point in the calculations, a small difference between two large quantities is involved, and also the exponential of a large quantity, if extreme accuracy is required, the working should be conducted to four or five places of decimals which involves laborious calculation. These results have been obtained with the aid of a 10-inch slide rule, and indicate what may be expected in the way of accuracy, viz. in this case 3 to 4 per cent.

$$i_1(19^\circ - 150^\circ) = 37\,\epsilon^{-1\cdot425\,\theta} + 21\cdot65 \sin\theta - 15\cdot4 \cos\theta$$
$$i_2(150^\circ - 199^\circ) = 92\cdot5\,\epsilon^{-\cdot5\,\theta}$$

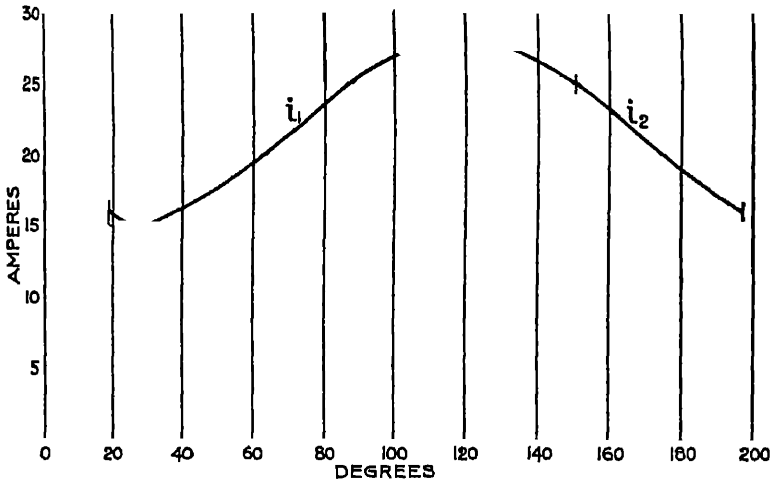

FIG. 72.—Constant current rectifier—wave form.

It is important, if the rectifier is required to supply a current in which the maximum and minimum limits do not differ greatly from the mean, that the rotor should be accurately adjusted relatively to the brush gear, i.e. that the angles θ_1 and θ_2 should be properly marked off and registered against the central or zero position. In a three-phase supply this can be accomplished by disconnecting one phase, and passing a small current through the remaining phase, which will ensure the rotor coming to a standstill at the midway position between the poles. This

position will enable the zero line XX in Fig. 69 to be registered, and θ_1 and θ_2 can be marked off on each side of this line.

In some manufactures of synchronous commutators a special timing gear is mounted on the shaft enabling the angle of advance or lead to be adjusted whilst the machine is in operation.

Transverter.—"Referring to Fig. 46, the diagrammatic representation of a Gramme ring, the usual arrangement is for the armature, containing the regularly spaced winding which is tapped at intervals and connected to its commutator in the case of a D.C. machine, to rotate between the field poles, and stationary brushes collect the continuous current. It will be apparent that the machine can be inverted, in which case the armature windings and commutator would be stationary, and the magnetic field and brushes would rotate. The rotating magnetic field need not be generated by any rotating pole pieces or field coils, as it is well known that a three-phase field system will engender a rotating flux, and provided that this flux rotates synchronously with the brush gear the same effect will be produced. Such a system is the basic principle on which the transverter is founded, and it will at once be apparent that the stationary windings can be insulated for high voltages and will not be subjected to centrifugal stresses.

"The first experimental Transverter has a nominal rating of 250 K.W., and when the primary winding of the transformer is supplied with three-phase current at 2000 volts and 50 cycles, the machine will deliver 2·5 amperes of direct current at 100,000 volts. It consists of a number of static transformers connected to the three-phase supply in such a manner that each goes through its normal magnetic cycle. All the transformers, however, have different phase displacements, and in only one is the flux passing through its maximum or minimum value at any one instant. The secondary coils, therefore, have voltages induced in them which are displaced in time in the same manner as the magnetic fluxes in the different cores. The secondary coils correspond to the armature coils of a direct

current generator of the Gramme ring type which all come in turn under the influence of the poles and also follow on in definite order through their positions of zero voltage.

"The experimental machine comprises six three-phase transformers, or the equivalent of eighteen single phase transformers. Each transformer leg carries two secondary coils, one of which is connected in reverse direction to the other. The result is that the secondary current consists of thirty-six distinct phases, evenly spaced in time, with a phase displacement between any one and

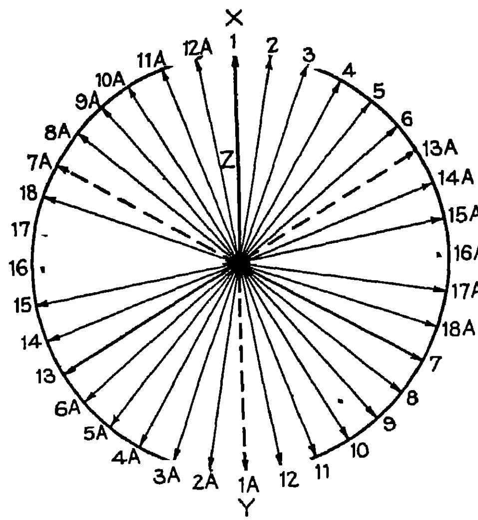

FIG. 73.—Phase relationships in transverter.

the next of ten electrical degrees. Reference to Fig. 73 will make clear the relationship between the secondary phases. The thirty-six radial lines are rotating vectors, representing the voltages induced in the secondary windings. The projection of each on the vertical XY gives the instantaneous value of the induced voltage in that particular winding at the moment considered. For example, the length OZ represents the voltage in the phase 13*a*. The number at the end of each vector is the phase number of the corresponding secondary coil, and it will be noticed that the numbers with the letter *a* affixed are opposite

to similar numbers without the affix. Each pair of coils with the same numbers are on the same core, but are connected in the reverse direction, as explained above, in order to get the required phase displacement of 180 electrical degrees.

"As the sequence of the numbers in Fig. 73 indicates, the phases are not grouped in numerical order in the transformers, but a three-phase grouping is employed on the primary side and a six-phase grouping on the secondary side. The six-phase secondary of the first of the six three-phase transformers which constitute the machine, comprises phases 1, 7, 13, 1*a*, 7*a*, and 13*a*; the next transformer carries the secondary phases 2, 8, 14, 2*a*, 8*a*, and 14*a*, and so on. The transformers themselves are arranged in two tiers of three transformers each, for immersion in the oil tank.

"The primary, or low tension, windings of each phase are all connected in series, the three phases being star-connected. The three cores of Transformer No. 1 carry one primary winding each. All other transformer cores are excited by two separate primary windings connected to two different phases. The three-phase system being balanced, the current will have the same value in each phase. The magnetising force of each primary winding will, therefore, be proportioned to the number of turns, and in those cores carrying two primary windings the total magnetising forces will be the resultant of the magnetising forces produced by the separate phases. It is obvious that by combining magnetising forces of different values and phases, any required phase displacement of the magnetic cycle of any particular core may be obtained. The ease with which the phase displacements can be obtained will be understood from Figs. 74, 75, and 76. The first of these diagrams represents the magneto-motive force in any one of the three cores of transformer No. 1, which, it will be remembered, are respectively excited by the three phases of the main alternating current supply. In the corresponding core of transformer No. 2 an equal magneto-motive force is, of course required, but it has to be displaced by 10 degrees with reference to that in transformer No. 1. This is obtained by winding the second transformer

limbs with two coils, one composed of twenty-six turns of one of the primary phases, and the other composed of six turns of another of the other phases connected in a reverse direction.

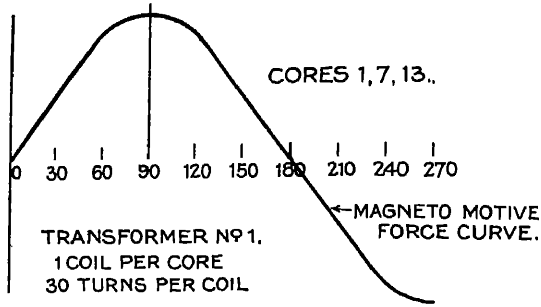

FIG. 74.

The total magneto-motive force of the two combined is, as will be seen from Fig. 75, equal in magnitude to that of the corresponding single coil on the first transformer, but displaced by 10 degrees. The limbs of the third transformer are wound with

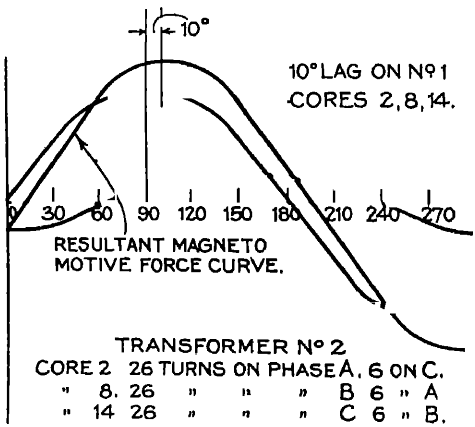

FIG. 75.

twenty-two turns of one primary phase and twelve reversed turns of another, thus giving a 20-degree phase displacement of the magneto-motive force. By similar methods the whole of the required thirty-six phases can be obtained.

"The coils forming the secondary, or high tension, windings are evenly distributed on all the eighteen cores, so that there are the same number of secondary turns on each transformer. The electro-motive forces induced in the secondary coils are all equal in value, but are displaced in phase from each other in exactly the same manner as the magneto-motive forces. The winding is a two-circuit one, and is electrically equivalent to a 2-pole Gramme ring armature although the resemblance is not recognisable in the actual construction. The two circuits correspond to diametrically opposite phases in Fig. 73, and

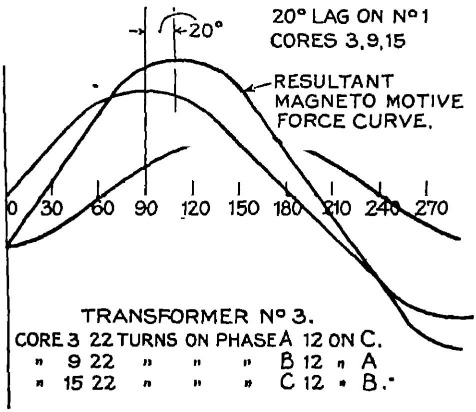

FIG. 76.—Magneto-motive force relationships in transverter.

they are connected to the commutator in opposite directions to form a singly-re-entrant winding.

"In the Transverter the secondary coils, which are opposite in polarity and voltage, are carried on the transformer cores, and are connected to the commutator in opposite directions. Thus coils 1 and 1*a* have their voltage induced by one primary winding; coils 2 and 2*a* by another primary winding, with the requisite phase displacement. The voltage distribution round the commutator at all loads is of a sinusoidal form, and as there can be no field distortion in the Transverter a higher voltage per bar is permissible on the commutator without danger of flashing over. Referring again to Fig. 73 which shows the

phase displacements of the secondary windings, at the instant depicted coils 16 and 16*a* are at zero potential and are therefore in a position suitable for commutation. The first-mentioned coil is changing from positive to negative and the second from negative to positive, and their positions correspond to the positions of the brushes on the commutator. The coils between the brushes are in series and their effects are cumulative, so that the resultant potential of the whole of the coils is obtained at those coils which are connected to the brushes at any instant.

"The coils are all going through their magnetic cycles with the frequency of supply, and in their proper order, so that if the coils are considered as fixed, as they are in the Transverter, it is evident that each brush must travel completely round all the coils in one cycle of the field. With a frequency of the current in the primary windings of 50 cycles per second, the brushes should therefore rotate at a speed of 3000 revolutions per minute. The connections to the commutator can, however, be so arranged that all the coils can be passed by the brushes in one-half or one-third of a revolution, when a brush speed of 1500 or 1000 revolutions per minute will be adequate for the respective cases.

"The secondary windings comprise 2296 sections of 30 turns each to give 100,000 volts at the terminals of the machine, and there are consequently 2296 commutator bars. In order to limit the voltage per commutator, and also to reduce the diameter which would otherwise be necessary, the commutator bars are arranged to form eight commutators, which are connected in series by the brushes. This arrangement of commutators has several advantages besides those mentioned, such as the dividing up of the whole voltage and the greatly increased sparking and creeping distances which the design permits. The secondary windings are also divided into eight entirely separate and distinct windings, each connected to its own commutator and thoroughly insulated from the next and from earth. The windings are only connected to each other through the commutator brushes. A point of mid-potential between the terminals is connected

FIG. 78.—Transverter commutator (stationary)

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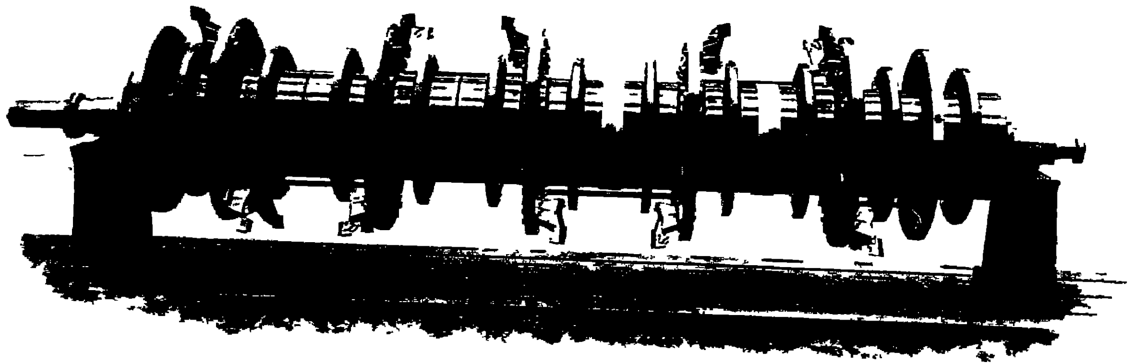

FIG. 79 —Transverter brush gear (rotary).

[*See page* 125.

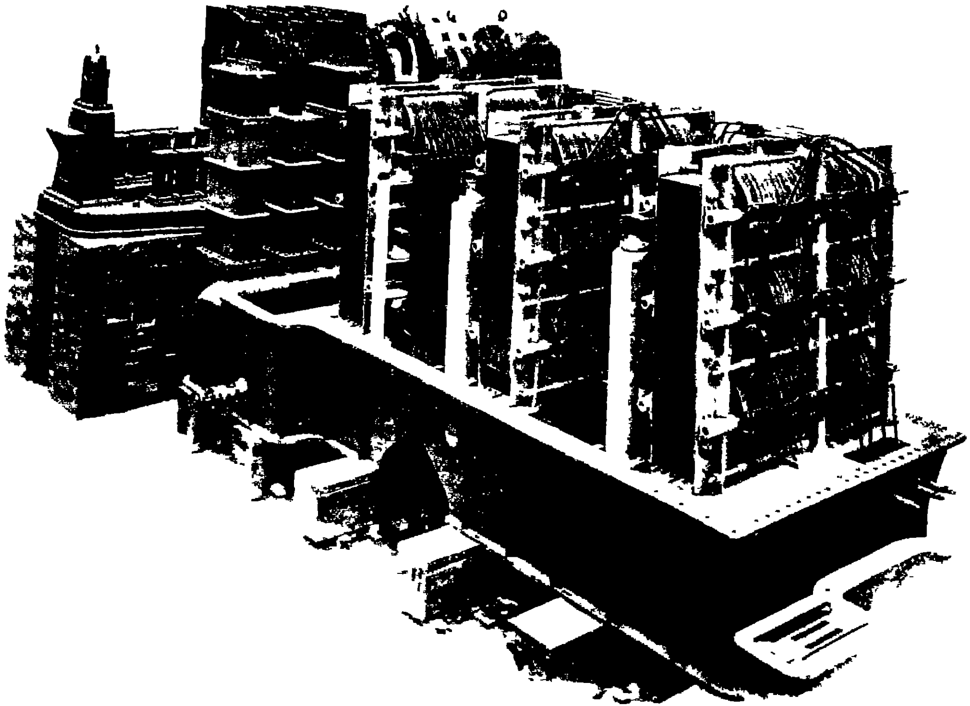

FIG. 80.—Transverter transformers and bedplate.

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through the commutator shaft to earth, so that the greatest difference of potential between any part of the windings and earth cannot exceed 50,000 volts. The arrangement of the eight commutators with their brush connections and the mid-point earthing connection is shown diagrammatically in Fig. 77.

"The commutators are of the disc type, the bars taking the form of copper studs (Fig. 78). They are arranged in circles of 24 inches diameter, with an air space of slightly over $\frac{1}{16}$ inch between successive bars. The maximum voltage per bar is 264 volts.

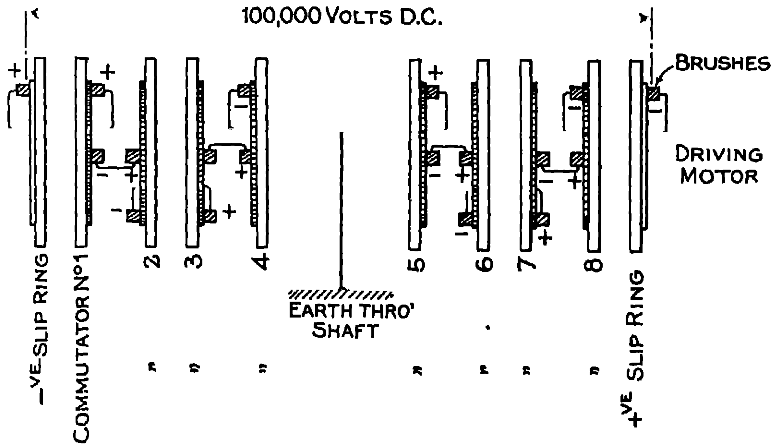


FIG. 77.—Transverter brush gear (diagrammatic).

"The brush gear is driven by a 15 H.P. synchronous motor, taking three-phase current 50 cycles at 2000 volts from the same supply as the Transverter. The motor is excited with direct current at 110 volts, and is fitted with dampers to prevent hunting and to enable it to start as an induction motor.

"The rotating brush gear is illustrated in Fig. 79, and the transformers and synchronous motor in Fig. 80."

Small Commutator Rectifier.—An example of a small type of synchronous rectifier is that made by the Crypto Electrical Co., which consists of a vertical shaft motor driving a special commutator for rectification. The motor is self-starting as an

induction motor, the rectified supply being used for the excitation of the synchronous motor.

The rectifier with its switchboard is illustrated in Fig. 82 *a* and *b*.

The commutator consists of two copper semicircular rings separated by two insulated sectors as illustrated in Fig. 81.

Four brushes press on to the commutator, two of which are

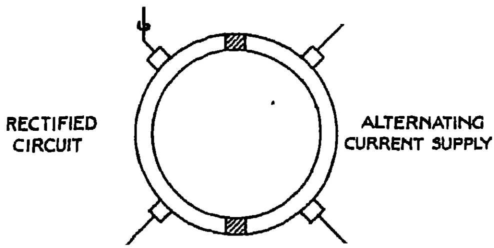


FIG. 81.—Connections of synchronous rectifier.

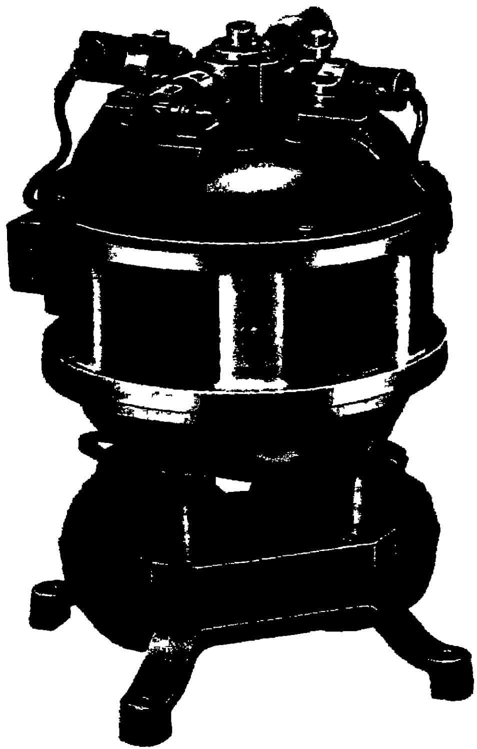
connected to the A.C. supply, and the remaining two to the rectifier circuit. No data is available as to the wave form of this particular machine, but there is no doubt that it will give a biphase rectification with a form factor approaching that of an ordinary sinusoidal wave.

A series of tests taken with this machine supplying current to an external non-inductive load gave the following results:—

TABLE XV.

Amperes on Moving Iron Meter.	Amperes on Moving Coil Meter.	Volts.	Form Factor.
2·05	1·75	17·6	1·17
2·45	2·10	17·6	1·16
2·80	2·40	17·6	1·16
3·40	2·90	17·4	1·17
4·10	3·50	17·2	1·17
4·30	3·75	17 0	1·15
4·92	4·25	17·0	1·16

The form factor for a pure sine wave is 1·11 and it is evident that the wave is approximately sinusoidal. It is noteworthy that the voltage drop on load, or in other words the regulation, is good and the rectifier would prove suitable for battery charging.



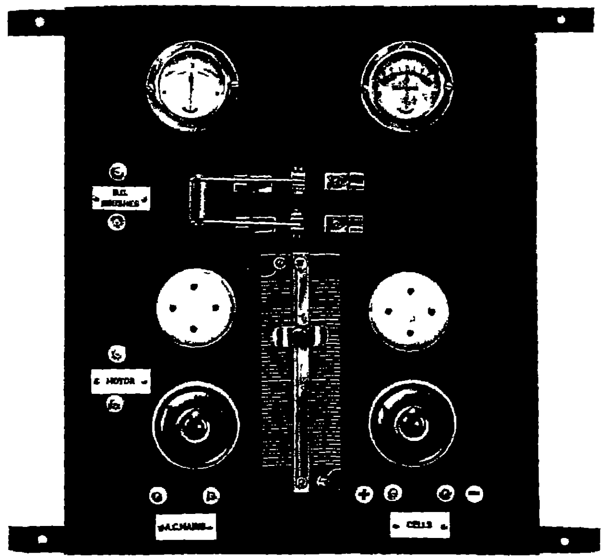


FIG. 82 *a* and *b*.—Small synchronous commutator rectifier set.

[*To face page* 126.

Mercury Jet Collector.—To overcome the difficulties attending the employment of brushes and their gear, Latour describes a method employing liquid jets impinging on the equivalent of a commutator, and the method which is described in the "Revue Générale de l'Électricité" has met with some success. The problem is, of course, bound up with the current-carrying capacity of liquid jets, which will therefore be briefly considered.

Let the length of the jet be . . .	l
and its cross section	s
The continuous current	I
The rise of temperature C degrees . .	θ
Velocity of jet	v
Specific heat of liquid	σ
Density of liquid	Δ
Resistivity	ρ
Resistance of length dl	R

Let an element of the jet dl travel the distance l in t seconds, then the heat generated is

$$Q = RI^2t \text{ watt seconds.}$$

$$= \frac{RI^2t}{J} \text{ calories}$$

where $J = 4{\cdot}2.$

But also $Q = s\,dl\,\theta\Delta\sigma.$

Therefore $RI^2t = Js\,dl\,\sigma\Delta\theta.$

Now $R = \frac{\rho\,dl}{s}$ and $t = \frac{l}{v}$

and therefore $I = s\sqrt{\frac{v\theta\Delta J\sigma}{\rho l}}$

or if $K = \sqrt{\frac{J\sigma\Delta}{\rho}}$

then $I = ks\sqrt{\frac{v\theta}{l}}$ . . . . . . (17)

For mercury $k = 141$ and hence if a temperature rise of 50° C. is specified in a jet of 4 mm. in length, the current density can

be 600 amperes per square millimetre, assuming such a pressure in the liquid that the jet would rise 10 metres in height if projected vertically.

Thus the use of a mercury jet will enable very large current densities to be employed with comparatively small temperature rises. In this particular case 1·4 K.W. of energy will be dissipated by the outflow of metal from the commutator, but no account is taken of the losses by radiation, conduction, and convection.

With regard to the embodiment of such jets in apparatus, Latour has constructed rectifiers up to 30 K.W. capacity, i.e. supplying currents of 250 amperes at 120 volts of a polyphase type; the efficiency of such a rectifier is of the order of 97 per cent.

In the example under consideration the losses were made up as follows:—

Mechanical losses in the jet .	330	watts
Joulean losses in the jet . .	230	„
Commutation losses . . .	165	„
Total	725	„

and with an output of 30 K.W. the efficiency is 97·6 per cent.

CHAPTER VI.

MECHANICAL RECTIFIERS (*cont.*).

EXTRA HIGH VOLTAGE RECTIFIERS, VIBRATING REEDS, ETC.

Description.—Extra high voltage, synchronous rectifiers of the spark type have a restricted use, on account of the small power outputs capable of rectification, and they have usually been replaced by the kenetron or thermionic tube, described in Chapter XII.

As the demand still exists, however, the principles involved will be discussed and several types of rectifier described.

Generally the high voltage, synchronous rectifier consists of a disc rotating synchronously with the supply, provided with one or more insulated segments, which are in some cases suitably connected to slip rings for leading away the current. Connection is made with the supply by the medium of a spark the equivalent of stationary brushgear, which in this case consists of metal spheres set at a predetermined distance from the periphery of the disc.

In order to overcome the possibility of a flash-over between the segments at the commencement of the period of rectification, the disc is made of large diameter and the segments are therefore of considerable circumferential length. When the metallic segments approach within sparking distance of the spheres, conduction takes place through the spark.

Diagrammatically the segments are arranged as in Fig. 83 which illustrates a design where no slip rings are required; a double spark being necessary for conduction in each supply lead.

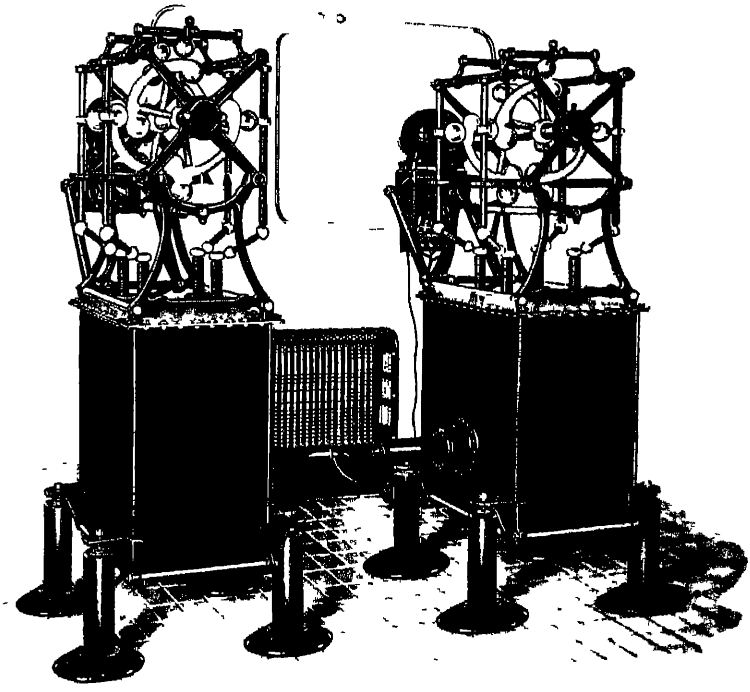


FIG. 84.—Acme rectifier.

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in the circuit by the inclusion of limiting resistances or reactances. In the case of dust precipitation by electrostatic means, however, the case is different. Flue gases, which it is desired to clean, and from which the valuable potash dust held in suspension can be recovered, are passed into a chamber which consists of a cylinder at earth potential, through which is threaded a fine wire at high potential above earth. The flue gases are arranged to come into contact with the wire, and the dust particles are charged positively in consequence; they are immediately attracted to the negatively charged cylinder walls where the charge is neutralised and the particles drop into a suitable container. The actual details of design are unimportant for this purpose, but it is necessary to note that it frequently happens that a chain of dust forms between the positive and negative electrodes through which a current can pass, charring the particles and causing what is practically a short-circuit. The transformer with the bad regulation is therefore desirable with this type of load as well as that of the X-ray tube.

Details of Construction.—In the construction of these rectifiers one of the greatest difficulties is that of insulation on account of the high voltages used. The Acme International X-Ray Co., Ltd., of Chicago have manufactured rotors which will stand up to 282 to 300 K.V. by making use of thin walled paper-shellac tubing for the shaft, instead of the waxed wood previously used. This form of shaft does not warp and retains its insulating properties in humid atmospheres. The cross arms in one particular design were about 3 feet long spaced 33 inches apart, and the total length was 12·5 feet. An illustration of this type of rectifier is given in Fig. 84 and it will be noted that for voltages above 200,000 two 100 K.V. rectifier units are connected in series.

If sharp edges or edges of small radius are used, corona effects will be inevitable at high potentials (see Chapter VII, page 166), and spheres and sections of anchor rings (spherical toroids) of large radius are often employed to eliminate loss from this cause. At the same time the approach of the toroid renders the gap equivalent to a sphere gap, rather than one of the needle

type; and it is well known that the former is much more reliable than the latter. During the operation of the rectifier the minimum air gap between the spheres and toroids is about $\frac{1}{16}$ inch, and arcing takes the form of short local sparks in place of the usual extensive brush discharge.

Wave Form Required in X-Ray Work.—The wave form of the current supply to X-ray tubes is of great importance, and has been the subject of research for some time past. The results of some of the investigations by M. A. Codd, where the wave forms of current from induction coils are compared with those from rectifiers, are reproduced.

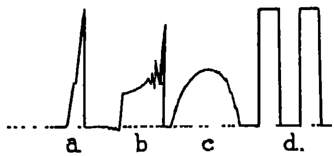

FIG. 85.—(*a*) Gas tube with coil and interrupter; (*b*) Coolidge tube and coil; (*c*) Coolidge tube with transformer and rectifier; (*d*) ideal with curve high peak and long duration.

In Fig. 85 four such curves are illustrated and their general shape has been substantiated by oscillographic records. It is well known that it is only the maximum or crest value of the current which produces useful X-rays, the remainder being spent in heating up the tube, which is an undesirable feature. For example, of the four curves above, (*d*) is the best wave form to use for the purpose, and if the supply is from a separate motor generator special designs must be prepared to attain this wave form, by modifying the windings (and thus the flux curve) and by means of special field poles, so as to induce a

FIG. 86.—Wave form with spark gap

FIG. 87.—Wave form with no spark gap.

flat, broad voltage wave. This is satisfactory to a certain extent, but the wave form of the supply voltage is usually beyond the control of the radiologist, and other means have to be adopted.

If in an X-ray circuit a plain spark gap is substituted for the tube, the wave form of Fig. 86 results, and if the gap is

short-circuited the wave is again modified as is depicted in Fig. 87.

In both of these oscillograms it will be noted that the voltage curve is not truly sinusoidal, but has a zero value for a short portion of the space axis. This is due to the geometry of the rectifier, whereby a short time elapses before the spark takes place between the spheres and the segments, after the voltage has risen above the zero value. The step which led to the improvement in design, by which the desired form was obtained, was the use of the spheres and toroids, which resulted in oscillograms of Fig. 88. The ascending steep portion of the wave is due to the extra curvature of the collectors, and the quick descent to the use of an air blower to extinguish the spark.

The gaps between the waves in Fig. 88 will be affected by a variation in the embracing angle of the sectors, which in practice may vary from 22° to 45°.

Elimination of Surges.—With all of these devices, oscillograph records reveal the presence of a high percentage of harmonics of which Fig. 89 is an example.

FIG. 88.—Wave form with spheres and toroids.

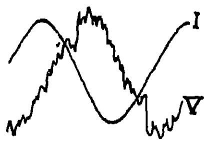

FIG. 89.—Harmonics in a rectifier circuit.

These harmonics are no doubt due to transients set up by the passage of the spark from the sphere to the segment. Any surges may be reflected in the rectified wave and should be eliminated where possible, as in the case of a resonant circuit, which may easily occur, if the inherent inductance and capacity of the plant and wiring have suitable values, the fuses or breakers may continuously cause trouble, apart from the more serious possibility of a breakdown in the insulation of the transformer or coil. This suggests the advisability of as small a spark gap as possible in the rectifier, and to the necessity of

restricting the energy dissipated in the spark to the minimum possible limits To this end the insertion of a capacity and resistance across each of the gaps would no doubt be beneficial, but the difficulty of applying this remedy is in the calculation of the correct values of such an impedance, as the insertion of incorrect values might have the opposite of the desired effect. Research on this point would be the only method to enable the correct values to be ascertained, and such results would be of importance as improvements in design would result.

Improvements in Design.—Considerable ingenuity has been exercised in the design of some of these spark rectifiers. The spherical toroid construction was an advance in the right direction, and Messrs. Lodge Cottrell have manufactured a synchronous rectifier suitable for operation directly from a three-phase supply, which removes a number of the difficulties due to sparking. An increase in the efficiency is also claimed in that the form factor approaches more nearly to unity : a desirable feature in cases of X-ray work and dust precipitation.

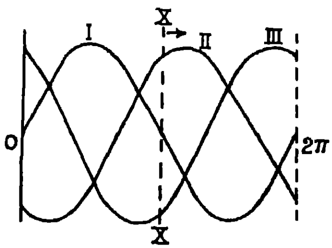

FIG. 90.—Three-phase current wave.

This device depends on the fact that when, in the case of the three-phase current wave of Fig. 90, the oscillation in I reaches its maximum, the currents in phases II and III are exactly equal in magnitude and direction. It is accordingly possible to connect them together for a short space of time. In Fig. 91 this arrangement is illustrated in greater detail. An insulated disc has two metal contactors embedded in its periphery, and rotates between three sparking spheres I, II, and III connected to the three phases. In the position shown the disc has been so synchronised with the supply, that the voltage of I is at a maximum, and the voltages of II and III are equal in magnitude and direction; the unidirectional current is taken from the two slip rings A, and when the disc rotates in the

direction of the arrow the instantaneous potentials of the three sparking points can be visualised by considering a line XX in Fig. 90 moving in the plane of the paper, and in the direction shown, and cutting the sinusoidal waves at ordinates representing the respective voltages. Thus from the position of $I_{Max.}$ the voltage of II decreases and the spark lengthens between II and

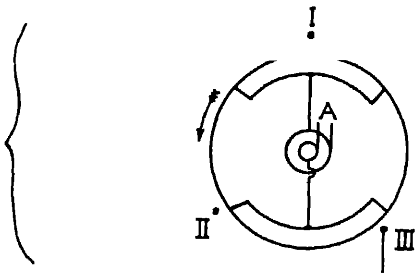

FIG. 91.—High voltage rectifier with three-phase supply.

the contactor. The speed of break can be so arranged by suitably designing the profile of the contactor that a current wave is obtained similar to that in Fig. 92.

Further improvement provides for the inclusion of a second disc on the same shaft and similar in shape to the first, but set so as to rotate 90° in advance. It is so connected electrically

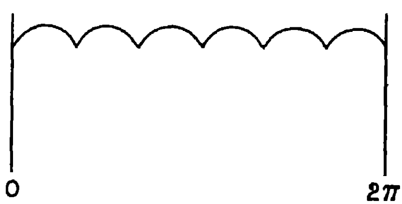

FIG. 92.—Current wave with improved contactor.

that after a period of short-circuit the current is diverted and passed through a resistance. Thus each phase of the transformer is delivering current throughout the cycle and many of the troubles due to a rapid building up of high iron saturation are thereby avoided.

This method is based on the arrangement of the mechanical

rectifier which has been treated in Chapter V, but it should not be assumed that the analysis there given is applicable in the present instance, although the general conclusions apply.

In Fig. 93 is shown a modern 100,000 volt, disc type rectifier driven by a synchronous motor; the disc is directly coupled to the motor, and the high tension terminals of the transformer are connected to the opposite horizontal brush arms. These arms can be adjusted for length of contact to suit the load requirements. The upper brush arms are connected through the control switches to the load, and the lower arms are earthed. Timing gear (not shown in the photograph) is used for setting the contacts whilst the machine is running. Dust-tight, oil bath bearings of large size are employed, and these machines will operate continuously without attention for many months.

Protective Resistances.—The inclusion of a series resistance in the high voltage circuit will afford a measure of protection by

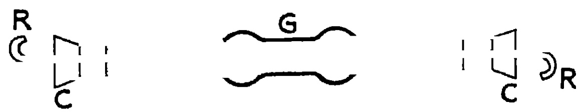

Fig. 94.—Protective resistance.

absorbing a percentage of the energy of a surge and by limiting the current, and where possible it is always advisable to include such protection. A suitable resistance for the purpose may be easily constructed at a small cost. A glass tube G about ¾ inch in diameter has several bulbs blown in it, and into the ends of the tube are inserted two corks through which are threaded copper wires which serve as electrodes, as shown in Fig. 94. The tube is filled with a saturated solution of copper sulphate in glycerine. The bulbs are provided to act as a fine adjustment of the resistance and the number required can only be ascertained by experiment. The current which the tube will safely carry is dependent on the heat dissipation, and as a guide to manufacture, one metre of tubing of internal sectional area equal to 10 square centimetres will have a resistance of approximately one megohm, and the maximum current which can be safely carried is about 30 milliamperes.

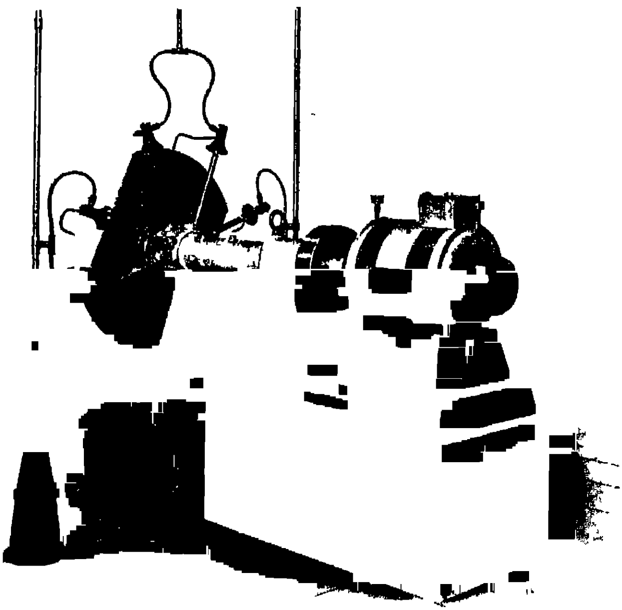

FIG. 93.—Synchronous spark rectifier.

[*To face page* 136.

VIBRATING REED RECTIFIER.

General.—This type of rectifier is shown diagrammatically in Fig. 95, and will be seen to consist of a coil C supplied from a tapping from the transformer, the primary of which is connected to the main supply. The terminal A of the transformer

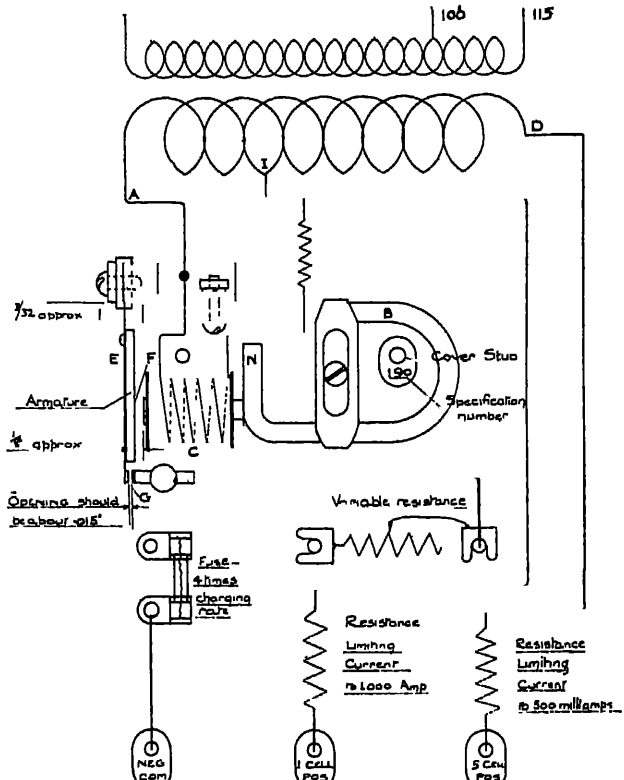

FIG. 95.—Diagram of a vibrating reed rectifier.

is also connected to the reed E, and through the contact G to the adjustable rheostat and thence to the battery.

If alternating current is supplied to the coil the force of attraction of the armature to the solenoid core will be

$$\frac{B^2 s}{8\pi}$$

where B is the induction and s the area of the core. Thus the armature will be attracted each half cycle, and alternating current will be allowed to pass of a wave form similar to that shown in Fig. 96.

The mean value of the current will be zero, and it is therefore necessary to eliminate a portion of the negative wave before rectification is accomplished. A permanent magnet BN is

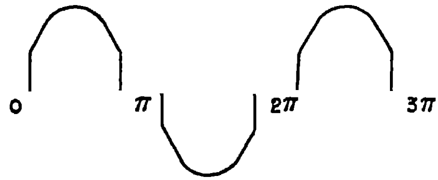

Fig. 96.

therefore included which neutralises the flux in the coil when the current is negative, polarises the device, and the relay will then deliver a current of wave form similar to that in Fig. 97.

It will be noticed that a definite point of cut-off is obtained, due to the fact that an appreciable time will elapse after the impulse from the magnet and before the reed makes contact. This cut-off is important in cases of battery charging as if it were not present discharge would take place during each cycle.

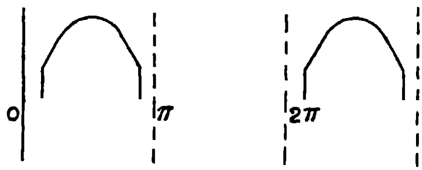

Fig. 97.—Wave form from a vibrating reed rectifier.

The type of rectifier shown in the illustration is obviously one which supplies a single phase wave form, i.e. one which provides a half wave of current followed by a half period when no current flows. It will be apparent that if the connections are as shown in Fig. 98, where the reed T vibrates synchronously owing to the force of attraction from coil C, then biphase rectification will take place as shown in Fig. 99, the reed acting

solely as a switch to change over from one half of the transformer secondary to the other; furthermore, no permanent polarising magnet is required.

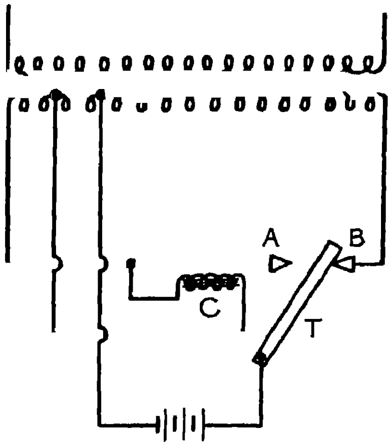

FIG. 98.—Biphase vibrating reed rectifier.

A further improvement has been embodied in some makes whereby in Fig. 98 the permanent magnet is replaced by an electromagnet supplied from the battery terminals, in which case the polarity of the rectifier suits itself to that of the battery on charge.

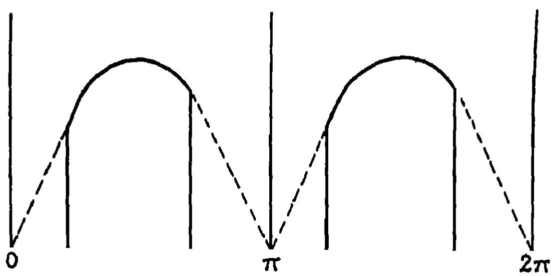

FIG. 99.—Biphase vibrating reed rectifier—wave form.

If a reed is made to vibrate by some external and periodic force, and its frequency is plotted against the displacement, the usual resonance curve, as shown in Fig. 100, is obtained, and

placement and the voltage in phase, and the secret of success is to make the current lead the velocity, in which case the reed will not function at a resonant frequency, and the amplitude will be reduced. If the frequency of supply is such that the reed operates at point X in Fig. 100 then the voltage and displacement may be brought into phase as shown in Fig. 101, see equation (6). Further, if the supply frequency is equivalent to point Y (Fig. 100) the force may lead the velocity by 90° and no electrical phase difference need be introduced, as is indicated in Fig. 102.

At the same time at point Y (Fig. 100) the amplitude is much less dependent on the variation of supply frequency. An

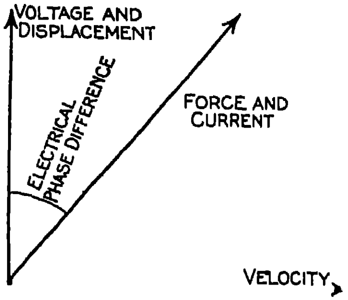

FIG. 101.—Phase relationships in vibrating reed rectifier.

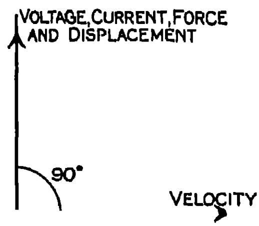

FIG. 102.—Phase relationships in vibrating reed rectifier.

additional advantage accrues from the fact that the correct point of cut-in of the reed is essential to the successful operation of the rectifier, and the likelihood is, that with big amplitudes and higher voltages the point of cut-in may not be satisfactory, and a certain amount of chattering may occur when the reed comes into contact with the stop. The best point of operation can only be obtained by trial.

Theoretical Analysis.—As it is possible that rectifiers of this type which are cheaply manufactured and not easily thrown out of order, will be used largely in the future for the charging of small wireless cells, the theory of their operation has been developed in some detail.

The mathematical analysis of such a rectifier presents certain

difficulties which are not easy of solution, and it is probable that a complete solution is unnecessary, so that the problem has been confined to the consideration of the reed alone leaving its adjustment, etc., to be settled by trial. The design of a reed, in so far as its natural period and relation to the elastic constants are concerned, is treated under the two headings of a loaded and an unloaded rod.

Case of a Loaded Rod.—In Fig. 103 a cantilever in the form of a metal spring is clamped to a heavy support, and to its extremity a mass W is attached, which is large compared with the mass of the spring itself. Let the mass of the spring be w per unit length. The deflection of the spring can then be con-

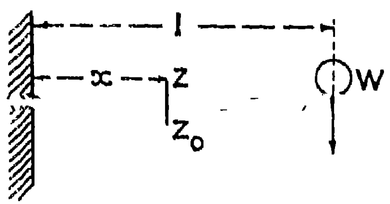

FIG. 103.—Loaded cantilever.

sidered to be due to the weight W alone. At any plane ZZ_0 the bending moment to the right of the section is

$$W(l - x)$$

and to the left

$$-EI\frac{d^2y}{dx^2}$$

where E is Young's Modulus and I the moment of inertia of the beam section. Equating and integrating, the slope of the curve is found to be

$$-EI\frac{dy}{dx} = Wlx - \tfrac{1}{2}Wx^2 + A$$

and as $\frac{dy}{dx} = 0$ for $x = 0$ therefore $A = 0$.

Integrating again, the deflection is seen to be

$$-EIy = \tfrac{1}{2}Wlx^2 - \tfrac{1}{6}Wx^3 + B$$

and as $y = 0$ for $x = 0$ therefore $B = 0$.

The deflection δ at the end of the spring is given by the terminal conditions

$$x = l \text{ when } y = \delta$$

whence $$-EI\delta = \tfrac{1}{3}Wl^3.$$

If this loaded rod is set into a state of vibration by some external force, the inertia of the spring itself will have a modifying effect on the energy of the system, and the expression for the kinetic energy of the rod will be

$$\frac{w}{2g}\int_0^l \dot{y}^2 dx + \frac{1}{2g}Wv^2$$

where v is the velocity of the weight W, and $\dot{y} = \dfrac{dy}{dt}$. But as the deflection is small it is legitimate to assume that

$$\dot{y} = \frac{y}{\delta} \times v$$

and the above expression is finally

$$\text{Kinetic Energy} = \frac{wv^2}{2g}\int_0^l \left\{\frac{\frac{1}{2}Wlx^2 - \frac{1}{6}Wx^3}{\frac{Wl^3}{3}}\right\}^2 dx + \frac{Wv^2}{2g}$$

$$= \frac{wv^2}{2g}\int_0^l \left\{\frac{3}{2}\cdot\frac{x^2}{l^2} - \frac{1}{2}\cdot\frac{x^3}{l^3}\right\}^2 dx + \frac{Wv^2}{2g}$$

$$= \frac{v^2}{2g}\left(\frac{33}{140}wl + W\right)$$

and it is seen that the effect of the inertia of the spring is such as to effectively increase the weight from W to $(W + \frac{33}{140}wl)$.

The frequency of oscillation depends on the stiffness of the spring; and if this is denoted by e the periodicity is obtained by substituting e for the vibrating mass and equating the deflection to unity.

Thus $$p = \frac{1}{2\pi}\sqrt{\frac{eg}{W + \frac{33}{140}wl}}$$

and $$\frac{el^3}{3EI} = 1 \quad \text{or} \quad e = \frac{3EI}{l^3}$$

and $$p = \frac{1}{2\pi}\sqrt{\frac{3EIg}{(W + \frac{33}{140}wl)l^3}} \quad . \quad . \quad (1)$$

Unloaded Rod.—Lord Rayleigh has shown that large variations in the curvature of the deflection of a rod which is vibrating transversely have little effect on the period of vibration. The case of an unloaded rod may therefore be considered as follows:—

Suppose that y is the deflection of a rod at any time t, at a distance x from the support, then the force acting towards the position of no deflection is

$$\frac{w}{g} \times \dot{y} \text{ per unit length.}$$

A proposition in the elastic properties of a beam states that

$$\frac{d^4y}{dx^4} = \frac{w}{EI}$$

where w is the loading, or in this case the weight of the rod per unit length. Therefore

$$EI\frac{d^4y}{dx^4} = -\frac{w}{g}\frac{d^2y}{dt}$$

or

$$\frac{d^4y}{dx^4} + a\frac{d^2y}{dt^2} = 0 \quad . \quad . \quad . \quad . \quad (2)$$

where

$$a = \frac{w}{EIg}.$$

At this point the assumption is made that the motion is harmonic, and it can be demonstrated that the error in so doing is small. It follows that

$$y = y' \cos m^2at$$

where m is any constant which must be evaluated; and y', which is the maximum value of y, is the excursion of the end of the rod. Substituting this value for y in the original equation

$$\frac{d^4y}{dx^4} = \frac{d^4y'}{dx^4}\cos m^2at$$

and

$$\frac{d^2y}{dt^2} = -m^4a^2y' \cos m^2at$$

or further substituting in equation (2)

$$\frac{d^4y'}{dx^4} - m^4y' = 0 \quad . \quad . \quad . \quad . \quad (3)$$

The solution of equation (3) is

$$y' = A \cos mx + B \sin mx + C \cosh mx + D \sinh mx \quad . \; . \; . \quad (4)$$

There are four conditions appertaining to a rod clamped at one end and free at the other which will enable the constants A, B, C, and D to be eliminated, viz.

when
$$
\begin{array}{ll}
x = 0 & y' = 0 \\
\dfrac{dy'}{dx} = 0 & x = 0 \\
\dfrac{d^2y'}{dx^2} = 0 & x = l \\
\dfrac{d^3y'}{dx^3} = 0 & x = l
\end{array}
$$

and substituting these terminal conditions in the original equation (4)

$$\cos ml \cosh ml = -1.$$

Reference to tables gives the first value of

$$ml = 1{\cdot}875.$$

Hence the frequency of vibration is, from the equation

$$y = y' \cos m^2at$$

$$p = \frac{m^2a}{2\pi} = \frac{m^2}{2\pi}\sqrt{\frac{gEI}{w}}$$

and as

$$ml = 1{\cdot}875$$

$$p = \frac{(1{\cdot}875)^2}{2\pi l^2}\sqrt{\frac{gEI}{w}}.$$

Hence if a weight W be added to an unloaded rod such that $W = kwl$, the periodicity of free oscillation is altered in the ratio

$$\frac{\text{Periodicity (loaded)}}{\text{Periodicity (unloaded)}} = \frac{1}{\sqrt{1+4k}} \text{ approximately.}$$

Natural and Forced Vibrations.—The natural frequency has been calculated for the two cases of a loaded and unloaded rod, and it remains to be seen what the effect will be of applying these results to the case of rod vibrating under the influence of an external force.

The equation of motion of such a system is expressed by the differential equation

$$M\ddot{y} + b\dot{y} + \lambda y = F_0 \sin nt$$

where M is the mass, b the damping factor, and F_0 the external force acting on the mass according to a harmonic law and assuming the damping force varies as the velocity. In this case n is twice the frequency of supply as the force operates every half cycle.

Rewrite the equation in the form

$$\ddot{y} + 2\kappa\dot{y} + p^2 y = f \sin nt \quad . \quad . \quad . \quad (5)$$

where $$\kappa = \frac{b}{2M}, \ p^2 = \frac{\lambda}{M} \text{ and } f = \frac{F_0}{M}.$$

The solution of (5) is found to be

$$y = \frac{f \sin \delta}{2\kappa n} \sin (nt - \delta) + a\epsilon^{-\kappa t} \sin (qt + e)$$

where $$\sin \delta = \frac{2\kappa n}{\sqrt{(p^2 - n^2) + 4\kappa^2 n^2}}$$

and a and e are constants depending on the initial conditions. It can be seen that, owing to the inclusion of an exponential, the term

$$a\epsilon^{-\kappa t} \sin (qt + e)$$

represents a transient state of affairs, so that after a comparatively short length of time, it decreases and eventually becomes zero; after which a steady oscillation persists of the form

$$y = \frac{f \sin \delta}{2\kappa n} \sin (nt - \delta)$$

which may be written

$$y = \frac{f}{\sqrt{(p^2 - n^2) + 4\kappa^2 n^2}} \sin (nt - \delta) \quad . \quad . \quad (6)$$

If now the natural period of the rod be made equal to the period of the external force, i.e. if $p = n$

$$y = \frac{f}{2\kappa n} \sin (nt - \delta) = -\frac{f}{2\kappa n} \cos nt = -\frac{F_0}{bn} \cos nt$$

and the rod oscillates with a maximum amplitude and with the period of the applied force. The maximum amplitude depends therefore solely on the damping factor b, the force F_0 and the periodicity of the applied force and not on the mass M. The factor κ is one which it is impossible to calculate: it depends partly on temperature, the modulus of elasticity, and possibly on the actual amplitude itself; and further it also includes a term for the external air friction. So far as the writer is aware no

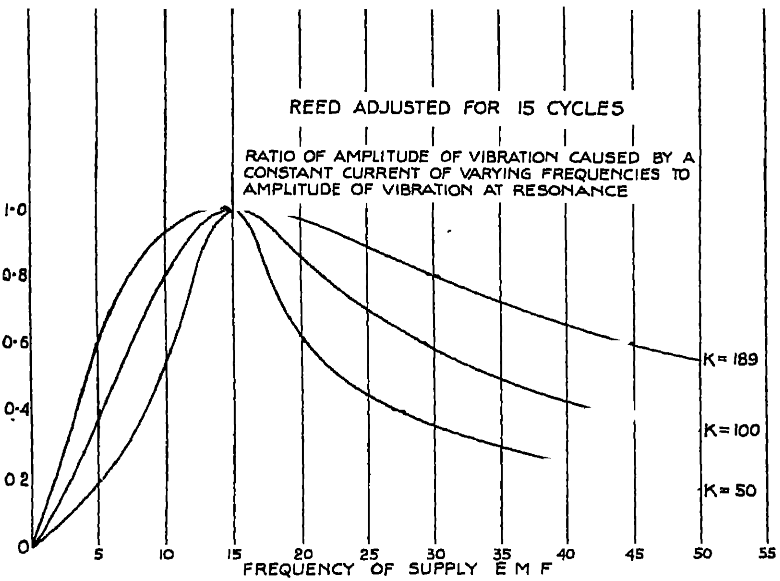

FIG. 104.—Reed resonance curve.

research has been conducted in this particular subject, although results have been published on an experiment for torsional vibrations.

It is seen from the above analysis that the amplitude can be completely determined but for the uncertainty of the constant κ. It will be noted from Fig. 104, which gives the amplitude of vibration of a reed for varying values of κ, that the sharpness of tuning, viz. the ratio of amplitude at other than resonant

frequencies to the amplitude at resonant frequencies, also depends solely on the damping.

With any one particular reed the value of κ can be predicted by measuring the amplitude at various frequencies up to resonance.

This will give various values of A in the expression

$$A = \frac{f}{\sqrt{(p^2 - n^2) + 4\kappa^2 n^2}}$$

where f and κ are kept constant, and only the applied frequency is varied. Thus if n_1 is the frequency when a maximum amplitude A_1 is obtained, and n_2 for an amplitude of A_2, then

$$\kappa^2 = \frac{p^2}{4} \cdot \frac{A_2{}^2 - A_1{}^2}{A_1{}^2 n_1{}^2 - A_2{}^2 n_2{}^2} + \frac{1}{4}.$$

Magnetic Constants.—The value of y can, therefore, be ascertained, which in the case under consideration is the airgap, or from which the airgap can be calculated. Usually, if g is the airgap,

$$y = g - \text{a constant}$$

and with any given conditions

$$g = \text{constant} \times \sin(nt - \delta) \times \mathrm{F}_0 + \text{constant}.$$

Now
$$F_0 = \frac{B^2 s}{8\pi}$$

where B is the induction, which also varies as the reluctance of the circuit.

The reluctance

$$\rho = \frac{l}{\mu s} + \frac{g}{s}$$

l being the length of the iron circuit and μ its permeability. Hence

$$\rho = \frac{l}{\mu s} + \frac{B^2}{8\pi} a \sin(nt - \delta) + \frac{\beta}{s}.$$

This function is complicated and cannot be solved by ordinary methods of analysis as μ varies with the induction B which again depends in an irregular fashion on the cyclic curve

of the iron. The problem can be approximately solved by employing the method (slightly modified) of motional circles which has been developed for the solution of equations on the vibrations of telephone diaphragms; but the complete analysis is lengthy, and reference should be made to the bibliography for further information.

Enough has been said to indicate that a vibrating reed rectifier equipment can be designed from actual calculation so far as the reed and its dimensions are concerned, and which will be suitable for any frequency of supply. Further help than this, mathematics can only supply at the expense of lengthy calculations, but the most difficult part of the problem has been solved, and the remainder can be left to trial of the experimental

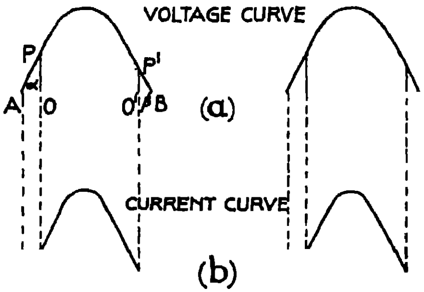

FIG. 105 *a* and *b*.—Voltage and current wave for reed rectifier.

model. The reed gap should be made adjustable and also the length should be variable. The number of ampere turns to provide the necessary mechanical pull will be calculable in the ordinary way.

Voltage and Current Curves.—Partly owing to the effect of the hysteresis cycle and partly to the fact that the value of y indicates that the variations in airgap are out of phase with the current, the point of cut-in will not be the same as the point of cut-off and the angle β ($=$ O′B) in Fig. 105 *a* is less than the angle a ($=$ OA), and the resulting current curve is similar to that in Fig. 105 *b*.

If the voltage curve is of the form

$$e = E \sin \theta$$

then the current wave will be

$$i = I \sin \theta - I \sin \alpha,$$

assuming for the time that the power factor is unity.

The mean value of the current over a complete cycle is therefore

$$\frac{I}{2\pi}\int_{\alpha}^{\pi-\beta}(\sin \theta - \sin \alpha)d\theta$$

$$= \frac{I}{2\pi}\left\{\cos \beta + \cos \alpha - \pi \sin \alpha + (\alpha + \beta) \sin \alpha\right\}$$

and the effective value is

$$I\sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi-\beta}(\sin^2 \theta - 2 \sin \theta \,.\, \sin \alpha + \sin^2 \alpha)d\theta}.$$

If it is known that $\beta = \alpha$ (a condition not often met with in practice) then the ratio of the readings given by a moving coil and a dynamometer will enable the angle α to be calculated.

If efficient working is desired, it is important that the point of cut-in, viz. P, and the point of cut-off P′ should be such that

$$OP = O'P' = E \sin \alpha$$

and this value of

$$E \sin \alpha$$

should equal the voltage of the battery under charge. If this is not the case the current curve in Fig. 105 *b* will represent the state of affairs and either the battery will discharge for a portion of the cycle, or a percentage of the effective charging current will be lost. In either case a loss of efficiency will result.

The control of the cut-in and cut-off voltage is determined in any one rectifier by the natural period of the reed and also by the adjustment of the minimum airgap. Both of these factors have an indirect effect on the magnetic constants of the circuit, and hence also on the periodicity, but as the magneto motive force required to force the flux across the airgap, depends largely on the size of this gap, it is advisable to set it to as small a length as possible. The fine adjustment can then be effected by varying the length of the reed. This will react on the natural period of vibration and on the amplitude as seen from equation (6). Thus a fine adjustment is obtained on the mean value of the

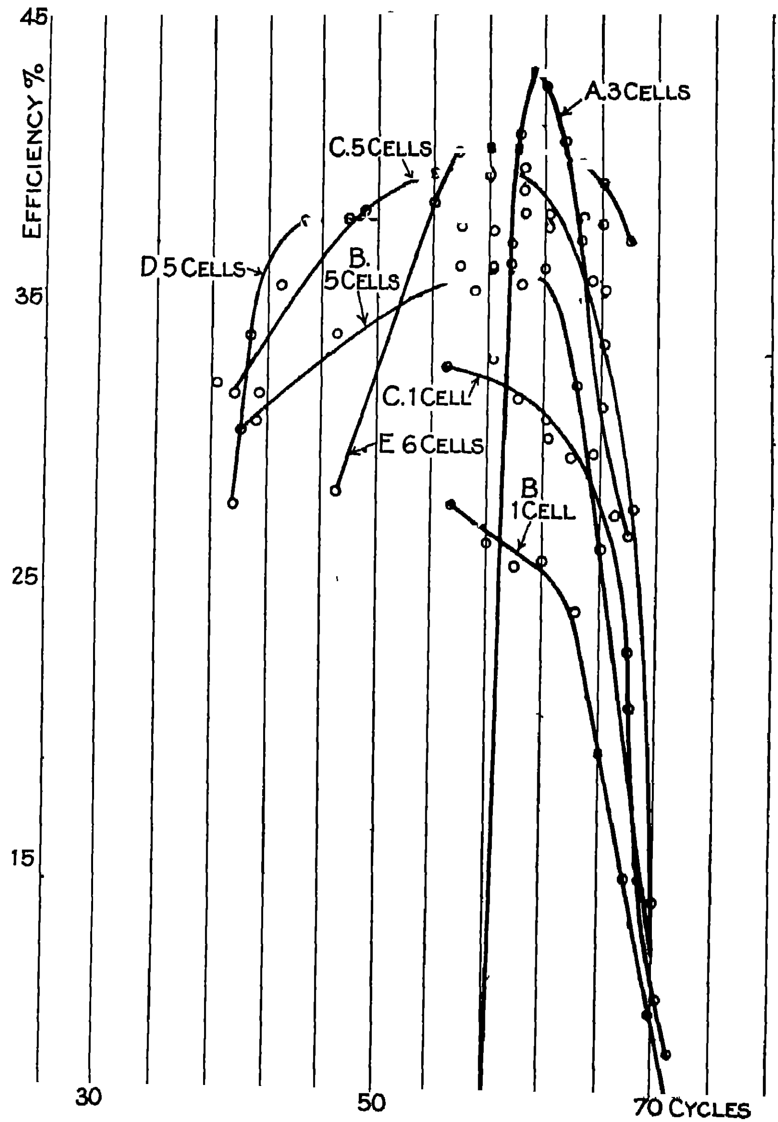

FIG. 106.—Frequency-efficiency curves of vibrating reed rectifier.

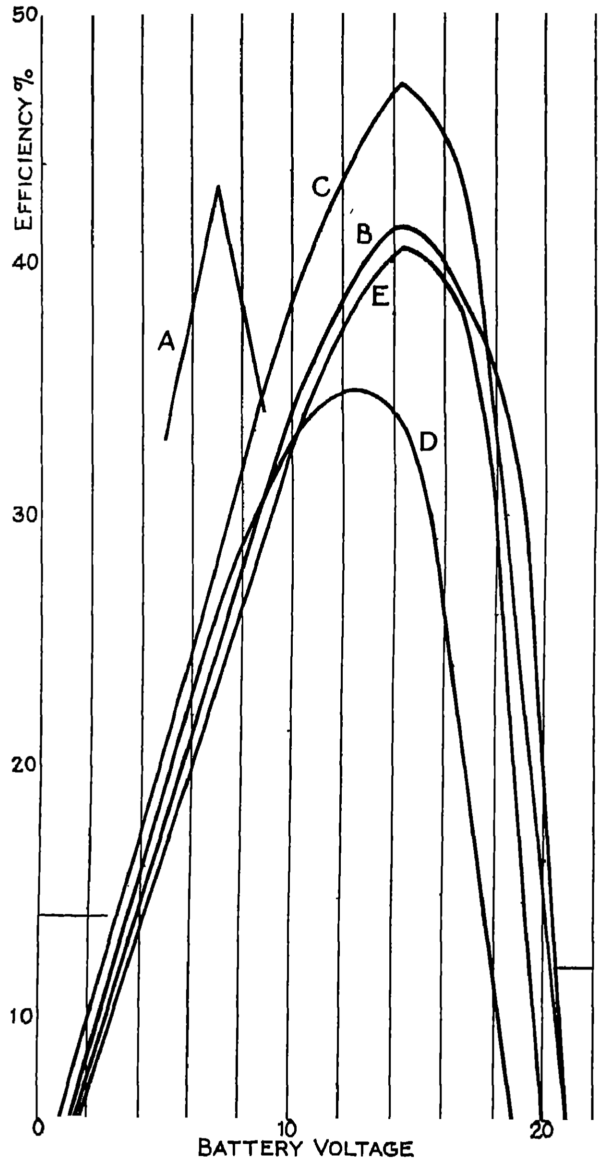

FIG. 107.—Efficiency-voltage curves of vibrating reed rectifier.

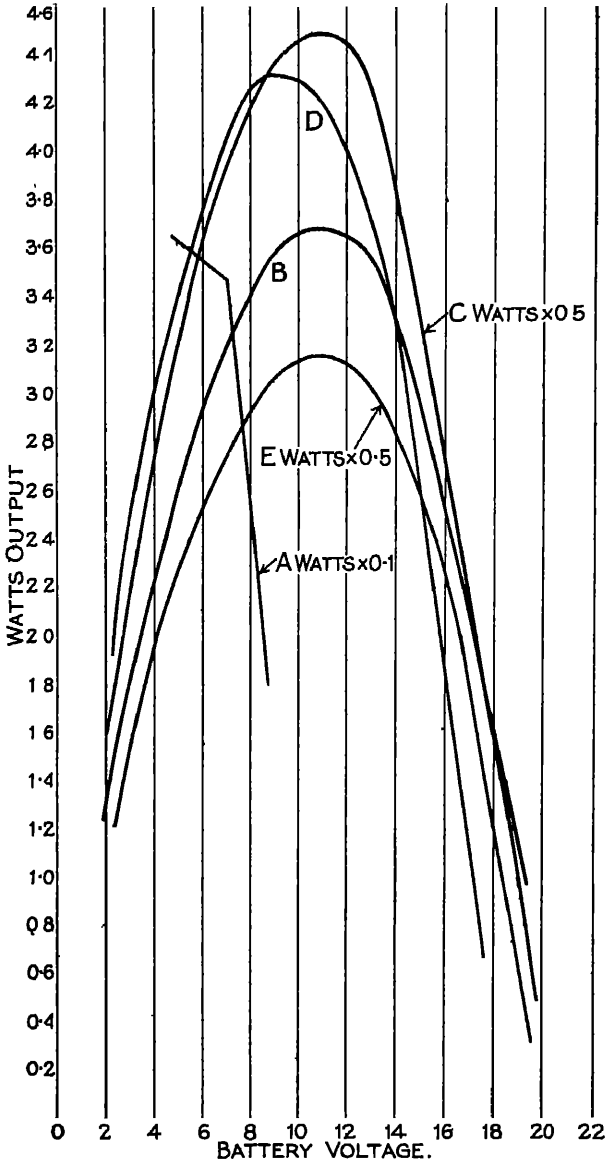

FIG. 108.—Output-voltage curves for vibrating reed rectifier.

current by altering the reed length, and this should be set to indicate a maximum reading on the moving coil meter.

This type of rectifier is convenient because of the latitude available in the number of cells it is possible to charge in series. For any given external resistance, i.e. the variable resistance in the rectifier and the internal resistance in the cells, the maximum value of the current I is fixed and the effect of varying the number of the cells is merely to alter the angle a which will in turn affect the mean value of the current.

The characteristic curves in Figs. 106, 107, and 108 are the result of a series of tests by the Bureau of Standards. Fig. 106

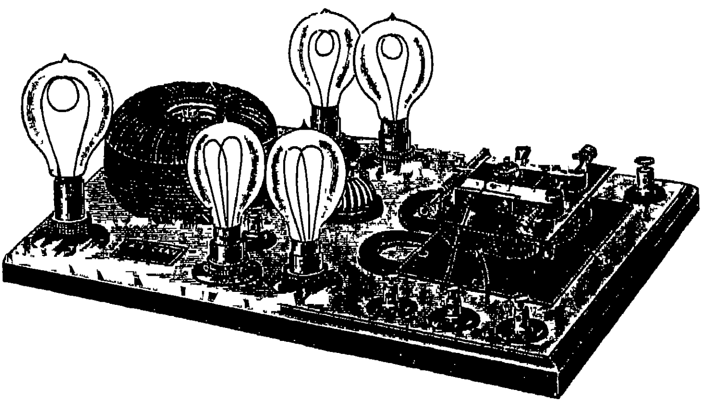

FIG. 109.

indicates the critical nature of some of the rectifiers (five types in number, A, B, C, D, and E), which is due solely to the point of operation on the frequency curve.

Fig. 107 is for similar rectifiers, but shows the variation of efficiency with battery voltage, and Fig. 108 the relative outputs for varying battery voltages.

Time Lag.—It will have been appreciated that there is a mechanical lag which is inevitable unless special devices are adopted, as the tongue of the relay takes an appreciable time to make contact. An ingenious device illustrated in Fig. 109 and developed by G. Sutton partially overcomes this difficulty.

In Fig. 110 the apparatus is shown diagrammatically, where

the main transformer 19 is fed via 1, 20, 19*a*, 21, and 2, and the secondary of this transformer supplies the rectified current to the batteries via 19*b*, 22, 23, 17 or 18 and 27 or 29, according to the position of the relay tongue.

The driving circuit is supplied from a separate choke and solenoid via 1, 4, 10, 8, 11, and 2. The lamp is inserted as a

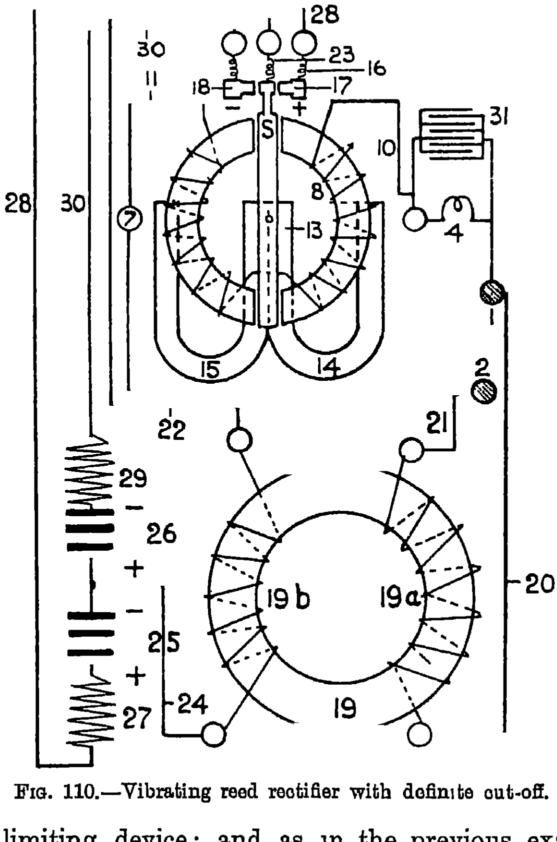

FIG. 110.—Vibrating reed rectifier with definite cut-off.

current limiting device; and as in the previous example, a permanent magnet is included to polarise the alternating current and provide a definite pull in either direction. The advantage over the former type is in the condenser which shunts the lamp 4. By varying the capacity of this condenser a definite leading current can be obtained in the driving circuit which will cause

the reed to operate in advance; and therefore when the voltage is at the correct or cut-in value the circuit is closed. This results in a wave form which closely approaches the ideal, and provides for the maximum efficiency. This is the alternative to operating the reed at a frequency other than resonant frequency, mentioned on page 140.

Pivoted Lever.—A type of mechanical rectifier which presents some novel features in that it employs a lever pivoted about its centre, and on which is threaded a solenoid, has been used in the United States with some success.

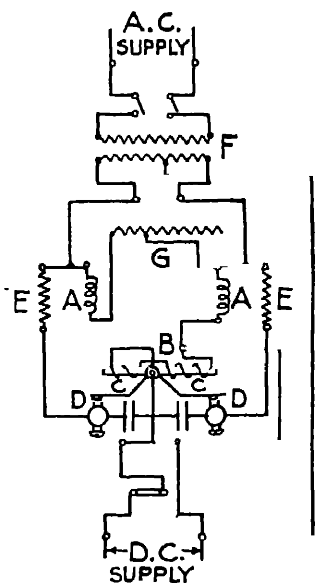

FIG. 111.—Pivoted lever rectifier.

Reference to Fig. 111 will show two alternating current solenoids AA which are supplied from a suitable transformer F. The flux from these solenoids passes through the lever on which the D.C. turns CC are wound. Thus the steady flux in B is in series with the alternating flux in AA and each end of the pivot is attracted in turn as one cycle of flux passes through the lever. The oscillation then is in synchronism with the supply frequency, and contacts DD are alternately made when the current is passing in such a direction as to cause a direct current to flow in the load circuit.

Three controls are provided which enable the wave form of the unidirectional current to be varied. The rheostat G will have a considerable effect on the impedance of the transformer secondary circuit Z (which equals $\sqrt{R^2 + p^2L^2}$) by increasing or decreasing the value of R; thus the power factor will also be affected, which in turn reacts on the point of cut-in and cut-off.

The regulating resistances EE are in the main D.C. supply before the interruption of the circuit, and will therefore affect the amplitude of the current wave only.

The analysis of this rectifier has not been included as it presents several uncertain features such as pivot friction, etc., and does not lend itself readily to treatment.

Oscillating Liquid Jet.*—An interesting form of mechanical rectifier which has, so far as the writer is aware, been tried on a large scale, is that described in British Patents 102977, 130936, 166654, 227720, and 227721.

It is impossible to describe the construction in detail, in fact the only information at present available is that contained in the specifications, but briefly the action depends on the deflection of a jet, presumably of mercury, and carrying an alternating current under the action of a magnetic field. The jet is normally directed to an electrode system consisting of two metal electrodes separated by an insulating partition. If the jet is directed to this partition, then at each half period the jet will impinge on first one and then the other of the electrodes, with a certain phase lag, and can thus be made to perform the function of a change-over switch or rectifier.

General.—The number of mechanical devices which may be applied to a process of rectification is considerable, and two of them have been treated in detail, as being representative cases of the two main types. All of them except the commutator type are difficult of complete analytical solution except by actual trial and error methods. One of the most interesting of the minor devices is that of the pendulum, which is arranged to swing in synchronism with the supply frequency. As the oscillations are maintained by the rectified current the device is reversible in action and may be used to supply alternating current from a direct current main, although the wave form will not be directly controllable; and the power which can be rectified is also limited.

* Since going to press a paper has been read at the British Association 1927 Meeting at Leeds on this form of rectifier.

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PART III.

GASEOUS CONDUCTION.

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CHAPTER VII.

CONDUCTION OF ELECTRICITY THROUGH GASES.

Scope of the Chapter.—It is not the intention to deal at any length with recent developments in the physical theories of gaseous conduction, and further it would not be within the province or power of the writer to undertake such a task. Nevertheless, if the reader wishes to appreciate the mechanism by which rectification is accomplished in cases of gaseous conduction, some knowledge of the underlying principles is necessary.

The difficulty and danger of including one chapter only with this title have not been overlooked, but it is more with the hope that its introduction will lead to further study that the impossible has been attempted. Moreover, the enquiring mind will seek in vain for an explanation of the simplest phenomena if the latest information is not available, and so far little has been published which the Engineer has time to read and study, that even an imperfect representation may be preferable to total omission.

Atoms and Electrons.—The atom is nowadays supposed to consist of a central nucleus surrounded by electrons revolving about it in various orbits. The nucleus itself consists of units of positive and negative electricity, there being a number Z of positives in excess of the number of nuclear electrons. In a neutral atom there are also Z external, orbital electrons which counterbalance the positive nuclear charge. Z is called the atomic number of the atom in question, and its value for an atom of a given element is determined by the position of that

element in the Periodic Table. Thus for Hydrogen $Z = 1$, for Helium $Z = 2$, Oxygen $Z = 8$, Neon $Z = 10$, and so on. For atoms of low atomic weight Z is equal to about one half the atomic weight.

The mass of an electron being only 1/1846 of that of the unit of positive electricity, it follows that practically the whole of the mass of the atom is in the nucleus, also since the diameter of a spherical charge is inversely proportional to its mass, the unit positive charge will have a diameter only 1/1846 of that of an electron. The diameter of the nucleus is extremely small, being never greater than 10^{-4} of the diameter of the atom itself.

Electrons are carriers of electricity and as each electron always bears the same charge, the number of electrons will be a measure of the current.

Ionisation.—If an electron moving with a sufficiently high velocity, encounters or passes very near one of the outer, orbital electrons of an atom, the repulsive forces between them may be such as to detach the orbital electron from the atomic system, which then remains with a unit positive charge. It is then said to be "simply" ionised.

It is found that if a stream of electrons is sent through a gas or vapour, then as long as their velocity is below a certain critical value, dependent on the nature of the gas or vapour, the collision between any one atom and an electron will be elastic, the electron rebounding from the atom with its original kinetic energy but with an alteration in direction. As soon as this critical velocity is exceeded, however, the collision is inelastic. The rapidly moving electron now succeeds in changing the orbit of one of the atomic electrons, which necessitates some energy being absorbed by the atom; so the original electron will rebound, having lost some kinetic energy. This critical velocity is referred to in terms of the potential through which the electron must fall in order to acquire the critical velocity, and this is spoken of as the resonance potential.

If now the velocity of the bombarding electron is still further increased, another critical value will be reached when the

electron succeeds in completely expelling one of the orbital electrons from the atomic system; the potential necessary to give an electron this velocity is called the Ionisation Potential, and is of primary importance in what follows. Actually, there are several ionisation potentials corresponding to the removal of one, two, or three or more electrons, but here the main concern is with simple ionisation.

After an inelastic collision, the energy of the atom will be increased by an amount equal to either eV or eV_1 where e is the electronic charge and V and V_1 are the resonance and ionisations potentials respectively

The atom then subsequently returns to its original state with an emission of this absorbed energy in the form of radiation which may or may not be visible. For example, when a free electron comes within the sphere of influence of the ionised atom which will now have a positive charge, mutual attraction will result, the free electron taking the place of the one which has been expelled previously, and the energy eV_1 will be emitted as radiation. This process is called recombination. The electron may not return immediately to the orbit from which one has been previously expelled, but may do so in steps, falling into a succession of possible orbits, which the modern theory of the atom seeks to define. The energy radiated during the different steps will be of different wave lengths, that from each step corresponding to a line in the spectrum of the gas.

Metallic Conduction.—In the case of metallic conduction there is a continual interchange of electrons between neighbouring atoms, and there is always a number of free electrons, each electron only being at liberty for a short space of time; whereas in a gas where there is no ionisation there is no such interchange.

When a potential is applied to a piece of metal, these free electrons move towards the positive pole, and constitute the electric current in the metal.

The actual number of electrons flowing is enormous—a current of one ampere corresponding to a flow of about $1{\cdot}6 \times 10^{19}$ electrons per second.

Corona Effect and High Voltage Discharge.—It is found that when two electrodes are a given distance apart, and a given voltage is applied between them, sufficient to produce a gradient such that the potential over the mean free path of the ions is equal to the ionisation potential, then if there are a few ions present, others will be produced by collision as described above, and the number tending to pass between the electrodes will tend to become infinite; this corresponds to a current flow manifested as a spark between the electrodes.

A few free ions are always present in gases due to the ionising radiations from the earth and radioactive substances. In general about six ions per cubic centimetre are generated per second due to this cause; the number present at any time depends on their rate of recombination. Electrons are also ejected from metals under the influence of ultra-violet light, and this may increase the number of free ions. Again, it is well known that sparks pass much more easily in the neighbourhood of flames due to the fact that they emit large quantities of free ions.

If the electrodes consist of fine wires or needle points the density of the electrostatic lines of force will be high, and the potential gradient correspondingly great; the electric field may not be sufficiently strong to produce ionisation the whole way between the electrodes but only in the vicinity of the pointed wire, giving rise to a local light emission, which is termed a corona. If the voltage is increased still further the increased electric field will ultimately cause a breakdown of the medium, and a spark will pass.

Glow Discharge Tubes.—If a cylindrical glass tube, about twenty centimetres long and two centimetres in diameter, having two metallic electrodes fused into its ends, is gradually evacuated, while the pressure of the air is in the region of atmospheric, it is found that no current will pass until the potential applied between the two electrodes is raised to several thousands of volts, and the discharge when it occurs, takes the form of a crackling spark. As the pressure is reduced, the potential required to start the discharge becomes less and less, and

the appearance of the discharge changes and soon takes the form of irregular, sinuous, purple coloured streamers going from one end of the tube to the other. On further reducing the pressure these streamers broaden and soon the tube is seen to be filled with a diffuse luminous glow extending from the positive electrode up to within a short distance of the cathode or negative electrode. This glow is known as the positive column.

The cathode is now found to be completely covered with a layer of bright luminosity, whose colour is slightly different from that of the positive column. This layer is called the negative glow, and on closer inspection it will be seen to be not quite in contact with the electrode, but is separated from it by a thin and very sharply defined dark region known as the Crookes or cathode dark space. Finally, a less well-defined region, the Faraday dark space, will be seen separating the negative glow and the positive column. Further reduction of the pressure results in a widening of both dark spaces and the negative glow, while the positive column becomes correspondingly shorter.

The value of the potential required to start the discharge reaches a minimum during this stage, and subsequently rises again continuously with decreasing pressure. At still higher rarefaction the cathode dark space increases until finally it fills the whole tube and no further luminosity of gas is observed. The glass walls of the tube, however, now begin to fluoresce with a colour depending upon the composition of the gas. If the exhaustion is carried to the extreme limit available the discharge refuses to pass at all.

Experiments have shown that for any given residual gas, the potential necessary to start the discharge depends upon the pressure of the gas, the form and material of the electrodes and the distance between them.

Having briefly explained the phenomenon, consider the movements of the atoms and electrons in the tube, and take the case given above where two electrodes are fused into a glass vessel which has been partially exhausted and across which a given potential is applied. As stated on page 166 a few ions

will always be present in the gas, so that on the application of the voltage, they will commence to move towards the anode and away from the cathode, under the influence of the electric field, their velocity increasing during flight. If their velocity is sufficiently great they will ionise the atoms previously described by expelling an electron from the influence of the positive nucleus, and the net result will be a general movement of electrons towards the anode, and positively charged atoms to the cathode. Recombination of certain electrons and positive atoms will necessarily take place, and this will conduce to a state of equilibrium, and the numbers of electrons moving in one direction and positive atoms in the other will be limited by this effect and also by the space charge (page 295). The net current flow will be the sum of the charges carried by the electrons to the anode and the positive atoms to the cathode.

Arc Conduction.—The conduction of electricity by an arc depends on different principles from the two cases above, the spark and the glow.

A necessary condition of arc conduction is that of at least one hot electrode: in the case of the arc lamp both electrodes are hot, but the positive crater is the chief source of illumination and is hotter than the cathode, whereas in the mercury vapour lamp the cathode is at a higher temperature than the anode, which is comparatively cool. The reason for this diversity is one of some complexity and depends on a number of variables, which manifestly cannot be discussed here; but in cases of rectification where conduction through gases is the modus operandi, the cathode is in general the hotter of the two electrodes and the anode is relatively cool.

The actual conduction of electricity is by means of the vapour of which the cathode is composed, but in order that this vapour may be first produced a connection has to be made between the electrodes which causes a heavy current to pass, and a high enough temperature to be reached for the purpose of vaporisation.

Thermionic Emission.—It has been stated on page 165, on metallic conduction, that there are large numbers of free electrons

present in metals. They are in violent agitation similarly to the molecules of a gas, their velocity depending on the temperature. Suppose two cold electrodes (one of which it is possible to heat electrically) are placed in a highly evacuated vessel. It might be supposed that on the application of a potential between them, some free electrons would leave the cathode and pass to the anode. As a matter of fact it is possible to obtain a current by such means, but potential gradients of some millions of volts per centimetre are required at the surface of the cathode before any measurable current flows. As soon as the cathode is heated to (say) 2000° C. the velocity of agitation of the electrons increases to such an extent that they can pass through the surface of the metal, and on the application of a potential difference between the electrodes, currents are observed to flow. Thus with a tungsten wire at 2000° C. a current of 0·004 ampere per square centimetre can be obtained and with thorium at the same temperature 30 amperes per square centimetre, while with sodium at only 370° C. 0·014 ampere per square centimetre will flow. With the alkali metals appreciable currents at quite low temperatures will be observed.

Valve Effect.—Having cursorily described the general principles of gaseous conduction it is possible to consider more closely the phenomenon of arc conduction, its application to the practical forms of rectifier being left over to a subsequent chapter.

It has been seen that the necessary condition for arc conduction is a cathode at a high temperature, which serves as a source of electrons. It follows that if one electrode is hot and the other relatively cool, a large current can only pass when the high temperature electrode is a cathode. Hence if an alternating voltage be applied across the two electrodes a large current will flow when the hot electrode is negative, and a very much smaller current when the cold electrode is negative. Thus arc conduction does not provide perfect rectification, although for all practical purposes with modern plant the current is unidirectional.

It is because of this rectifying property of the arc that

difficulties are experienced in its continuous operation on alternating current circuits. Thus during the half cycle when the hot electrode is negative, current can flow provided that the voltage is high enough, but when the current becomes zero the arc is extinguished, and usually cannot be restarted as the cathode temperature has fallen. If, however, the voltage

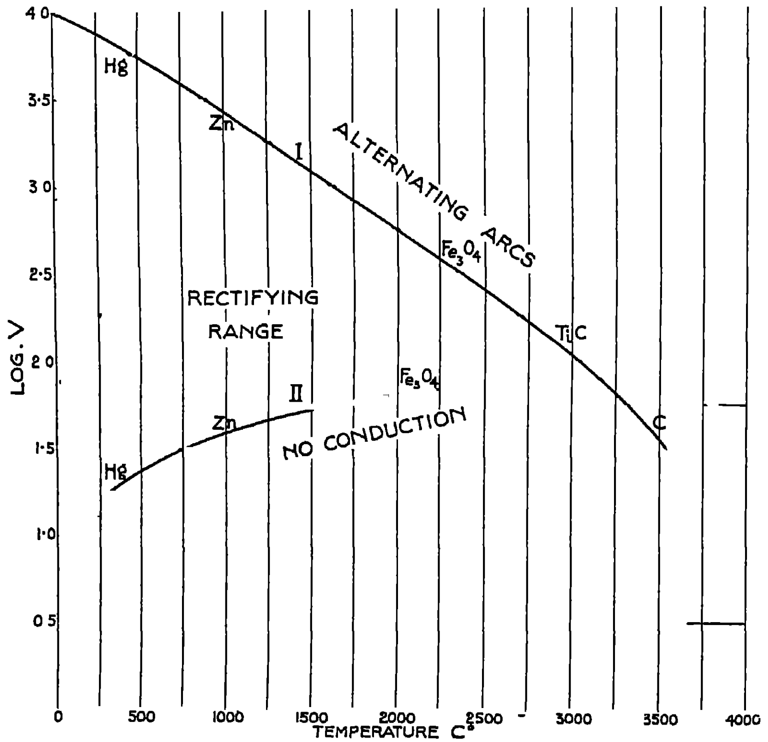

FIG. 112.—Arc characteristics of various materials.

is sufficient to cause an intense glow discharge to pass between the anode and cathode, this discharge, by heating the cathode, will enable the arc to be restarted, the anode for the previous half cycle becoming the cathode in the subsequent half cycle. If the voltage between the electrodes, sufficient to cause a spark to pass, is less than the voltage required to maintain the

arc, the arc will operate on alternating current but without rectification.

Fig. 112 illustrates this point where curve I shows the voltage required at various temperatures for a discharge to take place over a gap of 13 mm., and II the voltage required to maintain the arc. Thus above curve I alternating current arcs can exist, between I and II they can only exist provided some external means are available to maintain the temperature of the cathode; and this constitutes the rectifying range. Below curve II arcs cannot exist as the voltage is not sufficient to maintain them. Further, it is evident that if the electrodes are manufactured of a material such as carbon which occupies a position on the curves at a point beyond their intersection alternating current arcs can exist without external means, but also without rectification. The points roughly indicated by the symbols are approximately the boiling-points of the elements, and are also the temperatures of the vapour stream in the arc.

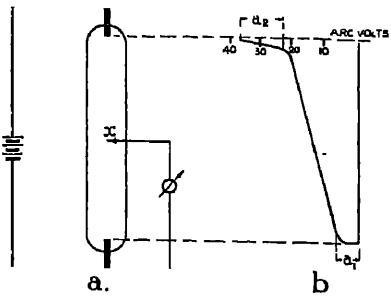

FIG. 113 *a* and *b*.—Voltage drop in arc stream.

Arc Voltage.—The voltage which is required to maintain an arc can be expressed by an empirical formula

$$e = a + \frac{c(l + d)}{\sqrt{i}} \quad . \quad . \quad . \quad . \quad (1)$$

where a, c, and d are constants, l is the length of the arc, and i the current flowing in the vapour stream. Thus the potential difference across the arc can be considered as the sum of two voltages, one of which is constant, and the other depending on the arc length and the current flowing.

If an exploring electrode be inserted in the arc stream as shown in Fig. 113 *a*, which can be moved longitudinally between the anode and the cathode, and if this be connected

through a galvanometer to the cathode, the voltage drop at different points along the stream can be ascertained. The curve obtained is shown in Fig. 113 *b*; it will be observed that there is a rapid fall of voltage near both the cathode and the anode, and it can be demonstrated that this fall is independent of the distance between the electrodes. Thus the sum of the cathode and anode falls, viz., $a_1 + a_2$ is equal to the constant term a in the above equation, whilst the term

$$\frac{c(l + d)}{\sqrt{i}}$$

represents the drop which is measured by the exploring electrode, and bears a linear relation to the length of the arc stream.

The constants in equation (1) have the following approximate values for various elements:—

a = 13 volts for mercury
16 ,, ,, zinc and cadmium
30 ,, ,, magnetite
36 ,, ,, carbon
c = 31 for magnetite
35 ,, carbon
d = 0·125 cm. for magnetite
0 8 cm. for carbon.

One of the characteristic features of arc conduction is that of the rising voltage curve with decrease of current, which infers that on a constant potential circuit the arc will be unstable, whereas on a constant current circuit the voltage of supply will suit itself to the current taken by the arc. On constant potential circuits this disadvantage is overcome by the insertion of a series resistance (in direct current circuits) or an inductance (in A.C. working).

The effect of such an impedance is shown in Fig. 114, where curves I *a* and *b* are the volt-ampere characteristics of a carbon arc at two different arc lengths, and II the voltage drop across the resistance or reactance. Curve III is the addition of I and II, and indicates that after passing through a minimum value of

56 volts, curve III has a rising characteristic, thus ensuring stability at a current of 30 amperes. At least 56 volts must be available from the supply with a resistance of 0·22 ohm in circuit.

Enclosed Arcs.—The above applies only to arcs in which the pressure and temperature are constant, viz. in air or in circumstances where similar conditions apply. In the case of an arc which is enclosed in a hermetically sealed tube, such as

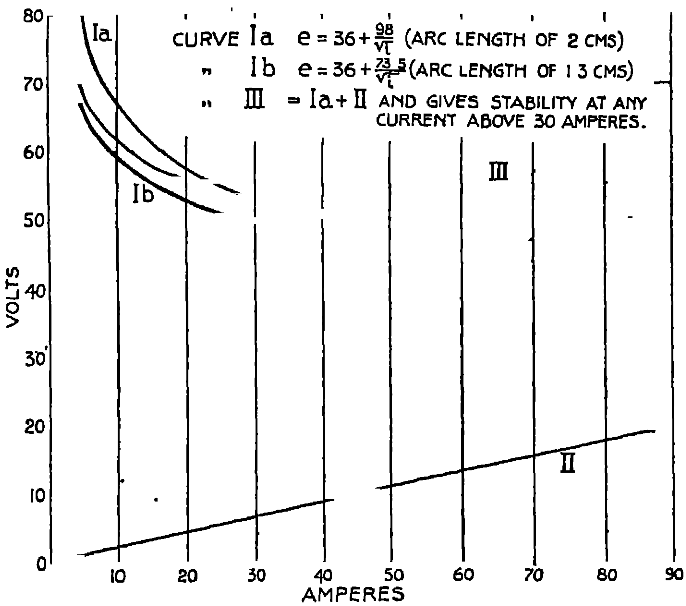

FIG. 114.—Curves of arc stabilising resistance.

in the mercury vapour rectifier, different conditions appertain. Thus in the case of a mercury vapour arc if the current is high enough in value for the whole section of the tube to be filled with the vapour stream, the radiation of power may be assumed to be proportional to the tube length, or

$$p = e_1 i = cli$$

where l is the tube length, and thus

$$e_1 = cl.$$

To this must be added a cathode and anode drop a and

$$e = a + cl.$$

If d is the tube diameter

$$e = 13 + \frac{1{\cdot}4l}{d}$$

for mercury vapour arcs in enclosed vessels.

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CHAPTER VIII.

MERCURY VAPOUR RECTIFIERS, THEORETICAL ANALYSIS.

Introduction.—It has been shown in Chapter VII. on the conduction of electricity through gases, that the rectifying properties of the arc are primarily due to the presence of a hot cathode, and, further, if rectification is to ensue, the voltage must be less than that required to start a spark discharge, but greater than that required to maintain an arc. This is the case with mercury vapour, and because of its physical properties of low boiling-point, together with its electrical conductivity it is the only element which has been tried with success on a large scale for commercial arc rectifiers.

One of the difficulties encountered in the manufacture of such rectifiers is the attainment and maintenance of a sufficiently high vacuum. This is overcome in various ways—in one case a blast of superheated vapour is caused to sweep by the hot cathode, the action, presumably, being mechanical, in that the kinetic energy of the hot gases is sufficient to impart a high velocity to the molecules of residual gas and thus sweep them out of the rectifier.

There are, however, other defects which are difficult to surmount, and which are the subject of a series of patents emanating chiefly from America, Germany, Switzerland, and France. The fact that experimental work and improvements in construction have been initiated in these countries is due, not to any lack of appreciation of the value of a mercury vapour rectifier in England, but to the fact that alternating current distribution is more prevalent there at the present time than in this country. There is no doubt whatever that high voltage

distribution will have greater opportunities in Great Britain in the future; and a knowledge of power rectifiers is essential to electrical engineers who have control of commercial plants. The mercury vapour rectifier can be made in large sizes, and has the great advantage that it is a static plant and requires little attention. This latter point is important and is frequently overlooked in designing and estimating for installations. It is also possible that a popular prejudice exists in favour of rotary plant which renders the adoption of the newer form of rectifier somewhat slow.

General.—So far (with the exception of the vibrating reed type of rectifier) mechanical rectifiers have been seen to partake rather of the nature of reversing switches than of devices which

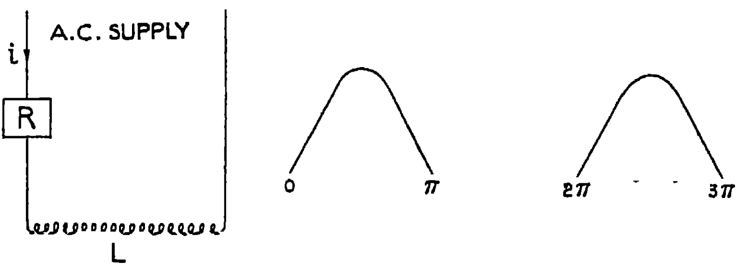


FIG. 115.—Single-phase rectifier.

have a definite and inherent valve effect. The rectifiers which are described in Part III., however, are definitely "valves," and it is advisable to recapitulate in greater detail what has so far been only briefly reviewed.

In its general form a rectifier circuit consists of an alternating current supply through a transformer with a middle tapping which is provided for a specific purpose, and it is important that this point should be understood at the outset. Consider the elementary circuit in Fig. 115.

The rectifier or valve R will allow current to pass to the load L in one direction only, and the wave form will be as shown to the right of the figure. This arrangement is usually called a single-phase system. The peak voltage of this wave will be equal to the peak voltage of the supply. Suppose

that a 1/1 ratio transformer is inserted in the circuit provided, with a middle tapping on the secondary; the diagram of connections will now be as shown in Fig. 116, and two rectifiers are required, one in each side of the secondary. When the potential of the secondary is such as to allow current to flow in the direction of the arrow, R_1 will pass current but R_2 will rectify, and any current flow through it will be stopped; the net effect is that current will pass to the load in the direction shown, and back to the middle tapping. When the current dies down to zero and commences to rise again in the opposite direction in the secondary, R_1 will rectify and R_2 will pass

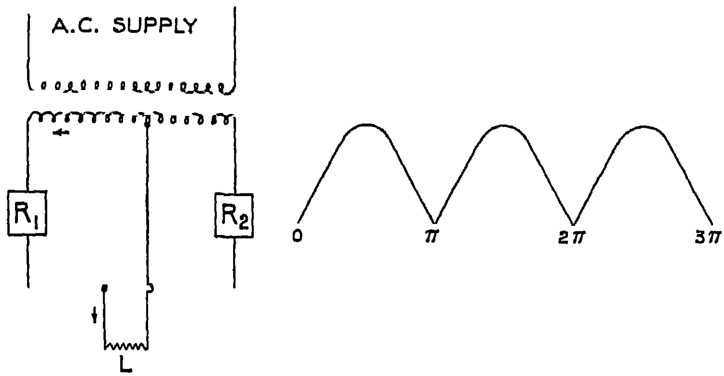


FIG. 116.—Biphase rectifier.

current, and the load is again supplied with current in the same direction as before. Thus the wave form will consist of a sine wave as in Fig. 115 with the gaps filled in by inverted loops 180° in advance. This arrangement is called "biphase" rectification, and always implies a reduction in voltage, since the voltage can never exceed (with this arrangement) that of the transformer secondary from the outer to the neutral point.

It has been shown in Chapter VII. that in the case of a rectified arc, the current must not be allowed to drop to zero as the arc will not restart of its own accord, unless some special starting devices are provided, and if the emission ceases for even as small a period of time as one hundred thousandth of a second it has been shown that the arc is extinguished; but if the loops

of the curves can be made to overlap, as is indicated in Fig. 117, the addition of the two curves will result in the full line curve with the dotted portion between the loops, and the required conditions of a continuous and positive current have been fulfilled; in addition a further advantage has been obtained of reducing the variation of the instantaneous current from the mean value. This result has been attained by the insertion of

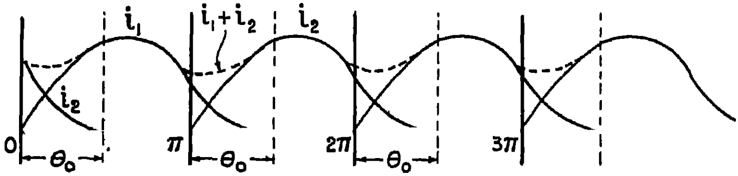


FIG. 117.—Biphase wave form showing smoothing by inclusion of reactance.

an inductance in each of the anode circuits, which acts as a reservoir of energy during the time that the current is increasing and restores it at a later period of the current wave. Thus if an inductance were inserted in the single-phase circuit shown in Fig. 115 the wave would be so affected that instead of lasting for a time represented by 180 electrical degrees it would persist for 180° plus a time depending on the circuit constants. This is indicated in Fig. 118.

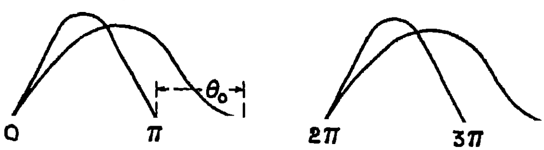


FIG. 118.—Single-phase wave form showing distortion by inclusion of reactance.

Rectification Theory.—The above effects of inductance in a biphase rectifier have been inserted at this juncture to indicate to the reader the effect produced by inductance, but the mechanism by which this is attained in multiphase rectifiers is now developed in greater detail.

It is first necessary to lay down two general principles which may be directly inferred from the fundamental theories of the "valve effect" enunciated in the previous chapter.

(1) An anode can only supply unidirectional current during

the time interval when its instantaneous potential is greater than that of the cathode, and at the same time greater than that of all other anodes.

(2) If more than one anode is supplying current, the potentials of each of such anodes must be equal to one another, and greater than that of the cathode or all other anodes.

These two principles and much that follows are in general perfectly true of all valves which operate by means of the conduction of electricity through gases; but the theories which have been developed are applicable more generally to the mercury vapour and thermionic rectifiers than to other forms.

Consider the direct effect of the above two principles on the performance and operation of these two forms of rectifier.

In every case it is at once apparent that a certain voltage drop will be developed across the rectifier; and this drop will vary with the type employed and its characteristics; for example, in the mercury rectifier this drop will be practically constant, but in the thermionic tube it will be proportional to the two-thirds power of the current flowing, and will depend partially on the characteristics of the tube.

As has been forecasted above, reactance in the anode circuit will have a considerable effect on the number of anodes functioning at one and the same time, or in other words on the overlap of the current waves.

Two methods of analysis have been developed to solve the problem theoretically, and although both of these methods are of interest, and are discussed at some length, they must not be accepted as a perfect solution of the problem of the design of a rectifier, although they are of great help in consolidating one's ideas on the subject, and further, they afford an insight into what is actually taking place in the rectifier, or in other words they portray the mechanism of rectification.

The first method, which may be termed the method of direct analysis, has been developed by several Swiss and French writers, notably H. Giroz in the "Revue Générale d'Électricité." This analysis depends on the consideration of hypothetical circulating currents in the transformer windings and deduces

the characteristic equations. The second method determines the actual wave form of the rectified current, and of the anode current.

Before a detailed description is given of the two analyses, consider the case of a circuit containing no inductance—an ideal which is only approximately approached in practice.

Rectifier with no Inductance.—It is assumed that the reactance of the transformer windings is negligible, and the first point of note is that conduction cannot take place unless the anode under consideration is positive with respect to the cathode.

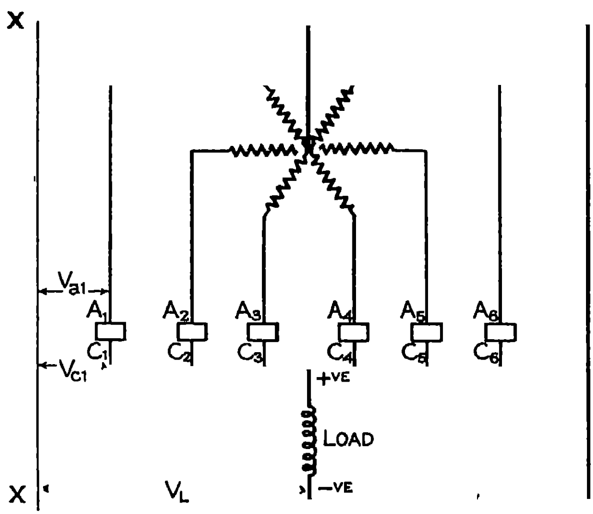


FIG. 119.—Rectifier voltages.

It will be shown that under these circumstances any overlap of the current waves will depend solely on the conditions of voltage drop in the rectifier.

In Fig. 119 if A_1 A_2 etc., C_1 C_2 etc., represent the anodes and the cathodes respectively, then in the case of thermionic rectifiers C_1 C_2 etc., will be situated in different containers, but in the mercury rectifier they will all be in the same tube; this difference, however, does not affect the argument. Further, all the cathodes at any one instant will be at the same potential above the datum line XX.

The voltage of supply is

$$V_{a_1} - V_L$$

the drop in the tube is

$$V_{a_1} - V_{c_1}$$

and across the load

$$V_{c_1} - V_L.$$

So that

$$-(V_{a_1} - V_L) + (V_{a_1} - V_{c_1}) + (V_{c_1} - V_L) = 0.$$

The voltage curves will appear as shown in Fig. 120 where

$$AC = V_{a_1} - V_L$$
$$AB = V_{a_1} - V_{c_1}$$
$$BC = V_{c_1} - V_L$$

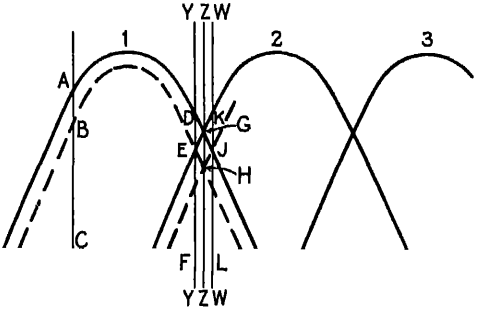


FIG. 120.—Voltage wave forms.

and the relation between the curves B and A may be assumed for the moment to be any function whatever.

When the voltage of phase 1 is decreasing the line YY is reached and therefore

$$DF = V_{a_1} - V_L$$
$$DE = V_{a_1} - V_{c_1}$$
$$EF = V_{c_1} - V_L \text{ the voltage across the load.}$$

At point E

$$EF = V_{a_2} - V_L$$

or

$$V_{a_2} = V_{c_1} = V_{c_2}$$

and anode 2 is in a condition to pass current provided that it can overcome the resistance of the tube, and hence conduction will tend to spread from anode A_1 to A_2, and the curve of $V_{a2} - V_{c2}$ will be as shown and will pass through the point E. Current will now flow from both anodes, the contribution from anode 1 still preponderating. At point G on ZZ where $V_{a1} = V_{a2}$ both arcs must pass the same current and

$$V_{a1} - V_{c1} = V_{a2} - V_{c2}$$

or the drop in the arc is the same in each case, viz. a voltage represented by the length GH. When the line WW is reached phase 1 ceases to function and phase 2 takes the load. The portion of the cycle over which both anodes can operate is represented by the length LF which will depend primarily upon the value of the volt drop in the rectifier, and secondarily on the prolongation of the voltage curve of the supply.

The above can only represent the true state of affairs provided, that the drop in the tube follows such a law, that each anode contributes a proportion of the current resulting in an equal cathode voltage in both phases. If the voltage drop in the tube is constant for any current, as is approximately the case for a mercury vapour rectifier, then although anode 2 can commence to conduct when point E is reached on the first cathode curve yet the voltage drop relation cannot be strictly fulfilled; and further, this is also true at the point J where anode 1 ceases to function. Thus the conclusion is inevitably reached that the change in operation occurs instantaneously when the point H is reached. This is true from the theoretical standpoint; in practice, however, the law governing the dependence of the voltage drop on the current rarely holds at low current values, and it may therefore happen that an overlap of currents will occur; but it is apparent that such overlap as does occur, is dependent solely on the relation between the voltage drop and the current, and cannot be maintained over any length of time.

Overlap with Inductance in Circuit.—If the anode circuit is considered to include reactance, or what is equivalent, the

transformer windings are so designed that their leakage reactance is appreciable, a totally different state of affairs is obtained.

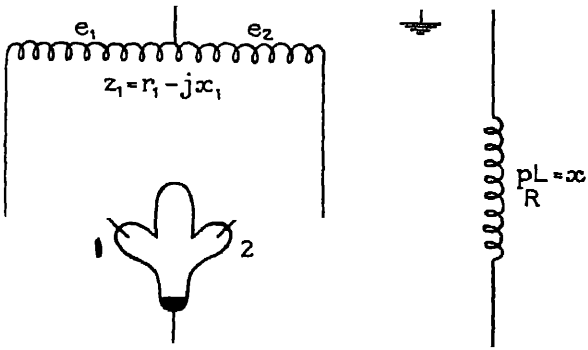


FIG. 121.—Biphase rectifier (mercury vapour).

Consider a biphase mercury vapour rectifier as is depicted in Fig. 121, where the load has an impedance of $\sqrt{R^2 + X^2}$, and the anode circuit has an inductance L_1 ($X_1 = pL_1$), then the

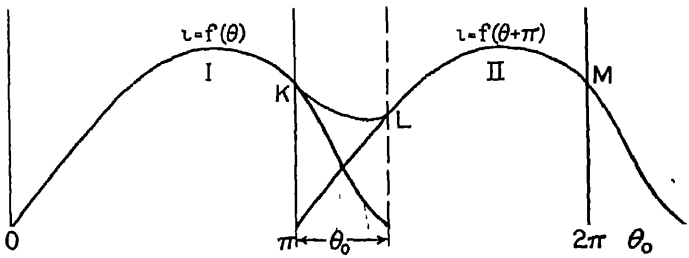


FIG. 122.—Current wave forms with overlap.

current curves will be distorted and will include an exponential factor, as will be shown later.

Assume for the moment that the resulting current waves take the form

$$i = f(\theta) \text{ and } f(\theta + \pi)$$

as shown in Fig. 122, there being an overlap of current waves to an extent of θ_0.

The normal voltage curves in Fig. 123 indicate the voltage of supply, the difference between the dotted and the full lines

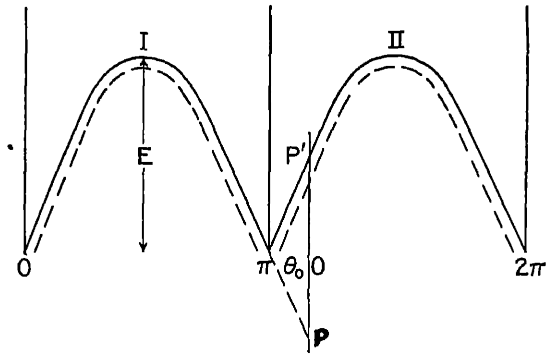


FIG. 123.—Normal voltage curves.

representing the drop in the rectifier, and the portion OP the potential of anode 1 below the neutral point, when the anode is

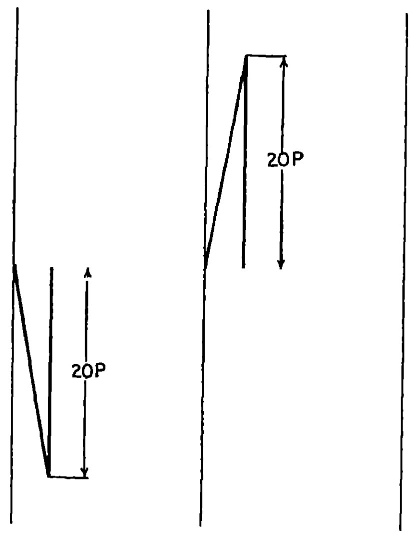


FIG. 124.—Voltage supplied by reactance.

normally non-conducting. If both anodes are conducting, and the net rectified current consists of the line OKLM in Fig. 122, and if duplicate conduction takes place during the period

of overlap θ_0, the initial conditions laid down on page 178 provide that the potential of anode 1 must be equal to that of anode 2, and of the same sense. This potential must, therefore, persist after the point π, in Fig. 123, and follow the curve of voltage pertaining to anode 2 to the point P′, which represents the end of the period of overlap. This excess voltage must be supplied from some extraneous source, which in practice consists of the counter E.M.F. of the inductance L'. This back E.M F. may, therefore, be represented by the peaky curves

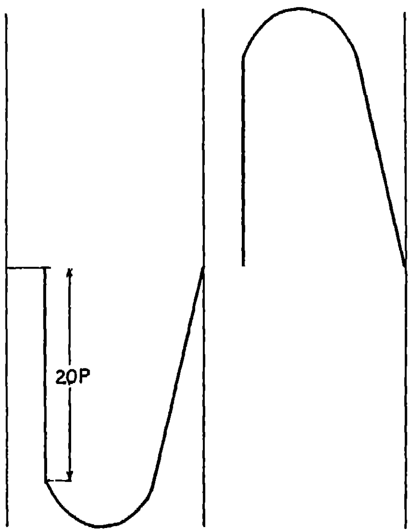


FIG. 125.—Voltage across anodes 1 and 2.

shown in Fig. 124, and the net voltage across the anodes 1 and 2 will take the form of the curves in Fig. 125.

The voltage of anode 1 to the neutral point of the transformer will be represented by the curve in Fig. 126.

If these theoretical curves are compared with the reproductions of actual oscillographs in Figs. 138 to 147 it will be observed that complete agreement with practical conditions is preserved.

If this reasoning is applied to a polyphase system where inductance is present in the anode circuit, whether due to an

external choke, or to the inherent leakage of the transformer windings, it will be noted that, provided that the inductance is great enough, current from any one anode will persist for a longer period than $2\pi/m$, where $2\pi/m$ is the period during which only one anode functions in the case of a non-reactive

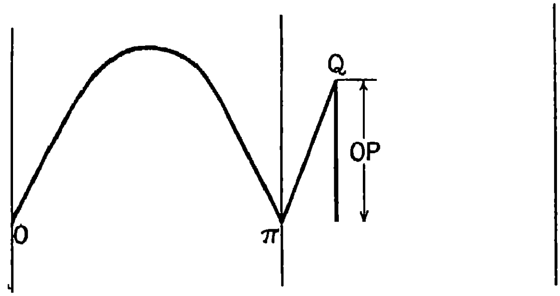


FIG. 126.—Voltage of anode to neutral.

circuit. The current may even continue for several periods of $2\pi/m$ in which case the inductance must supply the voltage to carry the current after the transformer winding has ceased to supply a sufficient E.M.F.

Referring to Fig. 127 the lines X and Y determine a complete

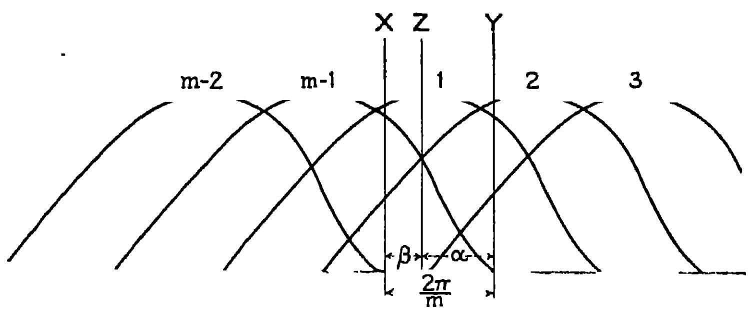


FIG. 127.—Polyphase currents.

sub-multiple $2\pi/m$ of the period of supply, and at the beginning of this interval at point X the $(m-2)$ anode current, which has been deformed from its true sinusoidal shape by the inductance, has fallen to zero, whilst the $(m-1)$ anode and the first and second anodes are still functioning. Thus the point X is a point of dis-

continuity in the wave form. Point Z where the third anode commences to function, marks another discontinuity, and point Y a third where the $(m-1)$ anode ceases to supply current. Thus if the problem is to be subjected to mathematical analysis, the complete interval $2\pi/m$, which represents an interval of supply which repeats itself regularly and systematically during the complete cycle, must be divided into two parts β and a. Over the portion β three anodes operate, whereas the second period a is distinguished by the fact that four anodes function. It will be apparent then that in the general case where p and $p+1$ anodes can operate in the intervals β and a respectively, the *modus operandi* can be completely determined by the scheme of Fig. 128.

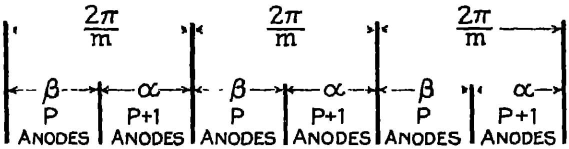


FIG. 128.—Scheme of functioning of anodes.

Further the overlap can at once be ascertained from the equation

$$\theta_0 = 2(p-1)\,\pi/m + a \quad . \quad . \quad . \quad (1)$$

A reference to Fig. 127 and putting $p = 3$ will show that this is true.

At the same time the total duration of time during which an anode supplies current is

$$2p\pi/m + a.$$

It is the purpose of this analysis, primarily to determine the characteristic of the rectifier; or in other words the curve connecting voltage drop and load. As, in an m phase system, there are only m channels through which current can pass, and the voltage drop in the rectifier is to an approximation independent of the current flowing, it follows that as the load increases, each anode must supply current for a longer period than it does on smaller loads, until at the short-circuit position all the anodes are passing current for the whole of the cycle.

As has been stated above, it is the function of the inductance in the anode circuit to prolong the voltage wave sufficiently to enable the voltages of all the anodes supplying current to be the same.

Thus it would be expected that as the load on a rectifier increases, the greater the number of anodes functioning at the same time, will become. The following scheme represents the state of affairs :—

No load :	1	anode	functioning	alone
	1 and 2	anodes	,,	alternately
Load	2	,,	,,	together
Increasing :	2 and 3	,,	,,	alternately
	3	,,	,,	together
	etc.			
Short-circuit :	m	,,	,,	,,

The period when p anodes are functioning represents a short transition stage between the condition of $p - 1$ and p anodes and p and $p + 1$ anodes functioning alternately, and although this period is only represented by a comparatively short period compared with the time of dual operation, it can be shown that between the point where p anodes and $p + 1$ anodes function together, the characteristic curve is linear, so that the ensuing calculations are simplified by ascertaining the positions of two adjacent points, where the conditions

$$a = o, \quad \theta_0 = 2(p - 1)2\pi/m, \quad \text{and} \quad \beta = 2\pi/m$$

apply, and thus the overlap is an integral multiple of the period $2\pi/m$.

Before proceeding to the general case, consider the no-load and short-circuit conditions, and in order that the problem shall be capable of mathematical treatment, it is necessary to make an assumption, which does not render the application of the analysis unsuitable for practical design. Assume that in Fig. 130 the reactance x_2 in the load circuit is of sufficient dimensions to render the wave form of the rectified current practically constant, and let I_M be the mean rectified current, i and i' the instantaneous currents in the load, i when $p + 1$

anodes are functioning, and i' when p functions, and i_0 the current at the time interval represented by $\theta = 0$.

Under these circumstances the wave form of an m phase system where $p = 1$ is as shown in Fig. 129, i.e. where two current loops overlap. These two currents during the period of overlap θ_0 add numerically to I_M, and thus the decreasing current of phase 1, and the increasing current of phase 2 are symmetrical about a horizontal line, represented by

$$i = I_M/2.$$

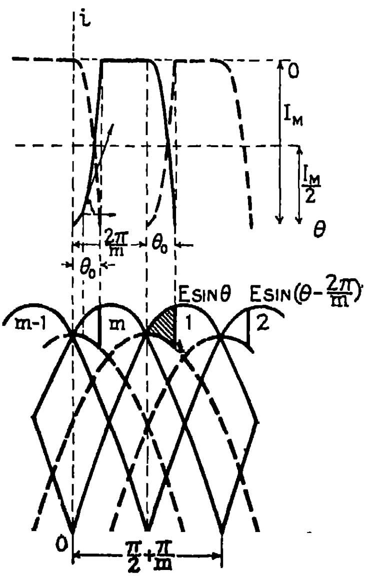


FIG. 129.—m phase rectifier wave form ($p = 1$).

Let these currents be i_1 and i_2 such that

$$i_1 + i_2 = I_M.$$

At this juncture it is convenient to assume that each anode during the period of overlap, contributes to the load a current $I_M/2$, and that the differences between the currents i_1 and i_2 and I_M are supplied by a circulating current in the closed circuit consisting of any two adjacent windings of the transformer and the corresponding anodes. These circulating currents must not

be considered as true short-circuit currents, although it will be appreciated that during the period of overlap two phases of the transformer are actually connected through the rectifier. They are represented in the following analysis by j_1 j_2 etc., to distinguish them from ordinary currents. Then

$$j_1 + j_2 = 0 \quad . \quad . \quad . \quad . \quad (2)$$

$$\left.\begin{aligned} i_1 &= j_1 + I_M/2 \\ i_2 &= j_2 + I_M/2 \end{aligned}\right\} \quad . \quad . \quad . \quad (3)$$

As the E.M.F. of the closed circuit of Fig. 130 is

$$e_1 - e_2 = 2x_1\frac{dj_1}{d\theta} \quad . \quad . \quad . \quad . \quad (4)$$

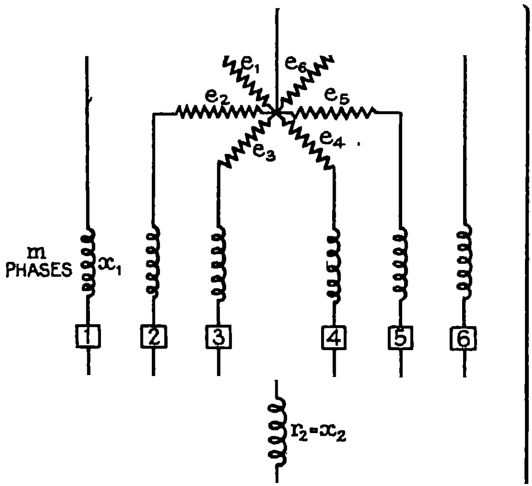


FIG. 130.—m phase rectifier system.

and as the various instantaneous voltages e_1 e_2, etc., are of the form

$$\begin{aligned} e_1 &= E \sin \theta \\ e_2 &= E \sin (\theta - 2\pi/m) \\ e_3 &= E \sin (\theta - 4\pi/m) \\ e_k &= E \sin \{\theta - 2(k - 1)\pi/m\}. \end{aligned}$$

then

$$\begin{aligned} j_1 &= \frac{E}{x_1} \sin \frac{\pi}{m} \int \cos \left(\theta - \frac{\pi}{m}\right) d\theta \\ &= \frac{E}{x_1} \sin \frac{\pi}{m} \sin \left(\theta - \frac{\pi}{m}\right) + C. \end{aligned}$$

The differential equation (4) assumes that the resistance of the closed circuit is negligible compared with its reactance, and it can be shown that to a first approximation this assumption is reasonable.

The constant C above is determined by the condition that when $j_1 = I_M/2$, $\theta = \pi/2 + \pi/m$ as the circulating currents only exist at the end of each interval of $\frac{2\pi}{m}$ degrees and therefore

$$C = \frac{I_M}{2} - \frac{E}{x_1}\sin\frac{\pi}{m}$$

and
$$j_1 = \frac{E}{x_1}\sin\frac{\pi}{m}\left[\sin\left(\theta - \frac{\pi}{m}\right) - 1\right] + \frac{I_M}{2}$$

Further when $j_1 = -I_M/2 \quad \theta = \pi/2 + \pi/m + \theta_0$,

whence
$$1 - \cos\theta_0 = \frac{x_1 I_M}{E\sin\frac{\pi}{m}} \quad . \quad . \quad . \quad (5)$$

and thus the overlap is determined.

The effect of the anode circuit inductance is manifested as a drop in the voltage of the rectified circuit, and to find an expression for it the following analysis is given.

It should be noted that the conception of the circulating currents j_1 and j_2 is important, in that their function is to provide the back E.M.F. in the inductance x_1, which carries on the E.M.F. wave past the point where it normally would be ineffectual in providing for rectification, thus

$$e_1 - x_1\frac{dj_1}{d\theta} = e_2 - x_1\frac{dj_2}{d\theta} \ldots = v \text{ (say)} \quad . \quad . \quad (6)$$

also
$$j_1 + j_2 = 0$$

and therefore
$$x_1\left(\frac{dj_1}{d\theta} + \frac{dj_2}{d\theta}\right) = 0$$

wherefore
$$v = \frac{e_1 + e_2}{2} = E\cos\frac{\pi}{m}\sin\left(\theta - \frac{\pi}{m}\right).$$

During the period of overlap θ_0, the rectified voltage is a mean of the voltages of the anodes which contribute current,

less the drop in the inductances, and therefore if E_M is the mean rectified voltage, and ΔE_M the drop due to the anode circuit inductance

$$\Delta E_M = \frac{m}{2\pi}\int_{\frac{\pi}{2}-\frac{\pi}{m}}^{\frac{\pi}{2}-\frac{\pi}{m}+\theta_0}\left\{e_2 - \frac{e_1 + e_2}{2}\right\}d\theta$$

$$= \frac{m}{2\pi}E \sin\frac{\pi}{m}\int_{\frac{\pi}{2}-\frac{\pi}{m}}^{\frac{\pi}{2}-\frac{\pi}{m}+\theta_0}\cos\left(\theta - \frac{\pi}{m}\right)d\theta$$

$$= \frac{m}{2\pi}E \sin\frac{\pi}{m}(1 - \cos\theta_0) \quad . \quad . \quad . \quad (7)$$

Eliminating θ_0 from equations (5) and (7)

$$\Delta E_M = \frac{m}{2\pi}x_1 I_M \quad . \quad . \quad . \quad . \quad (8)$$

It is interesting to note that in this case the rectifier behaves as if the drop in voltage on the rectified side were due to the passage of a current whose R.M.S. value is

$$\frac{mI_M}{2\pi}.$$

So far the example given is a particular case for an m phase circuit, where only two anodes function at one and the same time. Later this will be shown to determine the particular loading to which the rectifier is subjected.

Proceeding to the general case of all loads from equation (6) it is noted that the function of v is to supply a back E.M.F. which will render the separate voltages of all the anode circuits, which supply current simultaneously, equal, this being the condition of rectification; and the characteristic equation of v may therefore be written

$$v = \frac{1}{p}\sum_{k=1}^{p} e_k . \quad . \quad . \quad . \quad . \quad (9)$$

This equation is the general characteristic of the rectifier, and the others which have been mentioned above may be written in the more general forms

$$\sum_{k=1}^{p} j_k = 0 \qquad . \quad . \quad . \quad (10)$$

$$\left(j_1 + \frac{I_M}{p}\right)_{\theta'} = 0 \qquad . \quad . \quad . \quad (11)$$

$$x_1 \frac{dj_k}{d\theta} = e_k - v \qquad . \quad . \quad . \quad (12)$$

The last condition to be employed is the one which determines the fact that although the voltage wave forms are discontinuous, yet they have the same value at the discontinuity, whence

$$\left[\frac{1}{p}\sum_{k=1}^{p} e_k = \frac{1}{p+1}\sum_{k=1}^{p+1} e_k\right]_{\theta=\theta'} \qquad . \quad . \quad . \quad (13)$$

which determines θ', the point at which the discontinuity occurs; and in the case of the current waves, the same applies, which provides the equation

$$\left(j_{k-1} + \frac{I_M}{p}\right)_{\theta=\theta'-\frac{2\pi}{m}} = \left(j_k + \frac{I_M}{p}\right)_{\theta=\theta'} \qquad . \quad . \quad (14)$$

It is instructive to apply this analysis to the calculation of the short-circuit current of an m phase rectifier, in which case all phases will be operating together and $p = m$.

From equation (9) for $p = m$

$$\begin{aligned} v &= \frac{1}{m}\sum_{1}^{p} e_k \\ &= \frac{E}{m}\left[\sin\theta + \sin\left(\theta - \frac{2\pi}{m}\right) + \sin\left(\theta - \frac{4\pi}{m}\right) + \ldots \text{ to } m \text{ terms}\right] \\ &= 0. \end{aligned}$$

From equation (12)

$$x_1 \frac{dj_k}{d\theta} = e_k = E \sin\left\{\theta - 2(k-1)\frac{\pi}{m}\right\}$$

or

$$j_k = -\frac{E}{x_1}\cos\left\{\theta - 2(k-1)\frac{\pi}{m}\right\} + C.$$

To ascertain C from equation (10)

$$\sum_{k=1}^{m} j_k = -\frac{E}{x_1}\left\{\cos\theta + \cos\left(\theta - \frac{2\pi}{m}\right) + \ . \ . \ . \text{ to } m \text{ terms}\right\} + C$$
$$= 0 + C = 0$$

and hence $C = 0$, or

$$j_k = -\frac{E}{x_1}\cos\left\{\theta - 2(k-1)\frac{\pi}{m}\right\}.$$

To employ equation (13) to ascertain θ' for $p = m$

$$\frac{1}{m}\left[E\sin\theta' + E\sin\left(\theta' - \frac{2\pi}{m}\right) + \ . \ . \ . \ m \text{ terms}\right]$$
$$= \frac{1}{m+1}\left[E\sin\theta' + E\sin\left(\theta' - \frac{2\pi}{m}\right) + \ . \ . \ . \ (m+1) \text{ terms}\right]$$

whence

$$\sin(\theta' - 2\pi) = 0 \quad \text{and} \quad \theta' = 0.$$

Finally from equation (14)

$$\left(j_1 + \frac{I_M}{p}\right)\theta' = 0,$$

or

$$\left[-\frac{E}{x_1}\cos\left\{\theta - 2(k-1)\frac{\pi}{m}\right\} + \frac{I_M}{p}\right]_{\substack{p=m\\k=1\\\theta=\theta'=0}} = 0$$

or

$$\frac{E}{x_1} = \frac{I_{sc}}{m} \text{ where } I_{sc} \text{ is the short-circuit current}$$

and

$$I_{sc} = \frac{mE}{x_1} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

This last equation (15), represents the short-circuit current for an m phase rectifier.

To consider the wave form of the intermediate positions between no-load and short-circuit, a three-phase system is investigated and the voltage relationships are indicated in Fig. 131.

The curves 1, 2, and 3 are the open-circuit conditions, where

$$e_1 = E \sin \theta$$
$$e_2 = E \sin (\theta - 2\pi/3)$$
$$e_3 = E \sin (\theta - 4\pi/3)$$

and
$$E_M = \frac{Em}{\pi} \sin \frac{\pi}{m}$$

where $m = 3$, and only one anode functions at one and the same time.

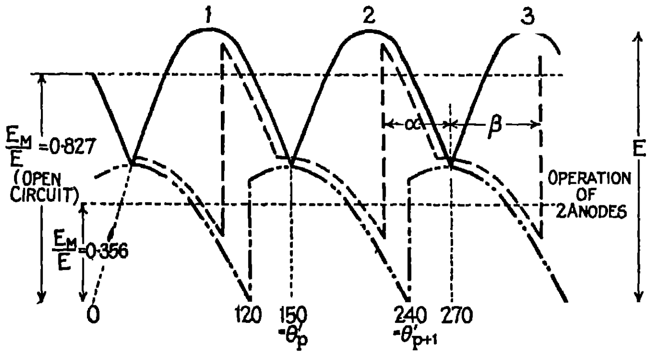


FIG. 131.—Voltage-load curves with a three-phase rectifier.

For the short-circuit state

$$I_{sc} = mE/x_1$$

and three anodes function together. At the point where two anodes operate the double-dot-dash curve is the wave form. In the intermediate stage where one and two anodes supply current alternately, viz. when $a \neq 0$, equation (1), the wave form will be as indicated by the dash curve with a much more peaky crest value. These curves exemplify the periods a and β in the latter case, and also conditions when $a = 0$. At the same time a most important point is indicated, viz. that when p anodes are functioning the equations are operable over a period $2\pi/m$ commencing at a particular value of θ' viz., an angle θ'_p obtained from equation 13, but when $p + 1$ anodes function this period $2\pi/m$ is moved forward, and a new calculation for θ',

viz. θ'_{p+1} is necessary, and finally limits are imposed which are

$$\theta'_{p+1} - 2\pi/m \text{ becomes } \theta'_p$$

and
$$\theta'_{p+1} \quad ,, \quad \theta'_p + 2\pi/m.$$

When the latter limits are employed, the lower values of the current and voltage are obtained; these values being termed I'_M and E'_M.

As this analysis is complicated, although by no means difficult to understand, an example of a three-phase supply is considered, as shown in Fig. 131.

Under these circumstances

$$\theta'_p = \pi/2 + \pi/3 = \frac{5\pi}{6}$$

and
$$\theta'_p - 2\pi/3 = \pi/6.$$

From equation (13) for $p = 2$

$$\frac{1}{2}\left\{E \sin \theta'_{p+1} + E \sin \left(\theta'_{p+1} - \frac{2\pi}{3}\right)\right\}$$
$$= \frac{1}{3}\left\{\sin \theta'_{p+1} + \sin \left(\theta'_{p+1} - \frac{2\pi}{3}\right) + \sin \left(\theta'_{p+1} - \frac{4\pi}{3}\right)\right\}$$

whence
$$\theta'_{p+1} = 4\pi/3$$

and
$$\theta'_{p+1} - 2\pi/3 = 2\pi/3.$$

This is the case when the wave forms are

$$e_1 = E \sin \theta$$
$$e_2 = E \sin (\theta - 2\pi/3)$$
$$e_3 = E \sin (\theta - 4\pi/3)$$

From equation (9)

$$v = \tfrac{1}{2}[E \sin \theta + E \sin (\theta - 2\pi/3)]$$
$$= E \cos \pi/3 \sin (\theta - \pi/3)$$

and from equation (12)

$$x_1\frac{dj_1}{d\theta} = E \sin \theta - E \cos \pi/3 \sin (\theta - \pi/3)$$

and
$$x_1\frac{dj_2}{d\theta} = E \sin (\theta - 2\pi/3) - E \cos \pi/3 \sin (\theta - \pi/3)$$

whence

$$\frac{x_1}{E}j_1 = \sin \pi/3 \sin (\theta - \pi/3) + C_1$$

$$\frac{x_1}{E}j_2 = -\sin \pi/3 \sin (\theta - \pi/3) + C_2.$$

From equation (10) $C_1 + C_2 = 0.$

In equation (11) putting $\theta' = \theta'_p + 2\pi/3$

$$\frac{E_1}{x_1}[\sin \pi/3 \sin (4\pi/3 - \pi/3) + C_1] + I_M/2 = 0,$$

whence $C_1 = -\dfrac{I_M x_1}{2E},$

and $C_2 = \dfrac{I_M x_1}{2E}.$

In equation (14) with the same value for θ'

$$\frac{E_1}{x_1}[\sin \pi/3 \sin \pi/3 + C_1] + I_M/2$$

$$= \frac{E}{x_1}[-\sin \pi/3 \sin \pi + C_2] + I_M/2$$

whence $C_1 - C_2 = -\sin^2 \pi/3,$

and eliminating the constants of integration

$$\frac{I_M x_1}{E} = 0{\cdot}75.$$

Now when $\theta' = \theta'_{p+1}$

it is found by the similar use of equations (10) and (14) that

$$C'_1 = -3/4 - \frac{I'_M x_1}{2E},$$

and $C'_1 - C'_2 = -\sqrt{3}/4,$

wherefore

$$\frac{I'_M x_1}{E} = \frac{3\sqrt{3}}{4} = 1{\cdot}29.$$

The value of the rectified voltage E_M is obtained in the usual way from the formula

$$E_M = \frac{m}{2\pi}\int_{\theta'-\frac{2\pi}{m}}^{\theta'} v\,d\theta = \frac{m}{2\pi}\int_{\theta'-\frac{2\pi}{m}}^{\theta} \frac{1}{p}\sum_1^p e_k\,d\theta$$

according to definition, and hence depending on the particular value of θ', viz. $\theta'_p + 2\pi/3$ or θ'_{p+1}, and for $p = 2$

$$E_M/E = 0{\cdot}356 \text{ and } E'_M/E = 0{\cdot}206.$$

The open-circuit value is

$$E_{MO}/E = 3/\pi \sin \pi/3 = 0{\cdot}827.$$

These results may be tabulated as follows:—

$$\begin{array}{ll}
E_{MO}/E = 0{\cdot}827 & I_{MO} = 0 \\
E_{M2}/E = 0{\cdot}356 & I_{M2}x_1/E = 0{\cdot}75 \\
E'_{M2}/E = 0{\cdot}206 & I'_{M2}x_1/E = 1{\cdot}29 \\
E_{Msc}/E = 0. & I_{Msc}x_1/E = 3{\cdot}0
\end{array}$$

where E_{MO} is the mean rectified voltage on open-circuit, and I_{Msc} is the mean rectified current on short-circuit. E_{M2}, I_{M2}, E'_{M2}, I'_{M2} are the mean rectified voltages and currents at intermediate loadings.

From these values of the load currents and voltages, the external characteristic of the rectifier may be ascertained, remembering that between points where only 1, 2, or 3 anodes are functioning during the sub-multiple period $2\pi/3$, the relation is linear.

Thus the open-circuit voltage determines the point where the load current is zero, and a straight line can be drawn through this point to the point whose co-ordinates are (1·29, 0·206): from the point (0·75, 0·356) a straight line should be drawn to the point (3, 0). (Fig. 132.)

The short vertical line connecting the two characteristics represents a true discontinuity which will be discussed later.

General Case.—It will be apparent that if an overlap exists, a discontinuity will occur in the scheme of operation on page 187, and therefore when p and $p + 1$ anodes function, the periodic interval of $2\pi/m$ must be separately considered from the double viewpoint of the angles α and β, and separate calculations made for each. It is convenient to employ the symbols above for the angle β, and the same symbols with a double dashed sign for the interval α, but in other respects the method is the same.

When the separated periods are employed it is evident that

the following relations also will apply

$$\sum_{k=1}^{p+1} j''_k = 0 \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$\left(j_k + \frac{I_M}{p}\right)_{\theta'} = \left(j''_k + \frac{I_M}{p+1}\right)_{\theta'} \quad . \quad . \quad (17)$$

$$\left(j_k + \frac{I_M}{p}\right)_{\theta'-\beta=\theta'-\frac{2\pi}{m}+a} = \left(j''_{k+1} + \frac{I_M}{p+1}\right)_{\theta'+a} \quad . \quad . \quad (18)$$

where as has been seen from equation (1) $\theta_0 = 2(p - 1)\pi/m + a$.

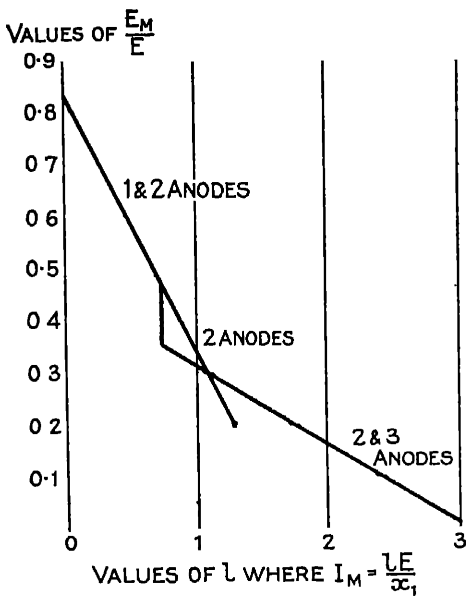


FIG. 132.—Three-phase rectifier characteristic.

As a further example of this method with separated periods consider the arrangement of one and two anodes supplying current, as shown in Fig. 129. From equation (9)

$$v = E \sin \theta$$

and

$$v'' = \tfrac{1}{2}E\{\sin \theta + \sin (\theta - 2\pi/m)\}$$
$$= E \cos \pi/m \sin (\theta - \pi/m)$$

from equation (1) $\qquad \theta_0 = a$

it is necessary to perform m complete separate calculations, in each analysis employing all the characteristic equations given. Thus, for instance, putting $p = 1$, the open-circuit condition is obtained, but there comes a time as the load increases, when the inductive E.M.F. increases to such an extent that the overlap lengthens and $p = 2$. For this state of affairs a new series of calculations is required, and at this point it will be necessary to take into consideration the divided period α and β, and the duplicate equations will be needed. The value so

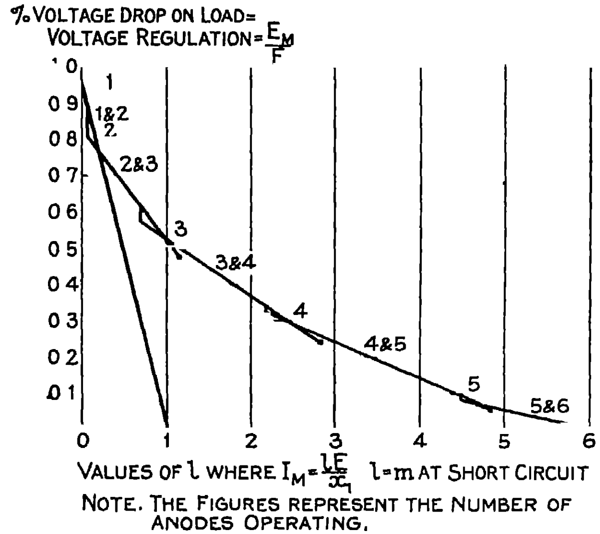


FIG. 133.—Six-phase rectifier characteristic.

obtained will persist for a state of loading when $p = 3$, and the process is repeated. Finally when $p = m$ the short-circuit condition above is reached. The calculation of the complete characteristic is thus a lengthy process, and will only rarely be necessary. The full calculations have been omitted and only the resulting curve is reproduced.

If the hexaphase case is investigated, the general curve of a rectifier will be obtained and is shown in Fig. 133, where it will be seen that as the load current increases, so the number of anodes p which function at one and the same time, also

increases to the short-circuit point, when all the six anodes supply current together. That this would happen could be deduced from the above analysis directly, where it is seen that as the load current I_M increases so the angle of overlap θ_0 increases, which from equation (14) infers that p also increases to a maximum value of m.

The above analysis results in a series of discontinuities in the characteristic curve, which are not observed in practice, and it is possible that their absence may be due to some delay action in the operation of the arc. Child has shown that a phase difference exists between the light intensity of the arc and the current, which may be as much as one-twentieth of a cycle. This infers also that the theory of the arc conductivity is not yet thoroughly explained, and it would appear probable that bound up with this experimental fact, is the unexplained absence of the discontinuities, which theory would expect to be present.

This analysis has been investigated at some length; but it is interesting to note that the rating of rectifiers is usually well below the short-circuit point; and thus the earlier calculations, embodied in equations (5) to (8), will meet most of the requirements up to full load conditions.

This naturally leads up to a further investigation by means of calculations of wave form, which assumes that at the most only two anodes function at one and the same time.

Single-Phase Rectification.—Having described in some detail the general method, it is interesting to apply the method of analysis, which results immediately in the wave form of the current. This treatment is not available for the general case, but it is useful to be able to employ it in certain particular cases. The equation for this deformed wave can be easily ascertained by referring to the conditions appertaining in Fig. 115.

Let the voltage of supply be denoted by

$$E \sin \theta$$

and r and x be the resistance and reactance of the load. Then if the drop across the rectifier is neglected,

$$E \sin \theta = ri + x \frac{di}{d\theta}$$

or
$$\left(D + \frac{r}{x}\right)i = \frac{E}{x}\sin\theta$$

The solution of this equation is
$$i = A\epsilon^{-\frac{r}{x}\theta} + \frac{r\sin\theta - x\cos\theta}{r^2 + x^2}E.$$

Now when $\theta = 0$, $i = 0$, which provides a value for the constant A and hence, writing
$$\tan a = \frac{x}{r}$$
the original equation becomes
$$i = \frac{E}{\sqrt{r^2 + x^2}}\left\{\sin(\theta - a) + \epsilon^{-\frac{r}{x}\theta}\sin a\right\} \qquad . \quad (19)$$
The term
$$\frac{E\sin a}{\sqrt{r^2 + x^2}}\epsilon^{-\frac{r}{x}\theta}$$
is the one which contributes to the spreading out of the current wave, and thus renders this type of rectification possible; at the same time a certain lag a is obtained, and the inductance, therefore, has the dual effect of introducing a wattless component as well as decreasing the undulatoriness of the wave.

Further, this effect is rendered greater or less according as x is large or small compared with r, and it would be anticipated that a load with an impedance with a preponderating inductive term would result in a smoother wave form than one where the resistance predominated. If the load is non-reactive the same effect can be obtained by the insertion of a reactance in the load circuit.

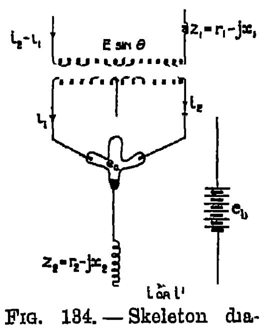


FIG. 134.—Skeleton diagram of mercury vapour rectifier circuit.

Biphase Rectification—Wave Form Method Theory.—It should be mentioned that whereas in Fig. 116 two separate rectifiers are shown, in actual practice in mercury vapour rectifiers two anodes at least must be included in one bulb, as shown in Fig. 134. Thus before one arc is extinguished the

other is prepared to take its share of the current. Under these circumstances let the impedance of the primary circuit be

$$Z_1 = r_1 + jx_1$$

and of the rectified circuit

$$Z_2 = r_2 + jx_2,$$

i_1 and i_2 will be the instantaneous currents in the two anode circuits, i and i' are the instantaneous currents in the rectified circuit, during the period when two anode currents are flowing, and when only one anode is functioning respectively, e_a the constant voltage drop across the arc, and e_b the counter E.M.F. of the load circuit. The net back E.M.F. is therefore

$$e = e_a + e_b.$$

Finally, θ_0 is the angle of overlap of the two current waves (see Fig. 117).

Firstly, consider the rectified circuit during the period θ_0 when both arcs exist at one and the same time; the equation to the voltage drop is

$$x_2\frac{d(i_1 + i_2)}{d\theta} + r_2(i_1 + i_2) + e = 0 \quad . \quad . \quad (20)$$

and during the same period a similar equation can be written down for the secondary circuit of the transformer, assuming in this case a 1/1 ratio of transformation, viz. :—

$$- E \sin \theta - r_1(i_2 - i_1) - x_1\frac{d(i_2 - i_1)}{d\theta} = 0 \quad . \quad (21)$$

Secondly, during the period from $\theta = \theta_0$ till $\theta = \pi$, i.e. when only one anode is functioning,

$$E \sin \theta - r_1 i' - x_1\frac{di'}{d\theta} = e + r_2 i' + x_2\frac{di'}{d\theta} \quad . \quad (22)$$

where i' is the instantaneous current in the rectifier circuit.

From the above it will be seen that the primary and the rectifier impedances can be considered as one by writing

$$Z = Z_1 + Z_2,$$

and therefore $\qquad r = r_1 + r_2 \quad \text{and} \quad x = x_1 + x_2.$

Thus equation (22) can be written

$$E \sin \theta - ri' - x \frac{di'}{d\theta} - e = 0,$$

and the solutions of these three equations are

$$i = i_1 + i_2 = A\epsilon^{-\frac{r}{x_2}\theta} - \frac{e}{r_2}$$

$$i_2 - i_1 = -E \frac{r_1 \sin \theta - x_1 \cos \theta}{r_1^2 + x_1^2} + B\epsilon^{-\frac{r_1}{x_1}\theta}$$

and $$i' = E \frac{r \sin \theta - x \cos \theta}{r^2 + x^2} - \frac{e}{r} + C\epsilon^{-\frac{r}{x}\theta}.$$

Let $$\frac{r_1}{x_1} = a, \quad \frac{r_2}{x_2} = b, \quad \text{and } \frac{r}{x} = c$$

and $$\frac{x_1}{r_1} = \tan \beta_1 \text{ and } \frac{x}{r} = \tan \beta.$$

The equations may now be written

$$i = i_1 + i_2 = A\epsilon^{-b\theta} - \frac{e}{r_2} \quad . \quad . \quad . \quad (23)$$

$$i_2 - i_1 = -\frac{E}{Z_1} \sin (\theta - \beta_1) + B\epsilon^{-a\theta} \quad . \quad . \quad (24)$$

and $$i' = \frac{E}{Z} \sin (\theta - \beta) - \frac{e}{r} + C\epsilon^{-c\theta} . \quad . \quad . \quad (25)$$

Now the rectified current when both arcs exist is

$$i_1 + i_2 = i,$$

and when only one arc exists it is i'; further, the constants are to be determined by the terminal conditions, which are

$$\text{for } \theta = 0, \quad i_2 = i, \quad \text{and } i_1 = 0,$$

and therefore in equation (4)

$$(i)_{\theta=0} = A - \frac{e}{r_2} \text{ or } A = (i)_{\theta=0} + \frac{e}{r_2}.$$

In equation (24) also for $\theta = 0$, $i_2 = i$ and $i_1 = 0$ and therefore

$$-(i)_{\theta=0} = \frac{E}{Z_1} \sin \beta_1 + B,$$

or $$B = (i)_{\theta=0} - \frac{E}{Z_1} \sin \beta_1.$$

Finally, in equation (25) for $\theta = \pi$, $i' = i$

$$(i)_{\theta=0} = \frac{E}{Z}\sin\beta - \frac{e}{r} + C\epsilon^{-c\pi}$$

or

$$C = \epsilon^{c\pi}\left\{(i)_{\theta=0} + \frac{e}{r} - \frac{E}{Z}\sin\beta\right\}.$$

The further condition for the end of the period of overlap is also available where

$$\theta = \theta_0, \quad i_2 = 0, \quad i_1 = i, \quad \text{and } i' = i$$

and hence, as

$$(i)_{\theta=0} = (i)_{\theta=\pi} = i_0 \text{ (say)}$$

$$A\epsilon^{-b\theta_0} - \frac{e}{r_2} = \frac{E}{Z_1}\sin(\theta_0 - \beta_1) - B\epsilon^{-a\theta_0}$$
$$= \frac{E}{Z}\sin(\theta_0 - \beta) - \frac{e}{r} + C\epsilon^{-c\theta_0}$$

Filling in the values for A, B, and C, as arrived at above

$$\left(i_0 + \frac{e}{r_2}\right)\epsilon^{-b\theta_0} - \frac{e}{r_2} = \frac{E}{Z_1}\sin(\theta_0 - \beta_1) - \epsilon^{-a\theta_0}\left(i_0 - \frac{E}{Z_1}\sin\beta_1\right)$$
$$= \frac{E}{Z}\sin(\theta_0 - \beta) - \frac{e}{r} + \epsilon^{c\pi}\left\{i_0 + \frac{e}{r} - \frac{E}{Z}\sin\beta\right\}\epsilon^{-c\theta_0} \qquad (26)$$

At this point it is necessary to stipulate whether the rectifier will be used on a constant voltage or constant current system. In the former case E will have been given and i_0 can be eliminated; if this operation is carried out, the following equations are obtained from which with a certain amount of trouble, the overlap can be calculated:—

$$\frac{\epsilon^{-b\theta_0} + \epsilon^{-a\theta_0}}{\epsilon^{-b\theta_0} - \epsilon^{c(\pi-\theta_0)}} = \frac{\dfrac{e}{r_2}(1 - \epsilon^{-b\theta_0}) + \dfrac{E}{Z_1}\{\epsilon^{-a\theta_0}\sin\beta_1 - \sin(\beta - \theta_0)\}}{\dfrac{e}{r_2}(1 - \epsilon^{-b\theta_0}) + \dfrac{E}{Z}\{\sin(\beta - \theta_0) + \epsilon^{c(\pi-\theta_0)}\sin\beta\} + \dfrac{e}{r}\{\epsilon^{c(\pi-\theta)} - 1\}} \qquad (27)$$

On the other hand, however, if the circuit is supplied from a constant current transformer the mean rectified current also will be constant, and it may be assumed that this mean current

will be approximately equal to the instantaneous value of the rectified current at the time when $\theta = 0$, and thus

$$I_M = i_0.$$

In this case E should be eliminated from the two equations (27) with the result

$$\frac{\epsilon^{c(\pi-\theta_0)} \sin \beta + \sin (\beta - \theta_0)}{\epsilon^{-a\theta_0} . \sin \beta_1 - \sin (\beta_1 - \theta_0)}$$

$$= \frac{Z}{Z_1} \cdot \frac{\epsilon^{c(\pi-\theta_0)} - \epsilon^{-b\theta_0} + \dfrac{e}{I_M r_2}(1 - \epsilon^{-b\theta_0}) + \dfrac{e}{I_M r}\{\epsilon^{c(\pi-\theta_0)} - 1\}}{\epsilon^{-b\theta_0} + \epsilon^{-a\theta_0} - \dfrac{e}{I_M r_2}(1 - \epsilon^{-b\theta_0})} \tag{28}$$

Returning to the equations of the currents

$$i_1 + i_2 = A\epsilon^{-b\theta} - \frac{e}{r_2}$$

$$i_2 - i_1 = -\frac{E}{Z_1} \sin (\theta - \beta_1) + B\epsilon^{-a\theta}$$

$$i' = \frac{E}{Z} \sin (\theta - \beta) - \frac{e}{r} + C\epsilon^{-c\theta}$$

and filling in the values obtained for the constants A, B, and C

$$i_1 + i_2 = \frac{e}{r_2}(\epsilon^{-b\theta} - 1) + i_0\epsilon^{-b\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \tag{29}$$

$$i_2 - i_1 = -\frac{E}{Z_1}\{\sin (\theta - \beta_1) + \sin \beta_1\epsilon^{-a\theta}\} + i_0\epsilon^{-a\theta} \quad . \quad . \tag{30}$$

$$i' = \frac{E}{Z}\{\sin (\theta - \beta) - \epsilon^{c(\pi-\theta)} \sin \beta\} + \frac{e}{r}\{\epsilon^{c(\pi-\theta)} - 1\} + i_0\epsilon^{c(\pi-\theta)} \tag{31}$$

Now the current equations can be completely determined when the circuit constants are settled, and also when E and i_0 are fixed by the conditions of supply. In the case of a constant current supply I_M is given, and E in the case of a constant voltage service, and hence, given one of these quantities, θ_0 can be determined from either of the equations (27) or (28). This value of θ_0 inserted in one of the equations (26) will enable the other to be determined, and from these two the current equations can be completed.

There is one case, however, viz. that where the current is given, where it may be of advantage to ascertain the voltage required to supply a certain current, as for instance in the case of battery charging. Here the charging current will be known before the set is designed, and hence the condition $I_M = i_0$ is available to an approximation. The procedure is then to eliminate E from the equations (26) and proceed to fill in the values of the constants and thus ascertain the current waves.

Biphase Rectification—Practical Example.—To make the matter clear a numerical example is given, but in choosing the values of the resistances and the reactances a rough calculation beforehand is advisable to make sure that an approximately correct value for the impedance is inserted.

Assume then that a 110 volt battery is to be charged with a normal current of 10 amperes, and thus $e = 124$ volts and

$$I_M = i_0 = 10.$$

Also assume

$$Z_1 = 0{\cdot}2 - 16\,j \text{ and } Z_2 = 3 - 15\,j,$$

whence $$Z = 3{\cdot}2 - 31\,j.$$

This at once gives the values of the constants

$$a = \frac{r_1}{x_1} = 0{\cdot}0125,\ b = \frac{r_2}{x_2} = 0{\cdot}2,\ c = \frac{r}{x} = 0{\cdot}103,$$

also $\tan\beta_1 = \dfrac{x_1}{r_1}$ and $\beta_1 = 89{\cdot}3^\circ$ and similarly $\beta = 84{\cdot}1^\circ$.

The equation (28) for θ_0 may then be written down

$$\frac{0{\cdot}995\epsilon^{0\cdot103(\pi-\theta_0)} + \sin(84{\cdot}1^\circ - \theta_0)}{0{\cdot}999\epsilon^{-0\cdot012\theta_0} - \sin(89{\cdot}3^\circ - \theta_0)} = \frac{9{\cdot}55\epsilon^{0\cdot103(\pi-\theta_0)} - 10\epsilon^{0\cdot2\theta_0} + 0{\cdot}49}{5{\cdot}13\epsilon^{-0\cdot2\theta_0} + \epsilon^{0\cdot012\theta_0} - 4{\cdot}13}.$$

This expression for θ_0 has now been reduced to its simplest form algebraically and it only remains to calculate it. Attempts have been made to evaluate an exponential equation of this type, but so far the writer has not discovered one which for speed in this particular case equals that of tabulating the right and left hand sides of the equation for various values of θ_0 and plotting the two curves so obtained, their point of intersection being the value sought. In this particular numerical instance one hour

sufficed for the calculations, using four points, viz. 30°, 40°, 60° and 70°. The first two values will indicate the relative slopes of the curves and these will give a rough idea of the next best values to assume. It is usually only necessary to take two more points to arrive at a result amply accurate for the purpose. It might be stated that such approximations as Newtons, Legrange's Theorem, etc., do not apply generally to functions containing exponentials, and if an equation can be arrived at which purports to give a true value, it itself can only be an approximation.

It will be shown later, however, that to a close approximation equation (5) may be employed to calculate θ_0 directly.

In this particular case θ_0 is found to be 56°. E can now be obtained from the equation (26) viz.:—

$$\left(i_0 + \frac{e}{r_2}\right)\epsilon^{-b\theta_0} - \frac{e}{r_2} = \frac{E}{Z_1}\sin(\theta_0 - \beta_1) - \left(i_0 - \frac{E}{Z_1}\sin\beta_1\right)\epsilon^{-a\theta_0}$$

whence

$$E = \frac{Z_1\left\{i_0\epsilon^{-a\theta_0} - \dfrac{e}{r_2} + \epsilon^{-b\theta_0}\left(i_0 + \dfrac{e}{r_2}\right)\right\}}{\sin(\theta_0 - \beta_1) + \sin\beta_1\epsilon^{-a\theta_0}}$$

$= 389$ volts (maximum value)

$= 276$ (R.M.S.) volts if a sine wave supply is available.

The current equations now become

$$i_1 = \tfrac{1}{2}\left[\frac{E}{Z_1}\{\sin(\theta - \beta_1) + \epsilon^{-a\theta}\sin\beta_1\} + i_0(\epsilon^{-b\theta} - \epsilon^{-a\theta}) + \frac{e}{r_2}(\epsilon^{-b\theta} - 1)\right]$$

and

$$i' = \frac{E}{Z}\{\sin(\theta - \beta) - \epsilon^{c(\pi-\theta)}\sin\beta\} + \frac{e}{r}(\epsilon^{c(\pi-\theta)} - 1) + i_0\epsilon^{c(\pi-\theta)}$$

whence filling in the values obtained

$$\begin{aligned} i_1 + i_2 &\quad 51{\cdot}3\,\epsilon^{-0\cdot2\theta} - 41{\cdot}3 \\ i' &\quad 12{\cdot}45\sin(\theta - 84{\cdot}1°) + 36{\cdot}9\,\epsilon^{0\cdot103(\pi-\theta)} - 38{\cdot}8 \\ i_1 &\quad 12{\cdot}15\sin(\theta - 89{\cdot}3°) + 7{\cdot}15\,\epsilon^{-0\cdot0125\theta} + 25{\cdot}6\epsilon^{-0\cdot2\theta} - 20{\cdot}6. \end{aligned}$$

These currents are plotted in Fig. 135 and it will be noted that the anode current i_1 has a negative value from 0 to 45°.

This in the nature of things cannot happen in practice because the anode is not a hot electrode, but theoretically no condition is laid down whereby i_1 cannot exist below the line of zero current: it may happen in an actual case that if sufficient inductance is not inserted to ensure a greater variation from the mean value than is the case here, the value i_1 will be zero until a considerable time has elapsed: in this case until 46° electrical degrees have elapsed. This modifies the current wave and the resultant rectified current will take the form i_2 from 0 to 46°

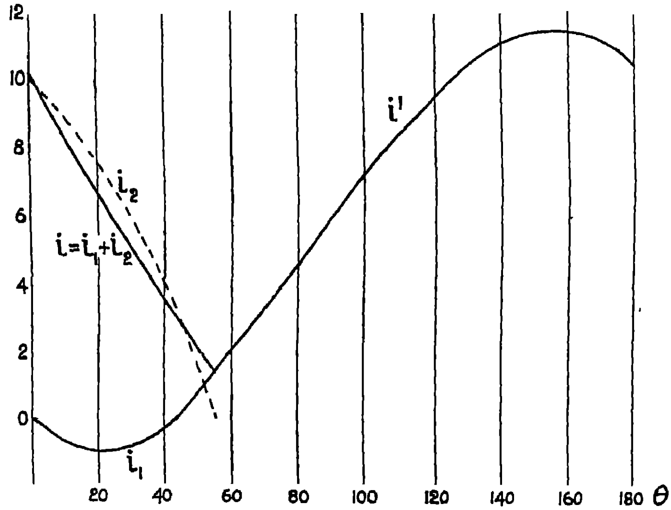


FIG. 185.—Current wave in biphase rectifier.

and $i_1 + i_2$ from 46° to 56°. However, in the ensuing calculations of wave form this negative value of i_1 has been assumed to be possible—the example being chosen to exhibit the theoretical possibility of such an occurrence. A further point is the assumption of the equality

$$i_0 = 10 = I_M.$$

It is seen that $i_0 = 10$ is true but on calculation

$$I_M = 7{\cdot}35$$

which is too low a value for the mean or equivalent charging current. The actual mean current equation is obviously

$$I_M = \frac{1}{\theta_0}\int_0^{\theta_0} \tfrac{1}{2}\left[\frac{E}{Z_1}\{\sin(\theta - \beta_1) + \epsilon^{-a\theta}\sin\beta_1\} + i_0(\epsilon^{-b\theta} - \epsilon^{-a\theta}) + \frac{e}{r_2}(\epsilon^{-b\theta} - 1)\right]d\theta$$

$$+ \frac{1}{\pi - \theta_0}\int_{\theta_0}^{\pi}\left[\frac{E}{Z}\{\sin(\theta - \beta) - \epsilon^{c(\pi-\theta)}\sin\beta\} + \frac{e}{r}(\epsilon^{c(\pi-\theta)} - 1) + i_0\epsilon^{c(\pi-\theta)}\right]d\theta$$

which on integration contains a number of constants, and an item i_0 which is the value $i_1 + i_2$ for $\theta = 0$, and also the overlap θ_0. Now I_M is determined by the external conditions and thus the equation provides a relation between E, i_0 and θ_0. A similar equation, viz. (26) has been arrived at, and if therefore an exact value is required one of the three variables must be fixed. This is a lengthy process and it is usually sufficiently accurate to assume

$$i_0 = I_M$$

and correct afterwards by increasing the supply voltage by successive approximations, and arranging the tappings on the transformer secondary accordingly.

As a check on the two methods, apply equation (5) of the direct method to ascertain the overlap. The equation is

$$1 - \cos\theta_0 = \frac{x_1 I_M}{E\sin\frac{\pi}{m}}.$$

In the example $x_1 = 16$, $I_M = 10$, $m = 2$ and $E = 389$, whence

$$\theta_0 = 58°,$$

as compared with 56° obtained by the wave form method.

It is advisable to ascertain the actual wave form of the rectified current and the above example has been carried further and the analysis performed by the First Ordinate method (Chapter II.).

Assume that harmonics higher than the seventh are absent, then the coefficients of the first seven harmonics are obtained by measurement. As the curve repeats itself every 180° all odd

harmonics are absent also, and it can be shown that even values of b_n become zero after $b = 2$. Thus

$$a_6 = \tfrac{1}{12}(10 - 4{\cdot}9 + 1{\cdot}9 - 5{\cdot}8 + 9{\cdot}6 - 11{\cdot}4 + 10 - 4{\cdot}9 + 1{\cdot}9 - 5{\cdot}8 + 9{\cdot}6 - 11{\cdot}4)$$
$$= -0{\cdot}1$$
$$a_4 = \tfrac{1}{8}(10 - 2{\cdot}8 + 5{\cdot}8 - 10{\cdot}8 + 10 - 2{\cdot}8 + 5{\cdot}8 - 10{\cdot}8)$$
$$= 0{\cdot}55$$
$$a_2 = \tfrac{1}{4}(10 - 5{\cdot}8 + 10 - 5{\cdot}8)$$
$$= 2{\cdot}1$$
$$b_2 = \tfrac{1}{4}(2{\cdot}8 - 10{\cdot}8 + 2{\cdot}8 - 10{\cdot}8)$$
$$= -4.$$

The corrections are as follows:—

$$a_2 = 2{\cdot}1 - a_6 = 2{\cdot}0$$
$$a_0 = 10 - 2 - 0{\cdot}55 - 0{\cdot}1$$
$$= 7{\cdot}35.$$

The wave form is then

$$7{\cdot}35 + 2{\cdot}1 \cos 2\theta - 4 \sin 2\theta + 0{\cdot}55 \cos 4\theta - 0{\cdot}1 \cos 6\theta$$

and the ordinates as calculated from this are as follows:—

$y_0 = 10$, $y_{20} = 6{\cdot}4$, $y_{40} = 3{\cdot}2$, $y_{60} = 2{\cdot}8$, $y_{80} = 4{\cdot}55$, $y_{100} = 7{\cdot}15$, $y_{120} = 9{\cdot}8$, $y_{140} = 11{\cdot}2$, $y_{160} = 11{\cdot}45$, and $y_{180} = 10$

which agree well with the plotted curve with the exception of the value for y_{60} where a point of discontinuity occurs, and accuracy cannot be expected with only seven sets of ordinates.

The above wave form may be written

$$7{\cdot}35 + 4{\cdot}52 \cos (2\theta + 65^\circ) + 0{\cdot}55 \cos 4\theta - 0{\cdot}1 \cos 6\theta$$

and is seen to consist chiefly of a harmonic of double frequency shifted a distance above the θ axis equal to the mean value 7·35, as would be expected. Further, the angle of lead of this double harmonic of 65° gives some clue to the angle of overlap θ_0 although it cannot be expected to be accurately equal to θ_0, which in the case under consideration equals 56°.

The primary current of the transformer is obtained by evaluating $(i_2 - i_1)$ for the period 0 to θ_0, and i_1 from θ_0 to π, reversing $(i_2 - i_1)$ from π to $\pi + \theta_0$ and also i_2 from $\pi + \theta_0$ to 2π and adding the two curves so obtained.

This curve is plotted in Fig. 136, and the supply voltage $E \sin \theta (= 389 \sin \theta)$ is also shown. The curve for the primary current is seen to be symmetrical about a vertical line through the value of π, and therefore contains no even har-

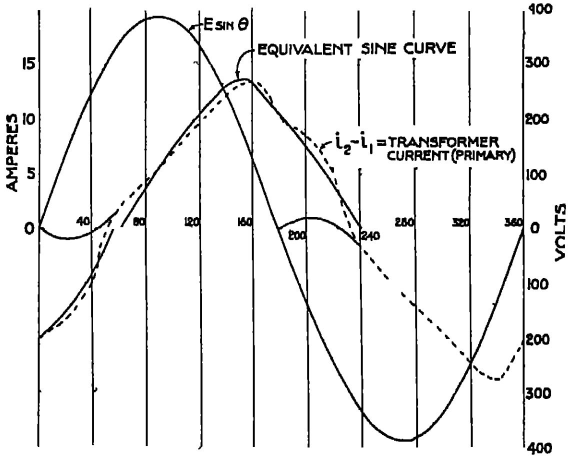


FIG. 136.—Supply current to mercury vapour rectifier.

monics. If it is analysed according to Thomson's rule the following values are obtained:—

$$a_7 = -0{\cdot}14$$
$$b_7 = +0{\cdot}31$$
$$a_5 = +0{\cdot}65$$
$$b_5 = -0{\cdot}4$$
$$a_3 = -0{\cdot}41$$
$$b_3 = -0{\cdot}43$$
$$a_1 = -10$$
$$b_1 = +5{\cdot}5.$$

Hence with corrections the values are as follows:—

$$\begin{aligned} a'_1 &= a_1 - a_3 - a_5 - a_7 \\ &= -10 + 0{\cdot}41 - 0{\cdot}65 + 0{\cdot}14 \\ &= -10{\cdot}1 \\ b'_1 &= b_1 + b_3 - b_5 + b_7 \\ &= +5{\cdot}78. \end{aligned}$$

The wave form is therefore

$$- 10{\cdot}1 \cos \theta + 5{\cdot}78 \sin \theta - 0{\cdot}41 \cos 3\theta - 0{\cdot}43 \sin 3\theta + 0{\cdot}65 \cos 5\theta - 0{\cdot}4 \sin 5\theta - 0{\cdot}14 \cos 7\theta + 0{\cdot}31 \sin 7\theta \ldots$$

This equation may be written in the form

$$11{\cdot}6 \sin (\theta - 60^\circ) - 0{\cdot}59 \sin (3\theta + 43^\circ) - 0{\cdot}76 \sin (5\theta - 58^\circ) + 0{\cdot}34 \sin (7\theta - 24^\circ) \ldots$$

and the effective value of the current is

$$\sqrt{\tfrac{1}{2}(11{\cdot}6^2 + 0{\cdot}59^2 + 0{\cdot}76^2 + 0{\cdot}34^2)}$$
$$= 8{\cdot}2 \text{ amperes.}$$

The maximum value of the equivalent sine curve is 13·7 amperes, and seeing that the E.M.F. wave has no harmonics, the harmonics in the current wave contribute nothing to the power consumed; and the power factor is a lagging one of 60°. The equivalent sine wave of current can then be drawn in Fig. 136 and although the curve appears to be irregular, in reality it differs little from that of a pure sine function.

An analysis of one anode current from 0 to 236° gives the following result:—

$$3{\cdot}74 - 5{\cdot}8 \cos (\theta + 31^\circ) + 2{\cdot}35 \cos (2\theta + 62^\circ) \ldots$$

If this curve is reversed and shifted forward 180° and the two curves so obtained added together the result agrees closely with the rectified current.

It is now possible to calculate the net efficiency in this particular case; the input is

$$2 \times 276 \times 8{\cdot}2 \times 0{\cdot}89 = 4{\cdot}0 \text{ K.W.}$$

and the output

$$250 \times 735 + \tfrac{1}{2} \times 164 \times 2{\cdot}1 \times 0{\cdot}42 = 1{\cdot}89 \text{ K.W.}$$ (page 221),

or an efficiency of 47 per cent. It must not be assumed from this example that this poor efficiency represents a normal case, as a considerable loss is incurred in the inductance and resistances. As stated above these constants have been introduced not with any idea of providing design data, but to indicate the theoretical process of analysis. As a matter of fact, as will

be seen later, very high efficiencies may be expected in mercury vapour rectifiers.

In the case of the primary impedance the voltage drop in the resistance r_1 which is in phase with the current is

$$(i_2 - i_1)r_1 \text{ for the period } 0 \text{ to } \theta_0$$

and

$$i'r_1 \text{ for the period } \theta_0 \text{ to } \pi.$$

The reactive drop at a quarter phase displacement from the current is

$$x_1 \frac{d(i_2 - i_1)}{d\theta} \text{ for the period } 0 \text{ to } \theta_0$$

and

$$x_1 \frac{di'}{d\theta} \text{ for the period } \theta_0 \text{ to } \pi.$$

In the rectified circuit similar values obtain with the substitution of the suffix 2 for that of 1 in the reactive component.

Thus all the voltage drops are known and can be plotted in curves, the ordinates of which should add algebraically to the supply voltage $E \sin \theta$.

Biphase Rectification—Wave Form.—Before leaving this analysis it is interesting to note the shape of the various wave forms involved, and in Fig 137 approximate curves are given

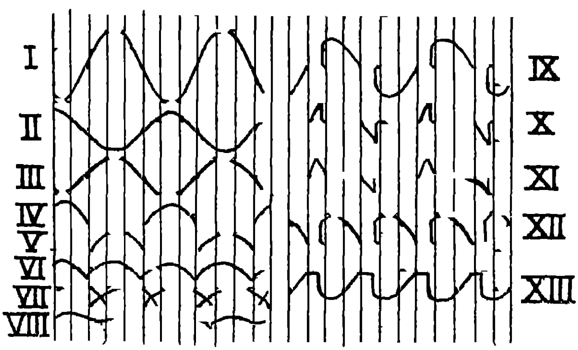


Fig. 137.

which are due to the late Dr. C. P. Steinmetz and which indicate the reactions very clearly.

I. Impressed A.C. voltage.

II and III. Voltages across each anode in turn: that is (with a biphase arrangement) one half the supply voltage.

IV and V. The anode currents (with no inductance), π electrical degrees apart.

VI. The net rectified current with no overlap, showing that as the current falls to zero every π degrees the arc will go out and will not restart.

VII. The two anode currents with overlap due to inductance.

VIII. The addition of the two anode currents, i.e. the net rectified current.

IX. When both arcs exist the anode potential must be the same in each case and hence the voltage across the two anodes is as shown in curve IX.

X. Hence the E.M.F. absorbed by the reactance must be the difference of the impressed voltage and that across the anode terminals, and the drop across the primary reactance is curve X.

XI. However, between the two peaks in X the voltage cannot actually be zero but takes the form XI.

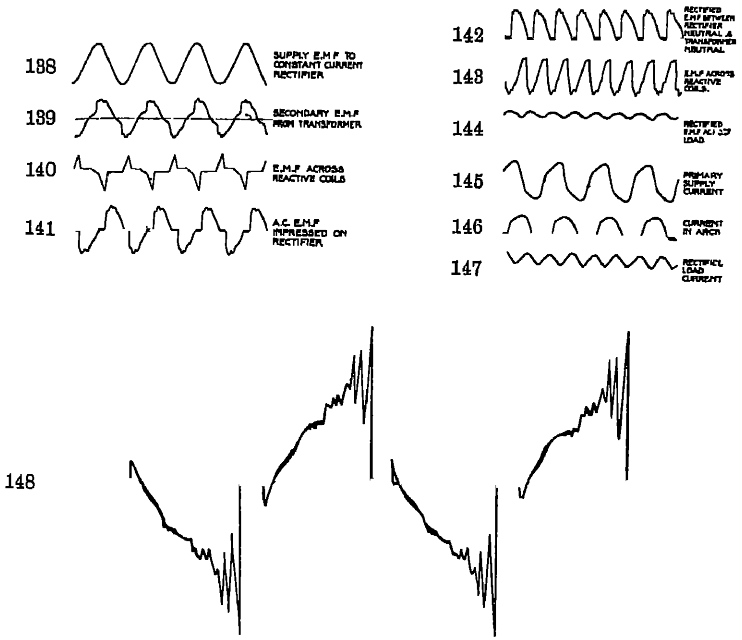
XII. The function of the biphase rectifier is to reverse the negative half of the wave and hence the voltage between the cathode and the mid point of the transformer is IX with the negatives reversed as shown in XII.

XIII. The rectified voltage must never fall below the arc drop and hence a reactive voltage must in the case of low resistance secondaries be inserted in the rectified circuit to retain curve XII above the datum line, and this is done by the inductance ($x_2\ r_2$), the drop across which is shown in XIII.

The above are only diagrammatic but indicate generally the shape of the various wave forms concerned. Actual oscillograms show a general agreement as the accompanying photographs will demonstrate (Figs. 138 to 147).

Finally there is an interesting point in the study of periodic transients which may occur in rectifiers, and will be due to the discontinuity in the current curve in Fig. 135 at the time when $\theta = \theta_0$ at the end of the period of overlap. It will be noticed in

Fig. 148 that a transient voltage of high frequency exists between the anodes at the end of the overlap period and this is due to the abrupt change in the reactive voltage $x\dfrac{di}{d\theta}$ at this point. Fig. 148, which is the outcome of a defective tube, should be compared with Fig. 141 where the transient term is non-existent. It is possible that the amplitude of such a transient



Figs. 138-148.

which is difficult to trace may be sufficient to cause a breakdown in the insulation of the transformer windings. The necessity for periodic testing of the rectifier plant is obvious and should not be neglected where reactive circuits are concerned.

Simplified Calculations.—So far the circuit has been considered from fundamental principles and the wave form has been arrived at having due regard to the various circuit constants.

The rectified wave form on analysis takes the form

$$I_M - I_2 \cos (2\theta + \phi)$$

plus higher harmonics. These higher harmonics, however, are of a lower order than the second and to a near approximation may be neglected; the wave form can then be assumed to comply with the equation

$$I_M - I_2 \cos (2\theta + \phi)$$

and the rectified voltage

$$E_M - E_2 \cos 2\theta.$$

Now in calculating for any one particular case it is usually necessary to commence with the voltage or current (or both) fixed, and one can also say what variation from the mean value is to be allowed.

Thus a curve such as Fig. 149 can be constructed from the

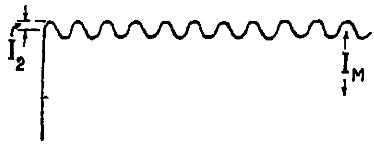


FIG. 149.—Undulatoriness of rectified wave form.

data available, and according to equation (3), page 71, the effective values of such a rectified current and voltage are

$$\mathscr{I} = \sqrt{I_M^2 + \frac{I_2^2}{2}} \text{ and } \mathscr{E} = \sqrt{E_M^2 + \frac{E_2^2}{2}} \quad . \quad (32)$$

and the mean values, or the equivalent readings on a moving coil instrument are I_M and E_M respectively. The ratio of the readings on moving iron and moving coil meters will be

$$\sqrt{1 + \left(\frac{I_2}{2I_M}\right)^2} \text{ and } \sqrt{1 + \left(\frac{E_2}{2E_M}\right)^2}.$$

Referring to Chapter I. (page 19), the assumption that the rectified current is equivalent to a wave form

$$I_M - I_2 \cos (2\theta + \phi)$$

is seen to be the same as assuming that each anode current is a single-phase wave of the form

$$\tfrac{1}{2}I_M + I_1 \cos(\theta + a) - \tfrac{1}{2}I_2 \cos(2\theta + \phi),$$

the effective value of which is

$$\mathscr{I}' = \sqrt{\tfrac{1}{4}I_M^2 + \tfrac{1}{2}I_1^2 + \tfrac{1}{8}I_2^2} \quad . \quad . \quad . \quad (33)$$

Further, the superposition of two approximate sine waves spaced π degrees apart involves the following relation :—

$$2\mathscr{I}' = \sqrt{2}\ \mathscr{I}$$

and hence from equations (32) and (33)

$$\mathscr{I}^2 = 2I_1^2$$

which gives a value for the amplitude of the fundamental in terms of the amplitude of the constant term and second harmonic, viz. :—

$$I_1^2 = \tfrac{1}{2}I_M^2 + \tfrac{1}{4}I_2^2 = \mathscr{I}^{2\prime}.$$

The current carrying capacity of each of the anode cables must therefore be 71 per cent. of that of the rectifier cables instead of 50 per cent. as might be expected.

As regards the supply voltage from the transformer outer to the neutral, the voltage is

$$E \cos \theta$$

where $$2\mathscr{E}^{2\prime} = E^2.$$

The rectified voltage (effective value) is

$$\mathscr{E}^2 = E_M^2 + \tfrac{1}{2}E_2^2 \quad . \quad . \quad . \quad (34)$$

as the wave form is biphase ; and moreover

$$\mathscr{E} = \mathscr{E}' \text{ and } EI_1 = \mathscr{E}\mathscr{I} \quad . \quad . \quad (35)$$

From equations (34) and (35) the relation

$$\mathscr{E}' = \sqrt{E_M^2 + \tfrac{1}{2}E_2^2} \quad . \quad . \quad . \quad (36)$$

is obtained, where E_M is the reading obtained on a moving coil meter. There are two factors which must be included in equation (36), viz. the drop across the arc stream and the drop

across the inductance, and hence

$$\mathscr{E}' = \sqrt{E_M^2 + \tfrac{1}{2}E_2^2} + e_a + c\mathscr{E}_s$$

where $\mathscr{E}_s$ is the volt drop across the reactance and $\mathscr{E}'$ is the voltage from each anode to the centre point of the transformer, and c is a constant.

It is important to note at this stage that if a sine wave is assumed $\mathscr{E}' = \frac{\pi}{2\sqrt{2}} E_M = 1{\cdot}11 E_M$ (see page 35), but that any departure from this will include another factor for the amplitude of the other harmonics. The above equation can, however, be written to an approximation

$$\mathscr{E}' = 1{\cdot}11kE_M + e_a + c\mathscr{E}_s.$$

The maximum variation of the rectified current from the mean value, viz. I_2 as indicated in Fig. 149 is obviously dependent on the angle of overlap θ_0 of the arcs; this can be calculated from the approximate solution as follows:—

$$i = \tfrac{1}{2}I_M + I_1 \cos(\theta + a) - \tfrac{1}{2}I_2 \cos(2\theta + \phi)$$

when $\quad i = 0,\ \theta = 0$ and hence

$$\tfrac{1}{2}I_M + I_1 \cos a - \tfrac{1}{2}I_2 \cos\phi = 0$$

also when $i = 0,\ \theta = \pi + \theta_0$ and therefore

$$\tfrac{1}{2}I_M - I_1 \cos(\theta_0 + a) - \tfrac{1}{2}I_2 \cos(2\theta_0 + \phi) = 0.$$

Subtracting

$$I_1 \{\cos(\theta_0 + a) + \cos a\} + \tfrac{1}{2}I_2 \{\cos(2\theta_0 + \phi) - \cos\phi\} = 0$$

and I_1 and I_2 are related by the equation

$$I_1^2 = \tfrac{1}{2}I_M^2 + \tfrac{1}{4}I_2^2.$$

I_M and I_2 are given from the current curve as are also a and ϕ and hence the relation between θ_0 and I_2 is obtained.

From these relations follow immediately

(1) Power input $= 2\mathscr{E}\mathscr{I} \cos a$,

(2) Useful power output $= E_M I_M + \tfrac{1}{2}E_2 I_2 \cos\phi$ (page 71),

(3) Efficiency $= \dfrac{E_M I_M + \frac{1}{2}E_2 I_2 \cos\phi}{2\,\mathscr{E}\mathscr{I}\,\cos a}$,

(4) Power factor $= \dfrac{E_M I_M + \frac{1}{2}E_2 I_2 \cos\phi}{2\sqrt{(E_M^2 + \frac{1}{2}E_2^2)(I_M^2 + \frac{1}{2}I_2^2)}}$,

and curves may be drawn connecting each of the quantities with the angle of overlap θ_0.

The power input is obtained as follows: the transformer current $= i_2 - i_1 = 2I_1 \cos(\theta + a)$. Similarly the transformer voltage $= E \cos\theta$ and the power input $= 2EI_1 \cos a$ because the arrangement is biphase (page 35).

Substituting the values for E and I_1 the input is

$$2\,\mathscr{E}\mathscr{I}\,\cos a$$

which may be written in the form

$$2\sqrt{(E_M^2 + \tfrac{1}{2}E_2^2)(I_M^2 + \tfrac{1}{2}I_2^2)}\cos a.$$

Polyphase Rectifiers.—The only circuits which have been considered up to the present are those which result from a simple star connection of the phases as shown in Fig. 150.

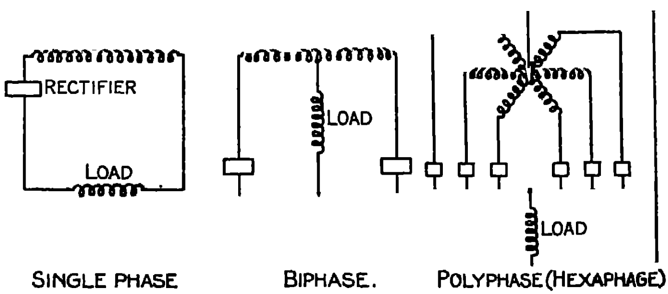


FIG. 150.—Simple rectifier circuit—star connected.

These systems have resulted in certain inherent voltage drops in the windings and the rectifier. It will be noted in what follows that the transformers supplying these rectifiers must be designed to meet the specialised demand of a discontinuous current, insofar as their secondary capacity is concerned. This results in an expensive type of construction, and attempts have been made to introduce different circuit

conditions to remedy this defect. At the same time it must be remembered that there are two methods by which a smooth rectified wave form can be attained, apart from the introduction of filters, viz. by the employment of high valued inductances, or by increasing the number of phases. The former method is not always the cheaper to instal, but results in a larger voltage drop on load; and in the latter case, the initial expense of a multiphase transformer of the order of eighteen phases has to be carefully considered.

The introduction of what is called a "regulating reactance" in the earth circuit has done much to palliate the effects of

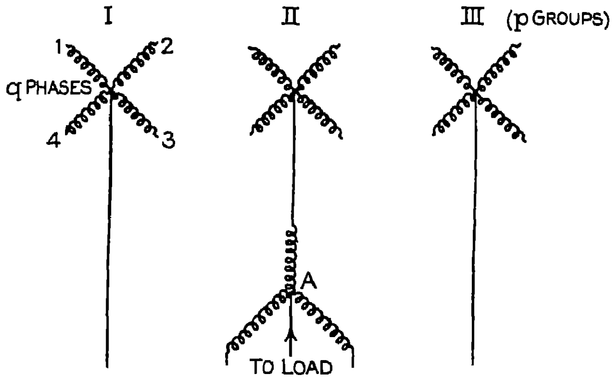


FIG. 151.—Multiphase circuit with regulating reactance.

voltage drop on multiphase circuits, at the same time retaining the advantages of using a multiphase system. These two advantages are coupled with one which is of great importance, viz. that the capacity of the transformer secondary is also reduced.

The general case is indicated in Fig. 151 where an m phase system is divided into p groups each consisting of q phases, so that $m = pq$. The centre point of each is brought to the leg of a p phase inductance A, the centre point of which is the negative terminal of the load.

The usual arrangement found in practice is a hexaphase system shown in Fig. 152. The two three-phase secondary

windings are evenly spaced, so that odd phases are brought out to one half of the negative of the regulating reactance A, and even phases to the other side. The middle point of the interconnection of these windings is the negative terminal of the rectifier, and the coils of the reactance are connected in opposition.

The rectified current arriving at point B divides equally into two parts, the one to the odd phase, and the other to the even phase secondary.

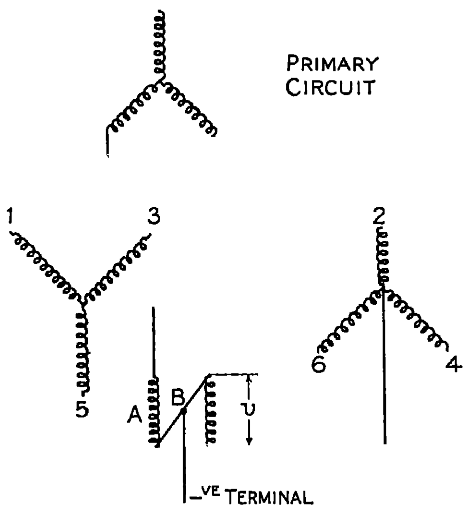


FIG. 152.—Hexaphase system with regulating reactance.

It will be remembered from Chapter I, that a three-phase rectified current consists of a wave form represented by

$$I_0 + I_3 \cos 3\theta + I_6 \cos 6\theta + \ldots \infty$$

and a hexaphase circuit has a wave form

$$I_0 + I_6 \cos 6\theta + I_{12} \cos 12\theta + \ldots \infty.$$

In the case under consideration each secondary supplies current of the three-phase variety, but the regulating reactance serves as a filter for the harmonics, which are odd multiples of three, and which circulate as a short-circuit magnetising current

in the reactance windings, resulting in an E.M.F. v being generated in the reactance proportional at any instant to the difference of voltages of the two circuits; thus

$$v = \tfrac{1}{2}(e_{135} - e_{246})$$

and

$$E_M = \tfrac{1}{2}(e_{135} + e_{246}).$$

At any instant there is always one odd and one even phase supplying current simultaneously; and each of three units, functions exactly as an ordinary three-phase rectifier, supplying one half the total current. For a large value of the regulating reactance, from equation (5)

$$1 - \cos\theta_0 = \frac{x_1 I_M}{E \sin\frac{\pi}{6}} = \frac{2x_1 I_M}{E}$$

gives the overlap θ_0 for a hexaphase system, but

$$1 - \cos\theta_0 = \frac{x_1 \frac{I_M}{2}}{E \sin\frac{\pi}{3}} = \frac{x_1 I_M}{\sqrt{3}E}$$

is the equation for the overlap for the new system. Thus the overlap is much less in the latter than in the former, and it follows that less inductance is required in the anode circuit; and as in an ordinary hexaphase circuit (equation 8, page 192).

$$\Delta E_M = \frac{3}{\pi} x_1 I_M$$

but for the divided phase system

$$\Delta E_M = \frac{3}{2\pi} x_1 \frac{I_M}{2} = \frac{3}{4\pi} x_1 I_M$$

thus the drop in the latter case is only one quarter of its value in the former.

In the general analysis of p groups of m phases

$$\Delta E_M = \frac{q}{2\pi} \cdot \frac{x_1 I_M}{p} = \frac{m}{2\pi p^2} x_1 I_M$$

showing that the drop in voltage is only $1/p^2$ the value in a simple case of m phases.

General Polyphase Current and Voltage Relationships.—The general calculations of the wave form of a polyphase system have been given in Chapter I., but they are recapitulated here for convenience.

Quite generally any loop of a polyphase system, which obeys a sine law, may be represented by

$$i = I \sin\left\{\theta - \frac{2(k-1)}{m}\pi\right\}$$

where k is any integer from 1 up to m, and the mean value of the rectified current will be

$$I_M = I \times \frac{m}{2\pi}\int_{\frac{\pi}{2}+\frac{2k-3}{m}\pi}^{\frac{\pi}{2}+\frac{2k-1}{m}\pi} \sin\left\{\theta - \frac{2(k-1)}{m}\pi\right\}d\theta = \frac{mI}{\pi}\sin\frac{\pi}{m}$$

and the effective value

$$\mathcal{I}^2 = I^2\left[\frac{m}{2\pi}\int_{\frac{\pi}{2}+\frac{2k-3}{m}\pi}^{\frac{\pi}{2}+\frac{2k-1}{m}\pi} \sin^2\left\{\theta - \frac{2(k-1)}{m}\pi\right\}d\theta\right] = I^2\left\{\frac{1}{2}+\frac{m}{4\pi}\sin\frac{2\pi}{m}\right\}.$$

If $\mathcal{I}'$ is the effective anode current per phase as before, the average of the ordinates squared is taken over a whole cycle and

$$\mathcal{I}^{2'} = I^2 \times \frac{1}{2\pi}\int_{\frac{\pi}{2}+\frac{2k-3}{m}\pi}^{\frac{\pi}{2}+\frac{2k-1}{m}\pi} \sin^2\left\{\theta - \frac{2(k-1)}{m}\pi\right\}d\theta$$

$$= \frac{I^2}{m}\left(\frac{1}{2} + \frac{m}{4\pi}\sin\frac{2\pi}{m}\right)$$

and therefore

$$\frac{\mathcal{I}'}{I_M} = \frac{1}{\sqrt{m}}\,\frac{\sqrt{1+\frac{m}{2\pi}\sin\frac{2\pi}{m}}}{\sqrt{2}\frac{m}{\pi}\sin\frac{\pi}{m}}.$$

If the number of phases is above three the expression

$$\frac{\sqrt{1+\frac{m}{2\pi}\sin\frac{2\pi}{m}}}{\sqrt{2}\frac{m}{\pi}\sin\frac{\pi}{m}}$$

approximates to unity and hence

$$\mathscr{I}' = \frac{I_M}{\sqrt{m}}.$$

In the case of the voltage the result is different as the E.M.F. continues to vary from the beginning to the end of each semi-oscillation whether current is flowing or not, and hence with the same notation as before

$$E = \sqrt{2}\mathscr{E}'$$

and also

$$E_M = \frac{mE}{\pi} \sin \frac{\pi}{m}$$

and hence E_M or its corrected substitute

$$E_M + e_a + cE_1 = \mathscr{E}' \sqrt{2}\frac{m}{\pi} \sin \frac{\pi}{m}$$

whence

$$\mathscr{E}' = \frac{E_M + e_a + cE_1}{\sqrt{2}\frac{m}{\pi} \sin \frac{\pi}{m}} \quad . \quad . \quad . \quad (37)$$

and for a six-phase supply

$$\mathscr{E}' = \frac{E_M + e_a + cE_1}{1{\cdot}35k} \quad . \quad . \quad . \quad (38)$$

where the constant k is introduced to allow for certain manufacturing approximations.

In an actual example quoted by the Brown Boveri Co. the nominal voltage across the inductances was 30 volts, the arc drop was 20 volts and c and k were found experimentally to be 1·2 and 0·97 respectively. A continuous voltage of 230 was required for the six-phase rectifier and the voltage was therefore

$$\mathscr{E}' = \frac{230 + 20 + 1{\cdot}2 \times 30}{1{\cdot}35 \times 0{\cdot}97} = 218 \text{ volts}$$

between the transformer neutral and each anode. There will also be a voltage drop on load which must be allowed for, and the power factor will be about 0·9. The no-load voltage of the rectifier will then be

$$1{\cdot}35 \times 0{\cdot}97 \times 231 - 20 = 282 \text{ volts}$$

and the total drop will be

$$282 - 230 = 52 \text{ volts} = 18{\cdot}5 \text{ per cent. regulation.}$$

If this drop is too great the chokes must be supplied with turns which can be short-circuited and then allow $c = 0{\cdot}2$ in place of $1{\cdot}2$. The transformer will now give 198 and 210 volts at full and no-load with a drop of 23 volts in the rectified circuit

(= 9 per cent. regulation).

With regard to the calculations for the currents,

$$\mathscr{J}' = \frac{I_M}{\sqrt{m}}$$

and in the case of a six-phase circuit $\sqrt{m} = 2{\cdot}45$, which in practical cases should be increased to $2{\cdot}8$ or 3 to allow for especial losses.

Power and Size of Transformers.—The size of the transformer required is determined from a consideration of the power output, but it also depends largely on the number of phases, since the anode current only flows when the voltage of one particular phase exceeds that of the others, and hence the ratio of the effective value of the current to the maximum or crest current is much lower than in an ordinary sine wave. This has a distinct bearing on the size of copper conductors and therefore on the ultimate size of the transformer. Thus if P_p is the primary rating, P_s the secondary rating and P the power absorbed in the rectified circuit, indicated in Fig. 130, then

$$P_s = m\mathscr{E}'\mathscr{J}'$$

$$= m\frac{E_M + e_a + cE_1}{\sqrt{2}\frac{m}{\pi}\sin\frac{\pi}{m}} \cdot \frac{I_M}{c_1\sqrt{m}} \quad . \quad . \quad (39)$$

where c_1 is a further constant introduced to allow for certain manufacturing difficulties.

For $m = 6$ this becomes

$$P_s = 6\frac{E_M + e_a + cE_1}{1{\cdot}35k} \cdot \frac{I_M}{c_1\sqrt{6}}.$$

Then for values of the constants $c = 0{\cdot}2$, $k = 0{\cdot}97$, $c_1 = 1{\cdot}15$, $e = 20$ volts and $E = 30$ volts, in a particular case, whence

$$P_s = 1{\cdot}61(E_M + 26)I_M$$

and thus the transformer should be designed for an output about 60 per cent. greater than the direct current load.

So far as the primary of the transformer is concerned if P_p is the K.V.A. rating

$$P_p = \frac{P}{\eta \cos \phi}$$

where η is the overall efficiency and $\cos \phi$ the power factor and $P = \sqrt{(E_M^2 + \frac{1}{2}E_2^2)(I_M^2 + \frac{1}{2}I_2^2)}$. If $\eta = \cos \phi = 90$ per cent., the primary power rating need only be 20 per cent. greater than the D.C. loading. This apparent paradox is worthy of careful attention as a transformer rated in the ordinary way may result in a lack of the power available and is likely to have serious results.

The secondary capacity of a transformer with regulating reactance has been stated above to be less than its corresponding equivalent in the simple star connection. It has been shown also that the equivalent current in each of the three phase secondaries is $I_M/2$, and that each bank behaves as a separate unit. In the general case, therefore, of p groups of q phases, the anode current in place of having a value

$$\frac{I^M}{\sqrt{m}} \text{ becomes } \frac{I_M}{p\sqrt{q}} = \frac{I_M}{\sqrt{pm}}$$

and if $m = 6$, and $p = 2$, as is illustrated above,

$$\mathscr{I}' = \frac{I_M}{2\sqrt{3}} \text{ in place of } \frac{I_M}{\sqrt{6}}$$

and instead of a rating 60 per cent. in excess of the primary, the transformer need only be rated 14 per cent. above the usual value. This results in a considerable saving in copper in the transformer, as well as in the leads to the rectifier.

Equation (39) under these conditions becomes

$$P_s = \frac{E_M + e_a + cE_1}{\frac{\sqrt{2}}{\pi} \sin \frac{\pi}{m}} \cdot \frac{I_M}{c_1\sqrt{pm}}.$$

Curves showing the effect of the change on the efficiency and power factor are reproduced in the next chapter.

Transformer Primary Currents.—The act of rectification causes a deformation of the supply current wave, and this consequently reacts on the power input and also on the power factor. This question which is important is outside the scope of this work, as it affects transformer design rather than that of the rectifier itself, but the matter has recently been investigated thoroughly by von Krijger and H. Jungmichl whose conclusions together with numerous oscillographs are contained in two articles in the E.T.Z. and E.u.M. (see Bibliography). The method which has been employed is as follows: with the same notation as before

$$\mathscr{E}\sqrt{2} = E$$

for the secondary voltage, and as the current only flows for a portion of the cycle, in a star-star connected three-phase system

$$\mathscr{I} = \sqrt{\frac{1}{2\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} I^2 \sin^2 \theta d\theta}$$

if a sinusoidal current wave is assumed. This can be reduced to

$$\mathscr{I} = \sqrt{\frac{1}{2\pi}\left(\frac{\sqrt{3}}{4} + \frac{\pi}{3}\right)} \,.\, I$$

and the K.V.A. output

$$= \mathscr{E}\mathscr{I} = EI\sqrt{\frac{1}{4\pi}\left(\frac{\sqrt{3}}{4} + \frac{\pi}{3}\right)} \text{ approximately.}$$

The power output, however, is

$$P_s = \frac{EI}{2\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin^2 \theta d\theta = \frac{EI}{2\pi}\left(\frac{\sqrt{3}}{4} + \frac{\pi}{3}\right).$$

Thus the power factor is

$$\frac{P_s}{\mathscr{E}\mathscr{I}} = \sqrt{\frac{1}{\pi}\left(\frac{\sqrt{3}}{4} + \frac{\pi}{3}\right)} = 0{\cdot}69.$$

Similar equations can be constructed for delta-star arrangements, etc., which incidentally has the same power factor as star-star, and for delta or star zig-zag systems of connection. The power factor varies between wide limits rising in certain cases to a value higher than 0 9.

CHAPTER IX.

MERCURY VAPOUR RECTIFIERS (*cont.*).

General Description.

General.—Theoretical analysis does not differentiate between the two types of mercury vapour rectifier, and in this chapter those aspects are considered which apply equally to the rectifying unit whether enclosed in a steel case or in a glass bulb.

Such differences as do exist are wrapt up chiefly in the maintenance of the necessarily high vacuum, but the two designs have diverged somewhat from one another on this account, and, therefore, two special chapters have been devoted to the consideration of the special features of each.

Continuity of service and reliability are likely to be the chief criteria when the installation of converter plant is considered. In this respect the mercury vapour rectifier is certainly not behind its competitors. Experience so far would appear to show quite conclusively, that a mercury vapour rectifier will function at least as long as rotary plant without overhaul, and on this account is likely to be favoured. Where it is at a disadvantage is in its limited overload capacity for long periods, a point which will be considered later, but the fact that overloads of 25 per cent. to 50 per cent. cannot be carried for long is likely to prove a serious drawback in a locality where the load is steadily growing over a term of years and fluctuating daily at the same time. On the other hand its capacity to bear short-circuit currents for a short time without damage largely compensates for the above disability. The only safeguard is in the installation of sufficient plant with an ample reserve in case of breakdown.

The complete rectifier plant consists of a transformer with no special design features, that excepting of the secondary

rating which has to be carefully determined (see Chapter VIII., page 228) for each case in turn, and which depends on the number of phases as well as the output required; and the rectifying unit, which may consist either of a steel container or a glass bulb controlled by switchgear either of the truck type or mounted on slate panels. No special switchgear is required beyond the ordinary circuit breakers and overload relays.

In the case of the steel container rectifiers, development in design has advanced along the lines of ease of renewal of the parts likely to require attention. This leads to the employment of a container, the cover of which can be easily removed, yet which will maintain a sufficiently high vacuum for long periods. At the same time a vacuum pump is required for the removal of the air from the container as well as the occluded gas in the metal itself: an effective metal to metal seal is also required if the case is to be quickly dismantled and re-erected. The glass bulb rectifier relies on the relatively long life of a bulb, and its comparatively small cost and ease of renewal, and its total replacement in case of failure.

There is a good deal to be said for each of these schools of thought, and the time is not yet ripe to state arbitrarily which is likely to supersede the other, or whether there will be a field for both. Certainly in the case of small plants of one or two kilowatts, the glass bulb rectifier has many advantages over any other competitor, but it may be that future developments will modify this state of affairs.

The two types are described in the two following chapters, and in the design of any new installation, which to install must be left to the personal predilections of the Engineer, as the advantages and disadvantages are almost equally balanced at the present time.

Physical Characteristics.—Gunthe Schulze has conducted interesting experiments on the physical characteristics of rectifiers both of the glass bulb type and on that enclosed in a metal container.

General particulars of three types of rectifier are given in Table XVI. where the arc temperatures are calculated.

TABLE XVI.

Type	Diameter of Anode in Cms.	Current in each Anode.	Volts Per Cm. of Arc	Temp. of Wall C *	Temp of Axis of Arc C.	Vapour Press. in Mm
Glass bulb for 100 amperes	5·5	0·5	2·20	200	850	0 120
Glass bulb for 100 amperes	5·5	16·6	0·58	350	2700	1·4
Glass bulb for 100 amperes	5·5	100·0	0·26	300	4800	—
Glass bulb for 500 amperes	6·5	500	0·2	300†	10900	—
Metal container	—	600	0·09	40†	7400	—

Tests were also made to ascertain the effect of the variation of current with constant vapour pressure, and in Table XVII. data are given with the bulb temperature kept constant at 200° C.

TABLE XVII.

Current in Amperes	Volts Per Cm of Arc.	Temp of Arc Degrees C
0·50	2·20	845
1·10	1·04	863
2·50	0·51	910
5·20	0·36	1108
7·50	0·31	1252
12·50	0·26	1487

In the results given in Table XVIII. the current was maintained constant and the voltage variation with pressure is indicated; at the same time the mean free path of electrons is given.

The variation in pressure was attained in part by loading up the other anodes and partly by inserting the whole rectifier in a heating zone.

This Table XVIII. is the result of calculation and is approximately accurate only.

At the same time evaporated mercury is condensing on the

* At anode. † With cooling.

TABLE XVIII.

Vapour Press. in Mm. Hg.	Volts Per Cm.	Temp. of Glass Wall Degrees C.*	Temp. of Arc Degrees C	Mean Free Path Cm.
0·003	1·42	250	2275	177
0·067	0·58	250	1418	5·30
0·081	0·52	250	1320	4·10
0·167	0·47	260	1256	1·90
0·290	0·52	260	1280	1·14
0·57	0·58	270	1418	0·622
0·690	0·65	280	1504	0·540
1·540	0·90	310	1800	0·286
4·160	1·18	350	2060	0·116
8·20	1·42	390	2317	0·0655
13·50	1·68	430	2542	0·0437

walls of the container, and in the case of a glass bulb it will be apparent that a variation of pressure and temperature will be found over the surface of the bulb. Thus in the case of a bulb 50 cm. long Schulze finds that the glass wall temperature varies from 100° C. at the cathode to 49° C. at the top, and the pressure from 0·160 mm. to 0·004 mm. mercury. This was the case of a bulb suitable for 100 amperes with cooling, and run at 30 amperes. Under these conditions 0·216 gramme of mercury condensed per second. If the bulb is loaded to 100 amperes and cooled to about the same temperature and pressure, approximately the same conditions appertain. In the anode space, however, the temperature and pressure are approximately constant.

The Arc.—At the liquid cathode there is a crater formed from which issues the stream of vapour; and round this crater the base of the arc travels at high speeds causing considerable eddies in the mercury pool. Above this crater there is a pale glow called the negative flame which is highly susceptible to the influence of a magnetic field. This negative flame is likely to play an undesirable part in the performance of the rectifier and should be kept to the smallest possible dimensions.

If the container is completely filled by the arc the voltage drop is independent of the current (page 173), and also if the

* At anode.

arc takes place in a medium at constant pressure the voltage drop varies inversely as the square root of the current. Neither of these conditions is strictly fulfilled in this particular case, and to a certain extent the voltage drop is affected by the amount of the current flowing. The effect, however, is not great and may be illustrated by the typical case of Fig. 153 where the upper curve (*a*) refers to the mercury vapour lamp, and the lower curve (*b*) to rectifiers.

Thus the arc will be seen to have a slightly negative temperature coefficient, which, if rectifiers are required to be worked in parallel, necessitates the insertion of a resistance or reactance to ensure stability by changing the slope of the characteristic. Further, it is important that the arc should commence

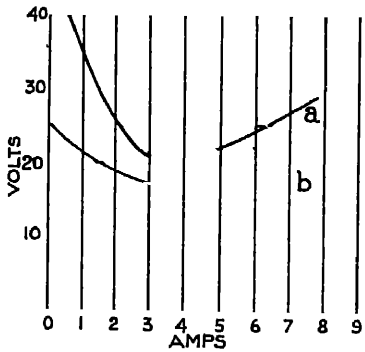


FIG. 153.—Voltage regulation of mercury vapour rectifiers.

with an anode glow, which necessitates a state of high evacuation in the container, as only in this case will the anode escape local heating and vaporisation leading to early disintegration and replacement. If the rectifiers are rarely used, or if for any cause the pressure rises in the bulb, the arc is likely to concentrate at one point of the anode and this will be the seat of local pitting.

Schulze has shown that the energy dissipated at the cathode, which is supplied by the bombardment of the positive ions, is made up from four sources, and although 58 per cent of the total current is carried by electrons, yet the far greater mass of the positive ions determines the increased energy due to their impact. (1) Thus 2·68 watts per ampere are dissipated by the wandering cathode spot. As the area of the spot is $2{\cdot}53 \times 10^{-4}$

square centimetres per ampere, the current density is 4000 amperes per square centimetre, which accounts for the high temperatures attained. (2) If the temperature of this spot is assumed to be 2000° C., the radiation is 0·0366 watt per ampere. (3) Due to the high temperature, the mercury evaporates to the extent of $2 \cdot 08 \times 10^{-3}$ grammes per second per ampere, and this requires 2·20 watts per ampere. (4) Finally the energy dissipated by the work done by the electrons during exit is $4 \cdot 1 \times 0 \cdot 58 = 2 \cdot 38$ watts per ampere. The total cathode loss is thus 7·3 watts per ampere.

The energy of the positive ions is so great and the temperature of the spot so high that mercury vapour is evaporated at a great rate and the evolution of molecules drives the incoming ions to the side and causes the spot to wander over the surface of the mercury.

Arc Voltage Drop.—It has been assumed up to the present that the arc drop is approximately constant, and of the value of about 20 volts. In actuality, however, it is not constant and both the electrode falls and the drop of voltage in the arc itself vary with the pressure of mercury in the container. Thus on page 172 the value of the constant *a* is only true for one particular pressure. In Table XIX., which has been compiled by Schulze, the particulars of arc drops are given for varying conditions.

TABLE XIX.

Condensation Temp. C.	Press. of Hg. in Mm.	Voltage Between Electrodes.	Anode Fall Volts.	Volt Drop in Arc.	Diam. of Arc Cm.	Luminosity.
80	0·004	22·0	13·0	3·7	—	Non-luminous
60	0·028	20·3	11·6	3·4	—	" "
106	0·36	14·6	5·8	3·5	—	" "
118	0·68	16·5	5·6	5·6	—	Luminous column at anode
123	1·07	18·0	5·5	7·7	8	Luminous column grows to cathode
140	1·82	21·0	6·0	9·7	7	Luminous column grows to cathode
150	2·73	24·5	6·8	12·4	6	Luminous column reaches cathode
160	4·08	23·5	6·8	11·4	5	Luminous column reaches cathode

These tests were not actually conducted in a proper rectifier but in a box cooled by a current of air from a fan.

Back-starting.—The two chief troubles experienced with mercury vapour rectifiers are (1) loss of vacuum, and (2) back-starting. The latter is the term applied to a passage of alternating current without rectification and may be due to several causes. One of these is given on page 234, but the others can be enumerated as follows :—

(1) An excessive voltage.

(2) An abnormal vapour pressure.

(3) Impurities on the cathode.

(4) Quantities of foreign gas.

The future development of the mercury vapour rectifier is probably bound up with this problem of back-starting, and it is the earnest desire of all designers to discover methods of overcoming the trouble.

Consider the four possibilities in greater detail :—

(1) *Excessive Voltage.*—The limit of voltage before back-starting takes place is a most uncertain quantity. Jotte has shown that with special precautions taken to prevent over-heating, voltages up to 20,000 can be rectified, but up to the present such is exceptional. Normally the anode is covered with a glow discharge which at high current densities tends to contract to a spot, and heats the metal to a high degree. The current then rises to a high value, and this is probably the most fruitful cause of the trouble.

(2) *Abnormal Vapour Pressure.*—This is obviously a cause of trouble as large currents follow a rise in pressure.

(3) *Impurities on the Cathode.*—A fortuitous impurity will cause back-starting, and to show the importance of cleanliness in assembly, it has been found that touching the anode with the fingers is often sufficient to introduce enough impurity to cause back-starting, but in addition the pressure of water vapour will cause the formation of iron oxide from the metal case which forms a sticky substance with mercury, which in time spreads over the surface of the cathode and causes trouble.

(4) *Presence of Foreign Gas.*—The presence of oxygen will

lead to the formation of iron oxide mentioned in item (3) above, but generally the presence of alien gases will increase the pressure which causes itself an increase in the current.

Although it is not safe to assume too rigid a relation for conditions of back-starting, yet Fig. 154 given by Schulze shows the connection between back-starting and load current. The indication is a rapid decrease in the voltage at which the phenomenon occurs with increase in load. This is due to the increase in the vapour pressure with the load, and shows the necessity for an increase in the size of the container with current strength. This curve only holds for continuous loading, back-

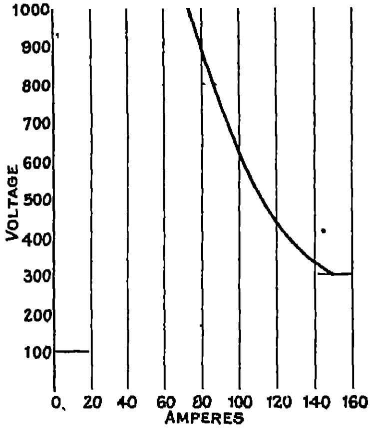


FIG. 154.—Voltage of back starting.

starting taking place at much higher voltages on big loads for short periods.

Wave Form.—The wave form of a rectifier has been discussed from the analytical standpoint in Chapter VIII., where the current curve has been calculated for an ideal case, and some oscillograms given of a particular rectifier. In Fig. 155 are given a series of curves showing how the undulatoriness of the rectified current may be lessened by increasing the number of phases. This is an analogous case to that of commutation whereby the amplitude of the commutator ripple may be decreased by increasing the number of commutator segments.

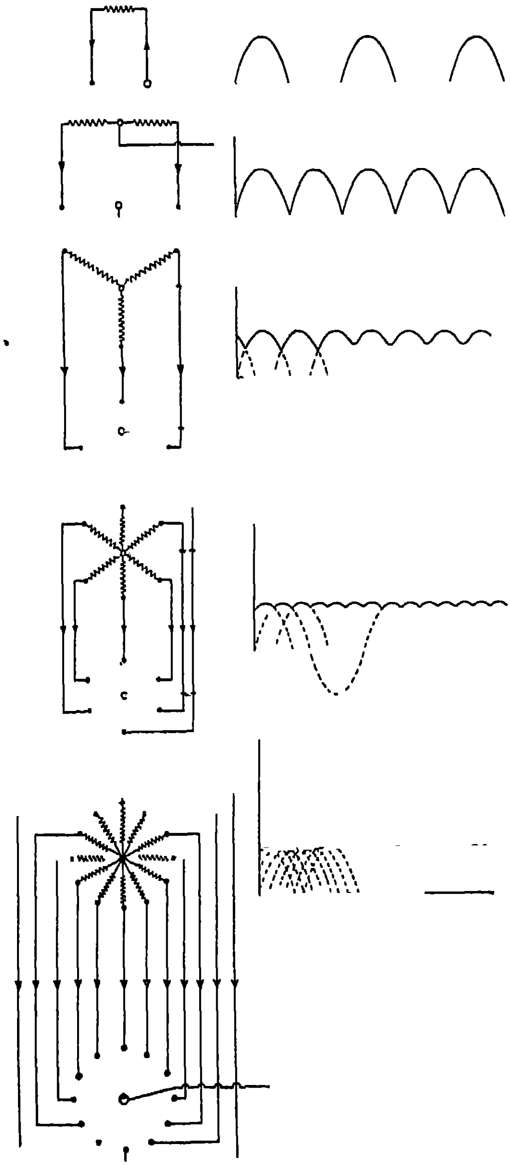


FIG. 155.—Effect of increasing the number of phases.

There are obvious and practical difficulties in the way of increasing the number of phases beyond twelve, and a six-phase system is the one more usually employed. The amplitude of the ripple may be further reduced by the insertion of choke coils, although the effect of such devices is more apparent in the case of single-phase circuits than where more phases are used.

Behaviour on Short-circuit.—If a rectifier is continuously overloaded (see page 242) the anode is liable to overheat and may function as a cathode, allowing current to flow in both directions. This will constitute a short-circuit of the rectifier unit, but not necessarily of the load, because a flash-over of the anode terminals is not a necessary corollary, but nevertheless serious damage to the plant will ensue if the rectifier is not cut out of circuit at once, and protective gear should be included in the switchgear to cope with such an emergency. On a short-circuit fault on the load or cable system, if the fault persists for a short time only, the rectifier is unaffected. For dead shorts, however, suitable protective gear should be provided in the usual way, but the fact of the rectifier being able to withstand heavy currents momentarily is of value in the case of transient surges.

Parallel Connection and Operation. — There are two methods by which the total existing capacity of a substation plant may be increased, (i) by running rectifiers in parallel, and (ii) using them in conjunction with rotary plant. The operation of rectifiers in parallel presents no difficulties provided (*a*) that the voltage drop in each rectifier is reasonably the same, or (*b*) that the falling volt-ampere characteristic is converted into one of opposite sense by the insertion of a suitable resistance or choke. Such a reactance may be made partially automatic in action by using a movable iron core, controlled by a solenoid, the position of the core being determined by the particular condition of the load; but in most cases no such complication is necessary. The method of connection of such a choke is described on page 244.

As regards the employment of rectifiers with other forms of

D.C. plant it is important that the voltage drop should be the same (within certain limits) for each piece of apparatus, otherwise the load will be unequally distributed. The drop in a large rectifier can be controlled to within 3 per cent. if necessary by automatic means, but in other cases where a poor regulation is preferable (such as in certain forms of battery charging apparatus) it can be increased to 20 per cent. (page 243)

The voltage drop is not constant in the case of new rectifiers, as it is higher when the plant is first installed. It is advisable, therefore, where parallel connection is required to ensure voltage stability by supplying current to an auxiliary load, so that the total output is constant, until the rectifier is sufficiently aged to cope with a varying load.

Intermittent Working. — Intermittent loading originally constituted one of the real disadvantages of rectification by means of mercury vapour. In modern plants if the load is likely to fall to zero or to a low value, a special form of excitation is provided to retain the vapour stream in being. This state of affairs will cause no trouble if only a short period of no-load exists, say from 15 to 20 minutes; but if the condition is likely to persist it is possible that mercury vapour may condense on the cooling anodes and give rise to back-starting; it is, therefore, advisable in that case to disconnect the main supply or alternatively to adopt some special means such as auxiliary excitation as described in pages 262 and 284. Devices can be employed to effect a complete shut down automatically (if so desired) by means of a time switch in the excitation circuit, but such a case is rarely likely to occur in practice where the load is only cut off for short spaces of time.

There is another point, however, in intermittent working which renders it difficult to obtain good regulation and efficiency —the voltage drop in the arc is assumed to be approximately constant, and hence the watts lost in the arc are proportional to

$$\int i\,dt,$$

but the losses in the auxiliary apparatus, such as resistances chokes and the like are proportional to

$$\int i^2 dt,$$

and the total loss is proportional to the sum of these two quantities, which with varying loads will assume different values.

Careful design, and if necessary subsequent modification in the light of experimental evidence after a period in service, are necessary to cope with problems such as these.

Overload Capacity.—The permissible continuous overload on a rectifier is less than that which can be allowed on rotary plant, because whereas the overload capacity of a rotary machine is limited solely by the heating of the windings, and any excessive heating can usually be detected before any damage is done, in the rectifier the heating of the anodes which is the natural sequence of overloading is not so apparent. The volt drop is approximately constant, but this is only true for normal currents flowing; if the current rises to too high a value the drop begins to increase rapidly with a consequent increase in the watts lost in the rectifier itself, and a general rise in temperature follows. Back-starting or shorting in the rectifier will probably ensue with serious consequences (see p. 237). To ensure continuous working without damage the overload ratings should be kept to the figures given in Table XX.

TABLE XX.

Overload Rating Per Cent.	Minutes of Permissible Overload.
16	60
25	30
45	20
50	17
70	10
100	5

As rectifiers are very efficient at even low loads, there is little excuse for a permanent overload, and it is always safer to install a large enough plant initially, taking a liberal view of any possible subsequent extensions.

On momentary overloads, however, these rectifiers have a

great advantage over rotary plant. In one instance of a steel container type of rectifier whose normal current was 400 amperes, a series of experiments was conducted to ascertain the permissible overload on short-circuit; and it was found that for a fraction of a second 8700 amperes could be carried without damage. These tests were repeated for sixty times on two consecutive days and when the rectifiers were opened up the anodes were found to be in good condition. A similar test on a rotary machine would probably result in the plant pulling up, and possibly damage to the windings as well.

Voltage Regulation.—From what has transpired it will be apparent that theoretically the only way of regulating the D.C voltage efficiently is by a change in the applied A.C. volts, various devices have been invented from time to time to provide a constant voltage on load; and a few of the most important are described. Such wasteful and obvious methods as rheostatic control are neglected, as in practice regulation is never effected in large plants in this way.

(i) Tappings on the transformer secondary may be provided, and this is commonly done in the plants where glass bulb rectifiers are used; but the arrangement is not very satisfactory because of the coarseness of adjustment.

(ii) A choke coil may be inserted in the rectified circuit with an arrangement for cutting in or out the last few turns of the choke. This method is open to objection on the same grounds as (i).

(iii) A choke with movable plunger which automatically varies the induction and therefore the counter E.M F. is a possibility but the regulating factor is not large for any given size of choke coil. The connections for such a device are illustrated in Fig. 156.

(iv) An induction regulator in the primary side of the transformer constitutes one of the best methods available, but is not often used on account of its expense. From the point of view of a voltage regulator it is an almost perfect device theoretically, and at the same time it works excellently in practice. The regulator may be automatically operated and as an infinite

number of steps are obtainable, the graduation of voltage change is exceedingly small. One objection to using such a regulator is the fact that the principle on which it works is that of an induction motor, and the power factor is therefore not high. Apart from this disadvantage the regulation is difficult to improve, and the watt losses are solely determined by the efficiency of the regulator.

(v) In the case of battery charging equipments, buffer cells may be provided with auto-cut-outs, as is frequently done in the case of D.C. sets. Here again the coarseness of adjustment is

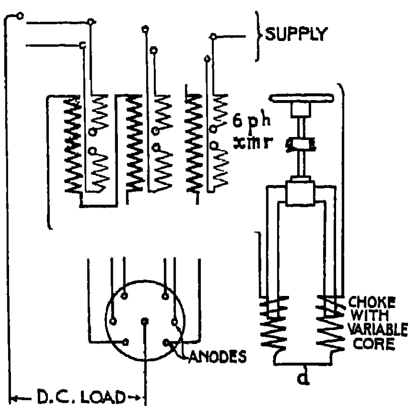


FIG. 156.—Voltage regulation by variable plunger in choke coil.

determined by two volt steps, and in battery charging alone this would not matter, but if the cells are connected permanently across the supply the adjustment would probably not be sufficiently fine.

(vi) A device due to Messrs. Brown Boveri is worthy of mention, its action will be explained for a single-phase supply, but the patent specification itself should be consulted for further details In a biphase arrangement, as shown in Fig. 157, P and S are the primary and split secondary of the supply transformer and CC are two chokes, one coil of each of which is connected to each anode; the other two coils (one in each

choke) are joined in series with an inductance and rheostat in shunt across the rectified circuit

Referring to Fig. 158, vertical ordinates represent the flux in the iron circuit of the choke, and abscissæ the net ampere

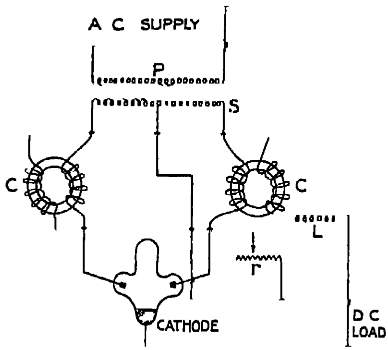


FIG. 157.—Brown Boveri regulator.

turns wound on the iron The voltage of the rectified circuit provides for the purpose of the choke a constant excitation (although it is subject to slight variations); but the anode current is varying, and hence the ampere turns contributed by the anode current vary proportionately. Let $MP = \phi$ be the flux

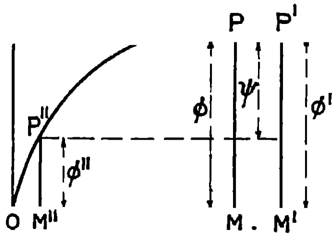


FIG. 158.—Theory of Brown Boveri regulator.

in the iron contributed by the anode ampere turns OM, then, as the choke is working at a point of high saturation, a slight increase in the ampere turns will produce only a small increase in flux. Let the constant D.C. excitation contribute ampere turns to the extent $OM'' = MM'$ and the resulting D.C. flux

will then be $\phi'' = M''P''$ and the total flux $\phi' = P'M'$, which is approximately equal to ϕ. The counter E.M.F. when D.C. excitation is provided is proportional to $\phi'T$ where T is the number of turns on the anode coil; but when the D.C. excitation is applied the total flux is $\phi' = \psi + \phi''$, and therefore as ϕ'' is contributed by the D.C. circuit, ψ must be contributed by the anode current, and the counter E.M.F. is reduced to a proportionality to ψT. Thus the introduction of a comparatively small D.C. excitation has a considerable effect on the counter E.M F. in the anode circuit and hence on the rectified voltage.

The absolute determination of the rectified voltage is dependent on the value of the resistance in the rheostat, which can be set to any desired figure. The chief factor in the successful operation of this regulator is in the degree of saturation of the iron, and it is important that the saturation should be high even for small values of the anode current.

It will be apparent that this method of regulation may be applied to three- or six-phase systems with equal success, and, further, if required the winding connected to the rectified circuit may consist of a compound coil, part of which is connected in series and part in shunt.

Over-compounding can be arranged to the extent of a 10 per cent. rise if desired by adding sufficient series turns at the expense of those in shunt.

A slight variation of this scheme has been devised by Messrs. Brown Boveri, in which a six-phase secondary is interlinked by means of the odd and even anode terminals, so as to provide, on a separate transformer, a voltage regulation within fine limits.

In some of these cases where choke coils are provided it will be apparent that the insertion of a counter E.M.F., whilst contributing to the better regulation of the circuit, also has a direct effect on the lengthening of the rectified wave supplied from each anode and hence reduces the undulatoriness of the rectified voltage

(vii) An ingenious device has been invented by Messrs. Siemens Schuckert for improving the regulation. It has been

stated that the arc is susceptible to the influence of a magnetic field, and in this scheme the anode consists of either a metal ring through which is threaded an electromagnet which is in series with the main rectified current, or a circular trough containing the mercury cathode with the electromagnet threaded through its centre and connected as in the previous arrangement, only in the latter case a potential winding is added to the magnet, which is connected in shunt with the rectified current. The arc travels in an erratic manner over the surface of the cathode, and if it is distended by the influence of the magnetic field, and moreover is constrained to terminate on a comparatively narrow circular annulus, it will travel round in a circle with a more or less constant angular velocity. On account of this circular motion a counter E M.F. will be generated in the core which will be proportional to the load on the rectifier because the flux is controlled by the series current. The tendency of any generator of electrical energy which operates on a falling characteristic is to take more and more of the load as the voltage falls until ultimately it denudes the other sources of energy of their proportion and itself ceases to function; therefore the mercury rectifier will have this tendency also unless it is corrected, and this device employs the counter E.M.F., which increases with the load to prevent it taking more than its fair share or, in other words, to give it the same effect as if it had a rising characteristic.

Efficiency and Power Factor.—The drop of voltage in the arc only varies at the most from 15 to 20 volts, and the arc losses are therefore a product of the mean current and the arc drop, or, in the case of an 800 ampere high voltage rectifier, about 14 K.W. To this loss has to be added the iron and copper losses in the transformer and in the other parts of the circuit. As the iron losses in the transformer are to all intents and purposes constant at all loads, the main losses above a certain limit (about ¼ load) are constant and the efficiency remains at a high figure which will increase up to a point with the voltage of supply.

The following table, which is the result of tests made by

Messrs. Brown Boveri, indicates how these losses are made up for a rectifier working at about 300 K.W.:—

TABLE XXI.

Item.	Apparatus.	Losses in Watts.	Per Cent. Losses.	Remarks.
1	Rectifier cylinder	10,000	3·10	Losses approximately constant at all loads.
2	Transformer	9,000	2·80	With partial load, efficiency dependent on iron losses more than on copper losses.
3	Anode choke	480	0·15	Losses vary with the load.
4	A.C. excitation	300-600	0·09	Losses commence at 600 and decrease to 300.
5	Auxiliaries (air pump, etc.)	900	0·28	When pump is stopped these losses are nil. Usually the pump runs continuously.
6	Voltage regulator	800	0·24	Efficiency depends on load.
	Total	21,480	6·66	Full load and overall efficiency 93·3 per cent. for a 300 K.W. rectifier.

(Items 4 and 5 are non-existent in a glass bulb rectifier.) From the above it will be seen that 50 per cent of the losses are

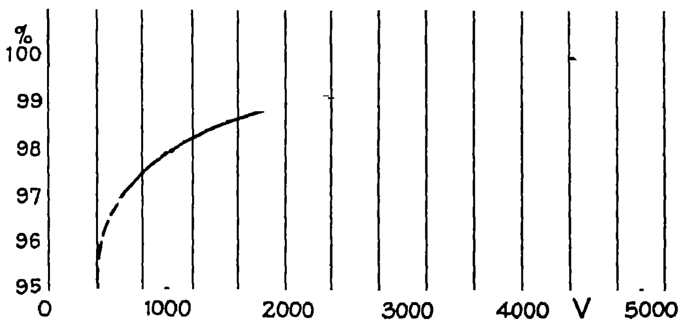


FIG. 159.—Effect of supply voltage on efficiency.

due to the volt drop in the arc, and as this is practically constant an increase in efficiency will accrue if the voltage of supply is increased as the current sensibly remains the same and the

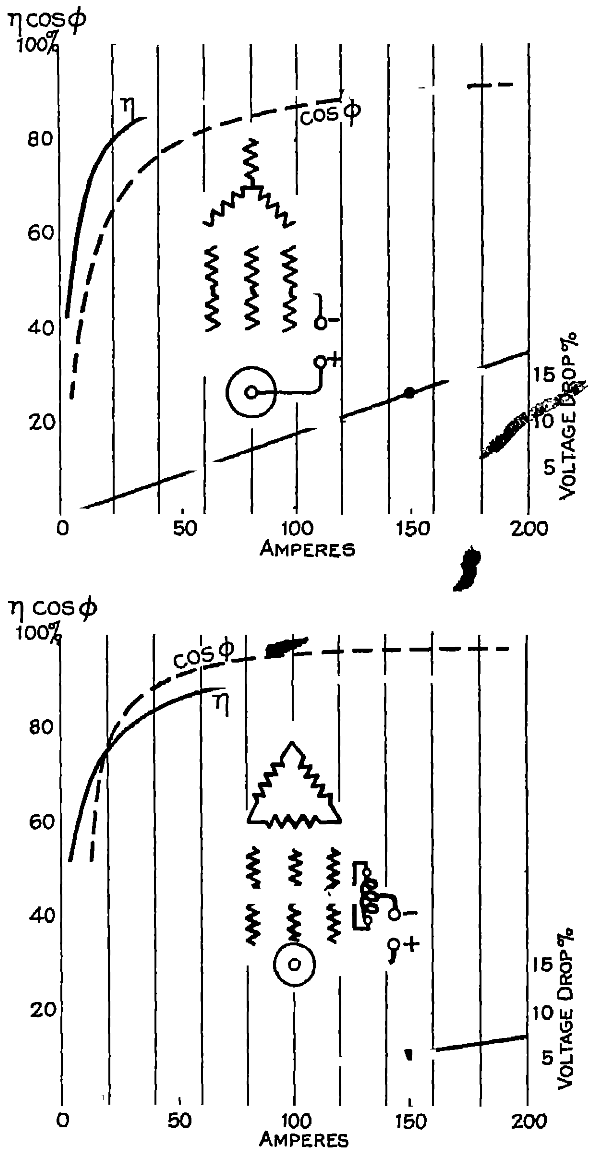


Fig. 160.—*a*, *b*. Effect of a regulating reactance in various circuits.

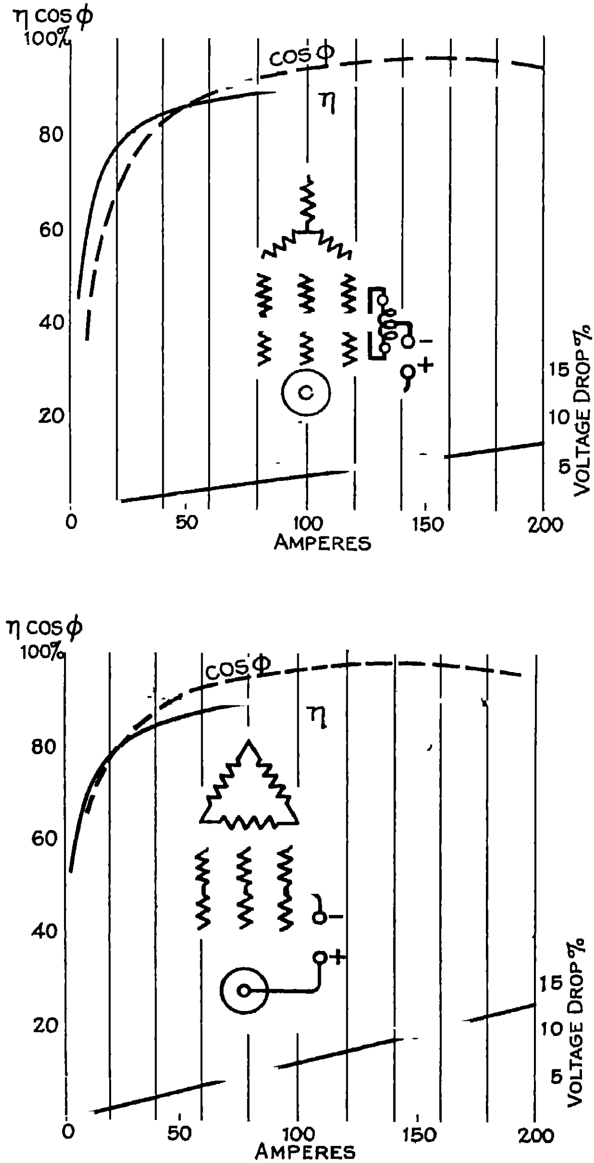


FIG. 160.—*c*, *d*. Effect of a regulating reactance in various circuits.

losses are therefore constant; Fig. 159 shows the effect of the variation of the supply voltage on the overall efficiency for supply voltages varying from 65 to 1000 volts, and it will be noted that the efficiency rises from 78 per cent. at 65 volts to 97 per cent. at 900 volts—a considerable gain—but that the efficiency does not increase markedly after a voltage of 500 is reached. Any improvement therefore should be directed firstly to a supply at this voltage.

It is necessary at this point to issue a word of caution as to the use of the word efficiency in rectified circuits generally. In the electrolytic sense, which includes the charging of batteries, etc., the efficiency is measured by the mean value of the current, whereas in most cases, such as the operation of motors, lighting, etc., the "energy" current or effective value is the one which is concerned. When, however, the rectified current pulsates with a small amplitude (i.e. when the undulatoriness is small) these two values are almost identical. It may therefore happen that the efficiency of a rectifier may appear to be different when supplying current to varying types of load (page 76).

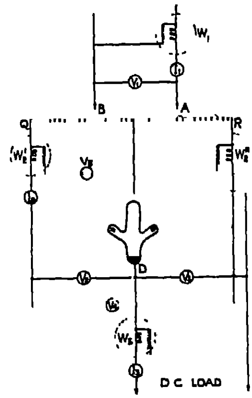


FIG. 161. — Experimental test circuit for mercury rectifier.

In the previous chapter the effect of the inclusion of a regulating reactance has been considered, and the mode of operation discussed. The characteristic curves of a rectifier with delta and star connection, both with and without a regulating reactance are shown in Fig. 160 and it is there seen that the voltage drop of the rectifier system is less than 50 per cent., where a reactance is employed, than it is without such a device.

So far the container loss, which is dependent on the drop in the arc, has been taken to be constant. This is not strictly true and it is interesting to compare the effect of this assumption with what is known to be the case. Dr. W. Tschudy has

conducted a series of classical experiments which elucidate this point and incidentally others at the same time.

In Fig. 161 I_1 and V_1 and W_1 indicate the effective current, voltage and power.

The total input $W_1' = W_c + W_i + W_g + W_3'$,

where W_c = transformer copper loss.
W_i = transformer iron loss.
W_g = bulb loss.
W_3' = power output.

The transformer output $= W_1' - W_c - W_i = W_2$

and $W_2 = W_2' + W_2''$ (corrected reading).

The bulb input $= W_2$

and the bulb output $= W_3'$ (corrected reading).

Rectifier efficiency $\eta_2 = \dfrac{W_3'}{W_2}$.

Transformer efficiency $\eta_1 = \dfrac{W_2}{W_1'}$.

Overall efficiency $\eta_3 = \dfrac{W_3'}{W_1'}$.

The bulb loss is calculated from the expression

$$W_g = W_2 - W_3'.$$

The values of these different instrument readings are given in Table XXII. and the curves are drawn in Fig. 162 connecting the various quantities. It must be remembered that the tests were taken on a rectifier of small capacity, the voltage of which was only 200, and the curve in Fig. 162 will indicate that this loss would be a very much smaller proportion at higher voltages; yet the curves point out very forcibly that there is a definite variation of bulb loss with load.

Further curves are given connecting the effect of the wave form on the bulb loss (see Fig. 163). The method by which Dr. Tschudy accomplishes a change in wave form and its effect on the bulb efficiency is described in U.S. Patent 1,189,887 of July 4, 1916, in which the mathematical analysis of the current and voltage waves is given.

TABLE XXII.

V_1.	I_1.	W_1'.	Cos ϕ_1.	V_4	I_2.	V_2.	W_2.	W_g	$\frac{W_g}{2}$.	V_3.	I_3.	W_3'.	$I_1 V_3$.	Average of two Columns Preceding.	η_1.	η_2.	η_3.
110	6·10	587	0·870	311 5	2 64	211·1	534	45	22	123·8	3·92	485	435	485	91·0	91 5	82·6
110	9·00	858	8 866	310·0	4·13	210·6	772	69	35	114·8	6·10	699	700	699	89·9	91·0	81·5
110	11·72	1090	0 847	306·5	5·40	210·0	986	100	50	111·0	7·95	882	882	882	90·4	89·9	81·0
110	14·05	1270	0·822	304·5	6·58	208·4	1138	129	64	106·8	9·45	1005	1005	1005	89·7	88·6	79·1
110	16·10	1434	0 810	301·3	7·48	206·5	1264	156	78	101·9	10 85	1104	1104	1104	88 2	87·6	77 0
110	18·28	1593	0·792	299 8	8·48	204·3	1396	189	95	98·8	12·20	1202	1205	1203	87·5	86·5	75·5
110	20·00	1712	0·778	298·0	9·15	202·0	1460	212	106	93·8	13·26	1244	1244	1244	85·2	85 5	72·6

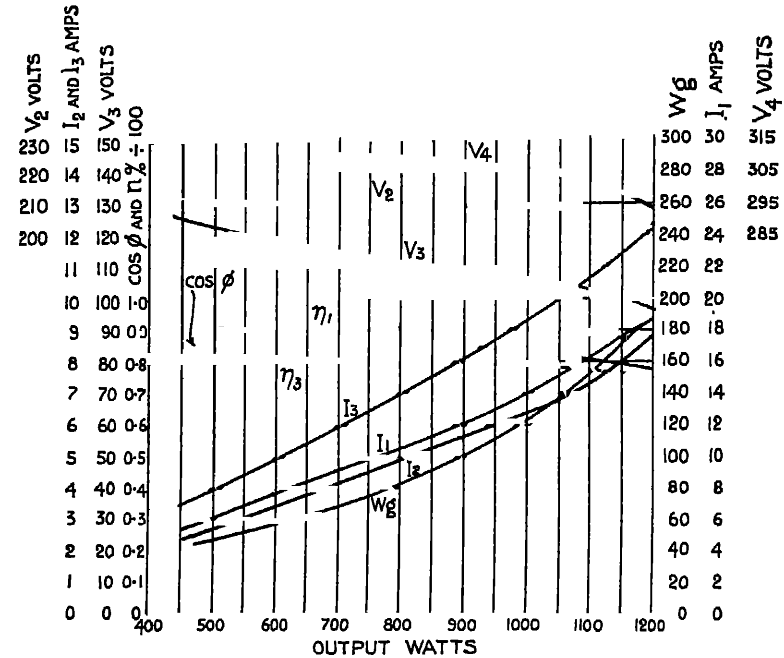


FIG. 162.—Mercury vapour bulb loss as a function of the voltage.

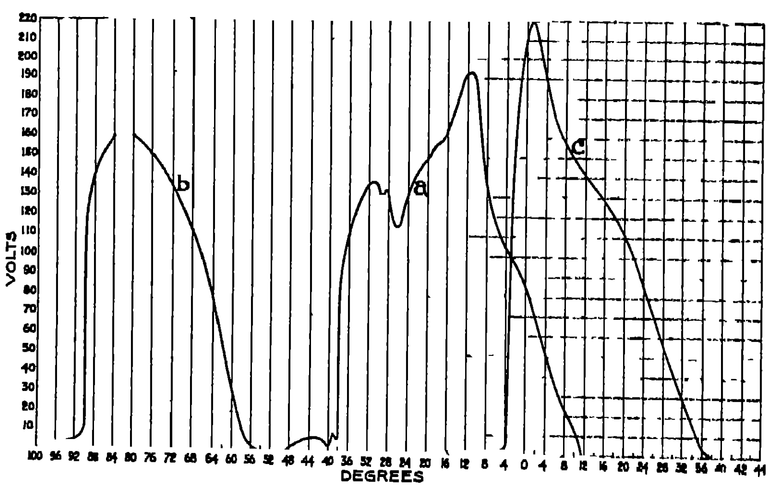
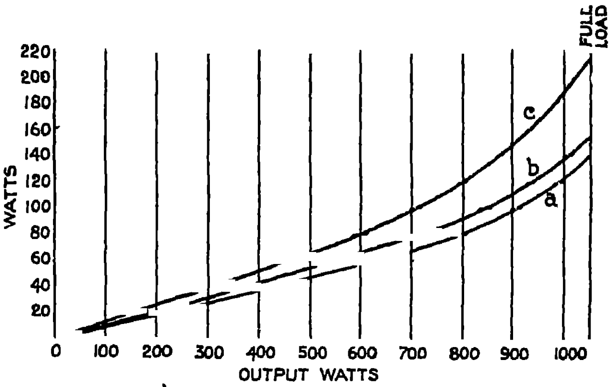


FIG. 163.—Mercury vapour rectifier bulb loss as a function of wave form.



CURVES (a),(b) AND (c) REFER TO THE WAVE FORMS IN FIG.163

FIG. 164.—Mercury vapour rectifier characteristic curves.

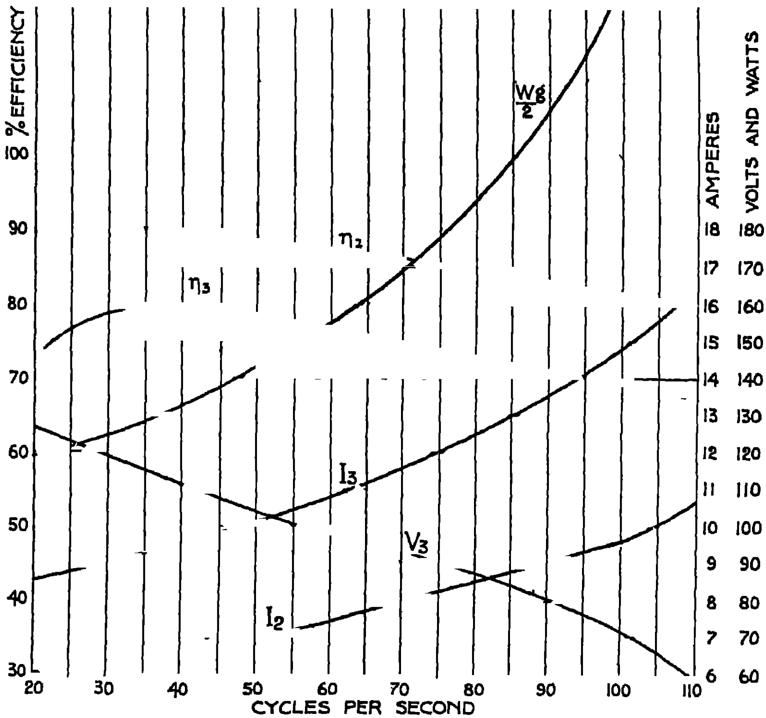


FIG. 165.—Mercury vapour rectifier bulb loss as a function of frequency.

The relation between anode and rectified watts output are given in Fig. 164.

Thus the wave form also has a very definite effect on the bulb output. The effect of frequency of supply voltage has been investigated and the relations are indicated in Fig. 165.

As regards power factor, the results in Table XXII. indicate a falling value for cos ϕ with load. This is for a small type of rectifier, but in the larger sizes the figure is greatly improved, as is seen by some tests carried out by Messrs. Brown Boveri and the results of which are given in Fig. 166.

These curves are for three rectifiers in parallel and working under test conditions.

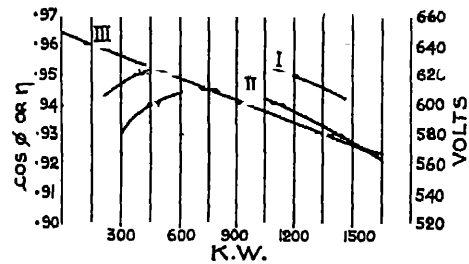


FIG. 166.—Test curves mercury vapour rectifier.

Curve I. Efficiency.
Curve II. Power factor.
Curve III. Voltage.

The power factor is dependent on the presence of higher harmonics in the rectified current wave, and therefore one would expect that in the case of three- or six-phase circuits where the rectified current begins to assume a less undulatory aspect that the power factor would improve. This is borne out by the facts as shown by a comparison between Dr. Tschudy's data on a single-phase rectifier and the larger six-phase arrangement of Messrs. Brown Boveri.

Anode Choke.—There are certain theoretical considerations in the design of choke coils for rectifier circuits which are worthy of attention. The rectified current undulates between a maximum current i_{max} and a minimum current i_{min}; this current passes through the windings of the choke and the consequent

induction varies from B_{max} to B_{min}, but these alternations are at double the supply frequency. Therefore if T is the number of turns on the choke and s the sectional area of the iron, the following relation for the counter E.M.F obtains

$$E_{max} = 2\pi(2f)\frac{B_{max} - B_{min}}{2} . sT \times 10^{-8} \text{ volts}$$

and hence

$$sT = \frac{\text{constant}}{B_{max} - B_{min}}$$

From this it is apparent that to obtain a perfectly smooth wave, i.e. when $B_{max} = B_{min}$ a choke of infinite size would be required, and as the whole of the rectified current has to pass through its windings only a poor compromise can be effected as to its smoothing action. Thus although the choke cannot be too large, yet on the other hand it must not be too small especially in biphase circuits as a useful current can only be obtained when the instantaneous value of the rectified circuit exceeds the counter E.M.F. of the load (if such there be). It is a fact, however, that a motor load will be quite effective with comparatively small chokes, i.e. of a size corresponding to 30 or 40 per cent. of the motor input. It has been suggested by Jonas that a shunt of no reactance across the choke will facilitate matters although the motor efficiency is reduced thereby by an extent of 3 to 4 per cent. yet the armature heating is also reduced. This device allows the constant current component i_g to pass through the choke, the alternating component taking the line of least resistance through the resistance.

Regulating Reactance.—The regulating reactance takes the form of an auto-transformer with duplicate windings in opposition so arranged as to have a high impedance with a small resistance. The use of this device reduces the voltage drop from 12 to 5 per cent.

Elimination of Harmonics.—In many cases of rectifier installation disturbances have been troublesome due to induced currents in telephone lines, and numerous devices have been tried in order to eliminate the objectionable harmonics.

One type of filter is shown on page 309, but as the main current has to pass through the choke coil the reactance will vary with the load. Messrs. Siemens Schuckert (British Patent 253914) have produced a type of filter which has none of these defects.

In Fig. 167 a rectifier R is supplying current through two chokes C_1 and C_2 to the direct current mains; and in parallel with these mains filter circuits F_1, F_2, etc., are placed, each one so designed as to tune out one particular harmonic.

This arrangement in which the two chokes C_1 and C_2 are magnetically separated, allows a very much smaller percentage of harmonics to pass into the direct current supply and the apparatus is much cheaper to manufacture.

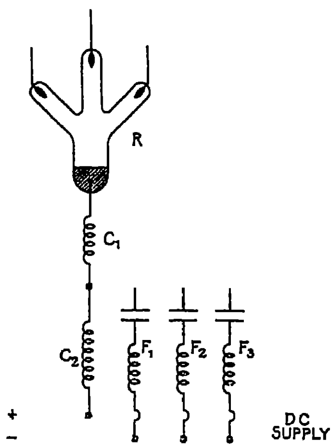


FIG. 167.—Siemens filter.

Battery Charging Plant.—If apparatus is required specially for battery charging it is advisable that the complete rectifier should have an inherently bad regulation to ensure a reduction of the voltage with increase of load. In special cases this may be accomplished by the use of a choke coil with an airgap in the iron circuit which may be adjusted as desired. If buffer cells are provided the supply transformer can have a number of tappings connected to a rotary switch with numbered contacts referring to the number of cells which can be cut out by any one tapping. Thus for instance if the switch is set to the number of cells to be charged, a current will flow at a voltage equivalent to the number of cells multiplied by 2·15, but if the voltage of the cells is 2·55 per cell only one-fifth of this current will flow. It can further be arranged that this current of one-fifth the normal current shall cause the arc to become extinguished, thus constituting a semi-automatic charging system.

Advantages of Mercury Vapour Rectifiers.—It is not the

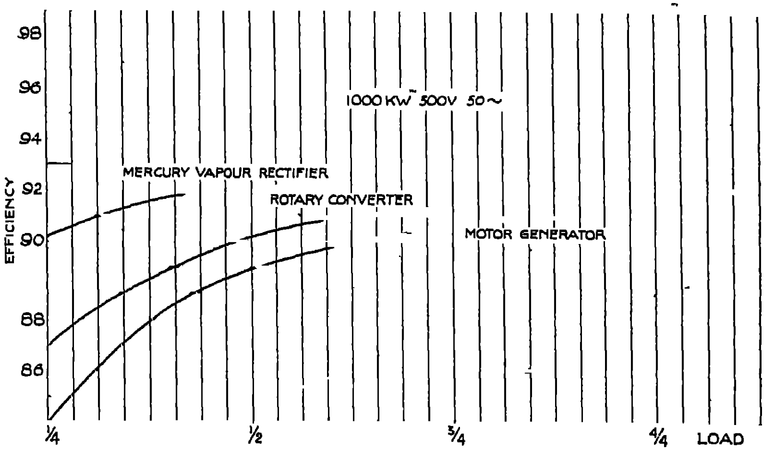


FIG. 168.—Comparative efficiencies and outputs of converters.

intention to attempt to depreciate the value of other types of alternating current converters, but it is interesting to compare

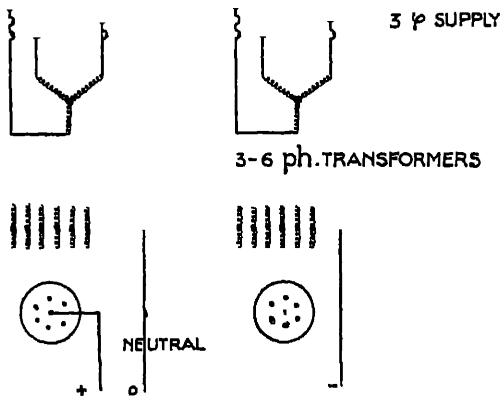


FIG. 169.—Three-wire plant.

the efficiencies of mercury vapour plants with rotating machines of the same size and capacity.

In Fig. 168 three curves are given of the efficiency of a mercury vapour rectifier including all accessories η_r, a rotary converter η_c, and a motor generator η_g. These curves must be accepted with reserve as regards their absolute values, and are only an indication as to what may be expected. The comparatively high efficiency of the rectifier at low loads is a noteworthy feature.

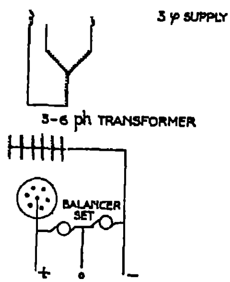


FIG. 170. — Three-wire plant with balancer set.

Three-Wire Plants. — Three-wire systems can readily be operated from mercury vapour rectifiers; and one arrangement with two rectifiers in series is shown in Fig. 169, but this has the disadvantage of causing the rectifier to work at a lower voltage and therefore a lower efficiency. A better method is to employ one rectifier and use a balancer set, whereby the condition of a higher voltage across the rectifier terminals is obtained (see Fig. 170).

CHAPTER X.

MERCURY VAPOUR RECTIFIERS (*cont.*).

METAL CASED RECTIFIERS.

General.—The metal cased mercury vapour rectifier has been developed on the Continent but so far has not been used largely in this country. There are several installations in England which appear to give satisfaction once the method of control and the special peculiarities of the plant are understood.

The mercury vapour rectifier depends for its certainty and constancy of operation on the state of the vacuum in the container In the case of the glass bulb, this is out of the control of the user, but with the steel container the maintenance of a vacuum, of an order much higher than is usually required commercially, is essential to good working. Thus special plant, more often found in a laboratory, than in substations, is a necessary constituent part of the equipment. It is not to be inferred that the pumps and gauges are too frail for commercial usage or that they cannot be controlled by the switchboard attendant, but nevertheless instruments of precision are required, and must be used as such if continuity of service is to be obtained.

Starting.—When small glass bulb rectifiers are put into service, the arc is struck by a tilting mechanism which employs a subsidiary cathode, but in metal containers of large size this is not a practicable proposition and an auxiliary device is used as shown in Fig. 171. This form of starter consists of a solenoid *a* operated by a push button *d* and supplied from a separate source. On contact being made by the push, the starting electrode *b* is drawn down and makes contact with the surface of the mercury cathode *c*; an arc is immediately struck

and the solenoid is short-circuited as shown in the diagram and the anode withdrawn. Thus if the push button is pressed continuously, a make and break will continue in consecutive order. Immediately an arc is formed at the starting anode the main arcs will also strike and the push button may be withdrawn. To actuate the solenoid about 500 watts are required, the supply for which is usually obtained from a small auxiliary D.C. motor generator set, but rotary plant can be dispensed with if preferred and be substituted by a small rectifier of the gas-filled type (Chapter XIV.) If the rectifier is required for battery charging only, the necessary starting current may be obtained from that source and no further supply is necessary.

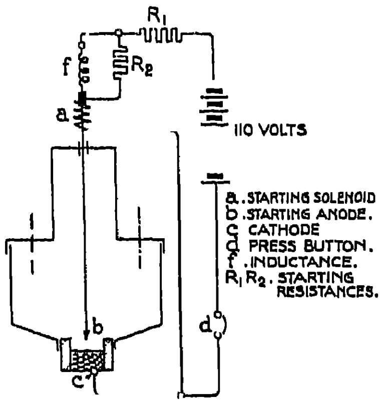


FIG. 171.—Mercury vapour rectifier—auxiliary starting device.

Back-starting is a phenomenon which is very detrimental to good performance. As may be expected, if for any reason the anode becomes hot, it will form a "cathode" spot and current will pass in the reverse direction. Once this state of affairs has commenced it will obviously become aggravated and will increase.

The special causes of back-starting in a metal cased rectifier (see also page 237) are:—

(i) If the arc concentrates at one point of the anode it is liable to cause vaporisation of the anode.

(ii) Drops of mercury may trickle down the tube on to the anode.

(iii) Ultra-violet rays which are present in large quantities may cause ionisation which may be sufficient to start an arc in either direction whether the cathode be hot or not, and current will pass in either direction from the anode to the cathode or conversely.

These difficulties have been completely eliminated in modern

design, and reference to Fig. 172 will show that the main anodes are covered by a protecting shield *g* which prevents mercury condensing or dropping on them; and also on this shield are

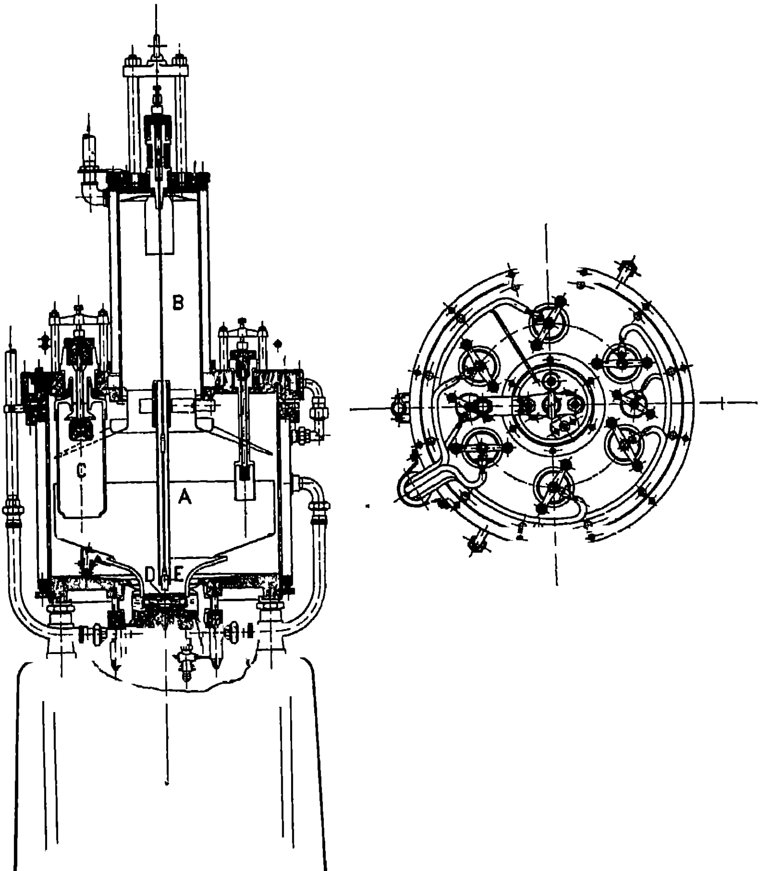


FIG. 172.—Section of steel case rectifier.

fixed louvre shaped metal pieces which so bend the arc as to prevent any injurious ultra-violet rays reaching the anode.

Excitation.—In cases where the loading on a rectifier is intermittent it is possible that the current may fall to zero or to

a very low value in which case unless some special arrangements are made, the rectifier will refuse to re-excite. Automatic working is the solution in some cases (page 274), but in the main, where large plants are involved, it is usual to provide auxiliary anodes which are supplied from a separate transformer. A current of about 5 amperes at 30 volts is required for the excitation of each small rectifying unit, and thus for a six-phase supply a transformer is advisable with a secondary output of 130 volts at 30 amperes per unit. This arrangement will reduce the losses in the exciter but is only available for use after the exhaustion of the rectifier chamber has been sufficiently advanced. It is usual to supply a choke coil and variable rheostat with each unit to ensure the current in the arc being well above the zero value.

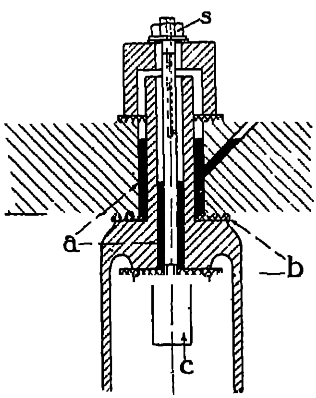


FIG. 173.—Mercury seal.

As a general rule it may be taken that, if the load is likely to fall below one-fifth of the normal full load, external excitation is advisable on account of the instability of the main arc.

Voltage.—One advantage of mercury vapour rectification is that one unit can work between such wide ranges of voltage as from 110 to 800 on the smaller sizes, and up to 5000 volts on the larger units. The criterion of size depends more on the maximum continuous current to be carried than on the voltage, and for very large currents parallel operation is resorted to.

For voltages above 600 specially designed units are available up to 5000 volts (see page 273), the sole trouble being, with the higher voltages, the provision of a suitable insulation to stand up to the high vacuum required.

Electrodes.—Numerous patents exist specifying various improvements in the shape and function of the anode and the cathode. Generally in the standard types of apparatus the

anode will consist of a solid iron cylinder 70 to 100 mm. in diameter, and it is important that sharp edges should be absent otherwise back-starting is liable to occur, but on the other hand it is equally important that as large a surface as possible should be available; therefore the anode is sometimes made out of cylindrical rod with a spherical indentation at the lower end to increase the surface area.

Various devices are used to increase the overall efficiency: for instance, one type provides for a construction similar to that of a petticoat insulator, with the various sheds insulated from each other for high voltage rectification, and so arranged that a high state of evacuation can be maintained for considerable periods.

Sealing.—Sealing presents one of the biggest of the problems in the design of an all-metal rectifier, but so far has been improved to such an extent as to render possible the building of large power units which up to a few years ago were an unknown quantity.

Fig. 173 illustrates a section of a simple form of mercury-asbestos seal employed by Messrs. Brown Boveri; *b*, *b* are two asbestos washers which can be tightly clamped to the frame by the nut *s*, this also forming the method of support of the anode insulator. Mercury is then poured into the inclined hole and finds its way into the annular space between the insulator and the walls of the container. It is found that the vacuum inside the container is high enough to cause the mercury to fill up the pores of the asbestos and also to cling very tightly to the sides of the container thus forming a tight and efficient seal. It is not safe to assume that this seal is so perfect as to permit of continuous working without the aid of a pump, but in view of the difficulties attending the design of a metal seal, the one illustrated is satisfactory with that proviso.

Cooling Arrangements.—The hottest spot in a rectifier is the cathode where a temperature of about 3000° C. is reached; the temperature gradient to the outside cover is fortunately high as the mean temperature does not usually exceed 60° C. At full load the anodes may be at a dull red heat, about 400° to

600° C., and as they form the principal item of renewal it is essential that they should be kept as cool as possible; there is also the necessity for preventing back-starting which accompanies an overheated anode. Water cooling is therefore essential if early and frequent renewals are to be avoided.

In the later models of rectifier automatic arrangements are included whereby the supply is switched off when the temperature of the anode plate reaches 53° C.

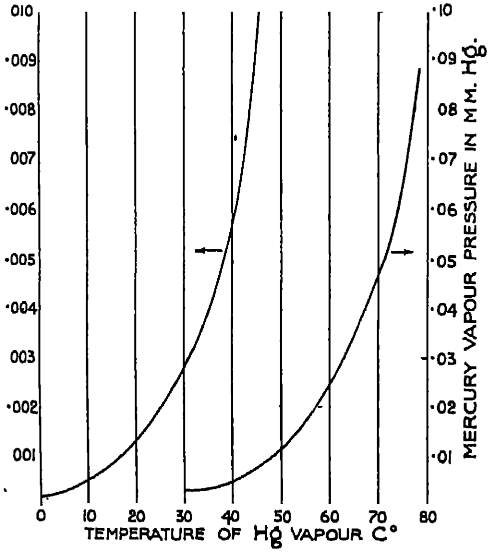


FIG. 174.—Temperature-vapour pressure chart for mercury.

In high voltage installation where it might be difficult to drop the voltage to earth through the cooling water, natural or forced draught may be employed.

It is impossible to describe these arrangements in detail, but an interesting article in the "Brown Boveri Review" for December, 1926, gives a detailed description of the various possibilities.

The usual figures for the quantity of water required for efficient cooling are 0·5 to 1·0 litres per minute for every 100 amperes of rectified current. Under these circumstances in

a good design of cooling system, the anode temperature should not exceed 50° C. This is in the case of a fresh water supply: if such is not available cooling tanks must be provided, together with the necessary pumping equipment or thermo-syphon.

Fig. 174 is a diagram which shows the temperature of mercury vapour plotted against the vapour pressure, so that with a pressure of 0·05 mm. the wall temperature should be 73° C. or a normal rise of about 60° C.

Air Pump.—The attainment of a high vacuum is essential to the good working of a rectifier, if only for one reason, viz. that of anode temperature; and to ensure that such is maintained at a low value a pump is necessary. When a rectifier is newly installed or has been opened up for repair there are considerable quantities of gas, consisting of air, carbon monoxide, or water vapour, etc., occluded in the metallic casing and in other metal parts. These gases will not be released by one simple pumping to a high state of vacuum, but will be drawn off from the walls continuously for some considerable period at a rate depending on the temperature; and thus even though the container is apparently perfectly gas-tight the vacuum will fall until all the occluded gas is drawn off. This process will last several weeks or months depending on the size of the load and heating of the rectifier, but it is not safe to cease from operating the pump until at least three months have elapsed from the date of installation, and at least two weeks should elapse before full load is put on the plant. Porosity of the container will possibly be another cause of loss of vacuum, and this is a much more difficult trouble to eradicate. There is a mean position of running at which the rectifier will attain its optimum vacuum in the quickest time and this can only be determined by a careful scrutiny of the actual temperature with the safe working temperature.

At full load it is never safe to run without the pump in operation, but in certain cases it is found possible to take half load current continuously without running the pump. As, however, the consumption of the pump motor is small it is not worth while incurring the risk of a breakdown; and

recommendations are usually to the effect of keeping the pump in continuous operation.

Fig 175 indicates a typical air pump consisting of a mechanical box pump driven by a $\frac{1}{2}$ H.P. motor and in series with it a mercury diffusion pump. This latter is in effect an ejector, but one in which the heavy molecules of vapourised mercury are used to bombard the lighter molecules of the gas and eject them from the container. This type of pump is often known by the name of the Langmuir pump and is highly efficient in operation. Experiment has shown that this combination of box pump and diffusion pump is capable of withdrawing 1400 c.c. of gas per second at a pressure of 0·02 mm. of mercury, and in the event of a stoppage a check valve is provided to shut off the vacuum pipes from the atmosphere automatically.

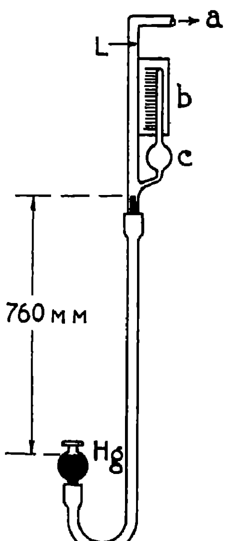


Fig. 176.—Diagram of McLeod gauge.

With these diffusion pumps in series with a box pump a vacuum of 10^{-3} mm. can be obtained in a commercial rectifier. In the laboratory it is possible in combination with a liquid air trap to increase this figure to 10^{-6} mm. or less if special precautions are taken.

Measurement of Vacuum.—The most sensitive method which is practicable, and one which is easily capable of recording a pressure of 10^{-3} mm. of mercury is the McLeod Gauge.

Fig. 176 shows this particular gauge in section. The glass tube *a* is connected to the container in which the pressure is to be measured, and provides a through passage to the bulb *c* and the top of the mercury column. Thus in the position indicated the bulb and the tubing are at the same pressure as the container. The mercury bulb is connected by a flexible tubing to the tube *a* and on raising the container, mercury passes the junction to the bulb and a portion of the gas, still at the same

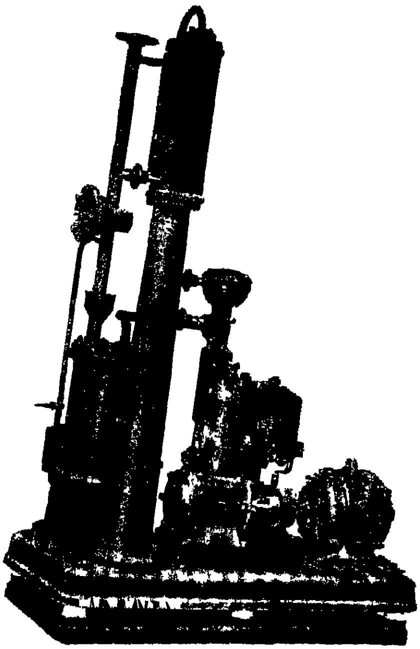


Fig. 175.—Air-pump for mercury vapour rectifier.

[To face page 268.

pressure as the container, is trapped, and on further raising the the column it is compressed into the bulb *c*.

The column is still further raised until the level of the mercury in the tube *a* is the same as that at the top of the tube *b*. By this time the mercury will have risen and filled the bulb and will have compressed the gas into a short length of the tube *b*. The difference in level of the two mercury columns (or what is the same thing, if the scale is correctly adjusted—the height of the mercury in the tube *b*) will be a measure of the pressure in the rectifier container. The calibration of this particular gauge depends on the cubic capacity of the bulb *c* compared with that of the tube *b* when the heights are correctly

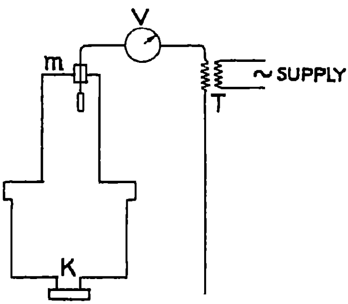


FIG. 177.—Vacuum gauge for rectifier.

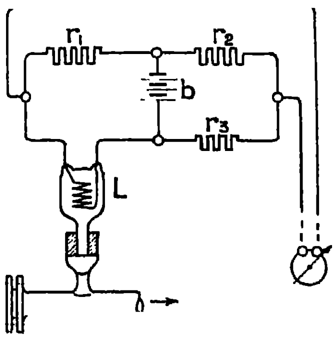


FIG. 178.—Vacuum gauge for rectifier.

adjusted. A simple application of Boyle's Law, which states that the product of the volume and pressure of a gas is constant when undergoing compression, enables the original pressure to be ascertained.

The disadvantages of this gauge are

(i) its fragility and delicacy of operation, and

(ii) the fact that it is not a continuously indicating recorder, as the mercury container has to be raised and lowered each time a reading is required. The first objection is overcome by enclosing the delicate glass parts in a metal case with special arrangements for raising the bulb, but the second objection is an inherent defect in the design of the apparatus itself.

Two methods are in use which enable continuous pressure readings to be obtained and also admit of an electrical remote control.

Fig. 177 indicates how the pressure may be measured directly in the rectifier container itself, the principle depending on the fact that there is a definite relation between the vapour pressure and the electrical conductivity of mercury vapour. A small auxiliary transformer T supplies alternating current to an auxiliary anode, and connection is made to a voltmeter which can be calibrated to read directly in millimeters of pressure.

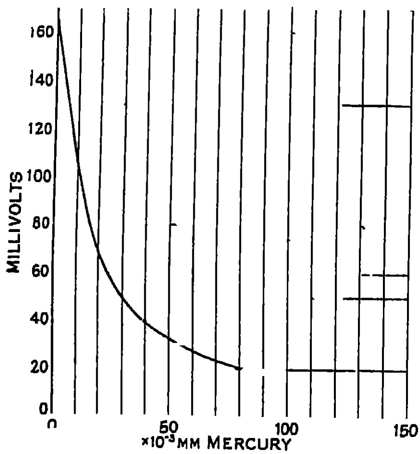


Fig. 179.—Calibration curve for pressure gauge illustrated in Fig. 178.

Unfortunately, however, the indications of this apparatus are not altogether independent of the rectifier load and it has, therefore, only a limited application.

Another method is based on the heat conductivity of rarified gas which varies with the temperature at high vacua, and also on the increase in resistance of a metal film exposed to high temperatures. In Fig. 178 the glass vessel L is connected by piping to the rectifier container, and the platinum foil in the vessel forms the fourth arm of a Wheatstone Bridge with a microammeter as shown. This instrument can be calibrated

to read pressures as low as 0·01 mm. of mercury, and the readings have the advantage of being independent of the load variations.

Fig. 179 shows the calibration curve for a gauge of this type.

Transformers.—It has been shown in Chapter VIII. (page 228) that the rating of the secondary must be so calculated that the copper conductors allow a greater current carrying capacity than those of the equivalent primary. The general conditions of input and output for various circuits have already been dealt with; but there is one specialised method of internal connection which is of value in reducing the number of coils in a polyphase transformer.

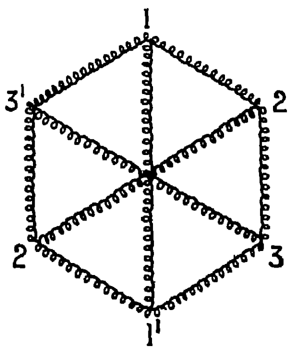


FIG. 180.—Six-phase transformer.

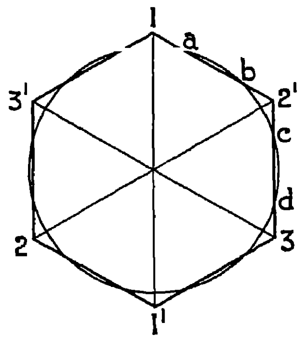


FIG. 181.—Division of winding in six-phase transformer.

In Fig. 180 six coils are shown spaced equally and joining the apices of a hexagon, and six other coils are also supplied to the neutral point. This will form a six-phase supply in which the currents are equally balanced. If now about a similar hexagon (Fig. 181) a circle is described so that the intersection of the hexagon and the circle takes place at points *a*, *b*, *c*, etc., then if $ab = bc = cd =$ etc., and tappings are taken from the coils at *a*, *b*, *c*, etc., a twelve-phase supply will be obtained with a similar current balance. If required this method can be continued to provide an eighteen-phase system, and in either case will result in an economy of copper in the transformer windings.

Description of Plant.—Individually the apparatus has been described in the foregoing paragraphs in detail, but some points are worthy of attention which have not been so far considered. It is not possible in the case of a mercury vapour rectifier to earth the metal container as is usually done with other forms of electrical plant, as if the interior of the case were at earth potential the arc might stray to its sides, and it is necessary, therefore, to construct the entire apparatus on an insulated base. The method by which this is achieved in the Brown Boveri Rectifier is shown in Fig. 182, where a 900 ampere unit is illustrated.

In the case of the rectifier in Fig. 183, the rectifier is situated abroad, and the regulations do not prescribe a containing and earthed metal shield, as is the case with the equipment illustrated in Fig. 182. (The radiating fins on the outside of the anode container are visible, which are themselves cooled by the water from the main cooling system.) The box pump, McLeod gauge and insulated stop valve controls are also shown. So far Messrs. Brown Boveri have standardised the following sizes of rectifier.—

300	ampere	up to	600 volts.
600	,,	,, ,,	600 ,,
1000	,,	,, ,,	600 ,,
250	,,	,, ,,	3000 to 5000 volts.
500	,,	,, ,,	3000 ,, 5000 ,,
750	,,	,, ,,	3000 ,, 5000 ,,

An increase in the capacity of mercury vapour rectifiers has recently been introduced by the Brown Boveri Co. The newer type is equipped with twelve anodes used in conjunction with transformers in double six-phase connection.

Fig. 184 illustrates the capacity of the plant, which has the following rating:—

Continuous rating	1500 K.W. at 1500 volts.
Overload rating	3000 K.W. for 5 minutes.
Overload rating	4500 K.W. for 1 minute.

Fig. 182.—Mercury vapour rectifier unit, 900 amperes.

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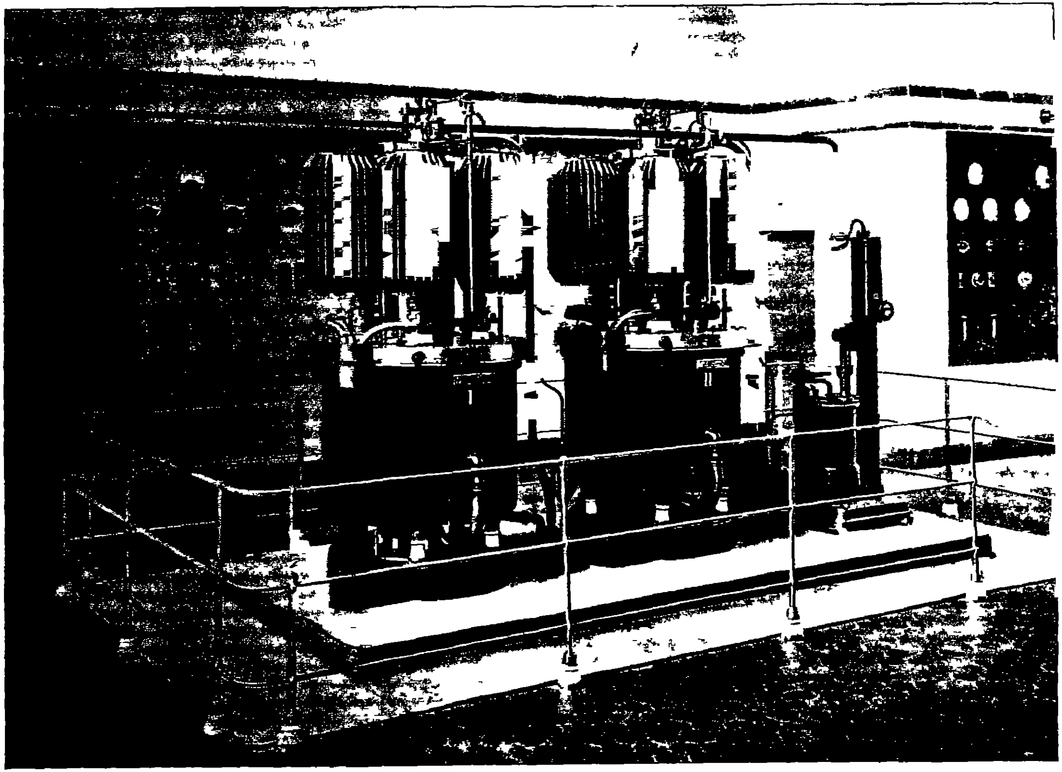

FIG. 183.—Mercury vapour rectifier, unshielded unit.

[See page 272.

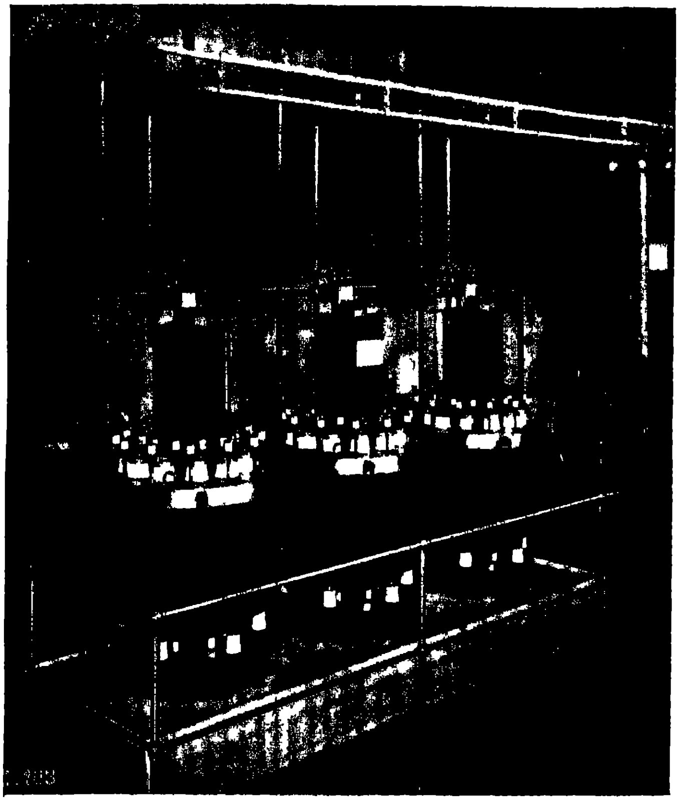

FIG. 185.—City of Berne substation.

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Large Plants.—Mercury vapour rectification has not been adopted so readily in this country as on the Continent; but in certain supply areas installations have found favour; at St. Albans and Hertford small rectifiers have been used to supply the whole D.C. load, thus showing considerable confidence in their reliability on the part of the Supply Authority. At Birmingham a small substation has been equipped with 250 K.W. of rectifier plant with such satisfactory results that the system is being extended. Again, in several of the Metropolitan Boroughs' and Supply Authorities' Power Stations and

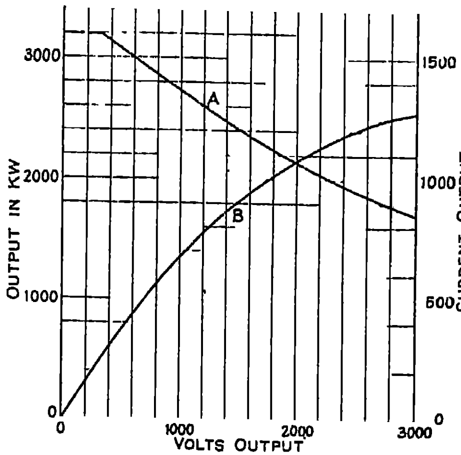

FIG. 184.—Capacity of Brown Boveri rectifiers.

Substations this type of plant has been tried. But it is to the Continent and to America that one must look for any extensive use of mercury vapour rectifiers.

The plant at the City of Berne Electricity Works has a capacity of 750 K.W. distributed over three units, the arrangement of which is shown in Fig. 185. The supply voltage is 3100 volts at 40 periods and is converted down by the transformer plant to 520 volts, the transformers being wound for a six-phase secondary with delta primary. Each cylinder has a normal current carrying capacity of 450 amperes and contains twelve anodes. In this particular station which supplies a tramway

network, the load curve indicates that the transient load never falls below one-fifth the full load, and it is therefore possible to work with the auxiliary excitation cut out of circuit. The starting of the plant is simple, consisting only of throwing in the main switch and pressing the starting push button on the switchboard.

About the same floor area is required as for rotary plant of the same capacity. A plan and cross section lay-out of the Pau Station of the Midi Railway, showing the transformers, cylinders and thermo-syphon coolers are illustrated in Fig. 186.

Automatic Operation.—Mercury vapour rectifiers above all other forms of converter appear to lend themselves to complete automatic operation. To describe the method by which such control is accomplished in general would be difficult; and a concrete case of a substation in this country has therefore been taken and is described below.

The first complete equipment was installed in the Harborne Substation of the City of Birmingham Electricity Supply. The method of control is by pilot cables from the power station, five in number, and no attendance is required at the substation for operational work. Indicating lamps at the power station inform the switchboard attendant when the rectifier is functioning correctly and whether the pumps are working satisfactorily. The only rotary plant which is needed is the D.C. motor generator and the pump, and as it has been found possible to supply the starting current from an alternating current source, the former has been dispensed with, leaving only the pump which will require periodic maintenance.

The operation of the various relays is shown in Fig. 187, and the sequence of events describing the control is as follows :—

By means of selector switches the rectifier can be left running continuously on the system, and under these conditions, will automatically be disconnected from the bus bars in the event of a fault or a failure of the E.H.T. supply, and started up again as soon as conditions are again normal. It can also be arranged for remote control from another substation, in which case the rectifier can be started up and shut down at will, and while

running will be automatically disconnected and reconnected as in the first condition.

The operation of the control gear is as follows, the action of the relays being described in their proper sequence of operation:—

(1) A switch operated by the secondary of a potential transformer, which when energised closes the circuit operating relay (2), and when de-energised due to the failure of the A.C. supply pressure, clears the set from the A.C. and D.C. bus bars.

(2) A time limit relay, which when energised closes contacts operating the oil switch closing solenoid (3), and when de-energised closes contacts which operate the oil switch operating solenoid (4). An interlock on this relay prevents the D.C. breaker from being closed or remaining closed when this relay is de-energised.

Immediately the oil switch closes, the pump and the ignition converter start up.

(5) The ignition relay which when energised through interlocks on relays (*5a*) and (6) closes the ignition arc circuit. The action of this relay is delayed 10 seconds to allow the ignition motor generator to excite fully.

(*5a*) If the ignition arc fails to strike, this relay comes into operation and de-energises (5), when the operation is repeated with an "in" and "out" movement until ignition is obtained.

(6) The coil of this relay is in series with the excitation circuit and operates immediately the ignition arc has been struck, by relay (5). When de-energised the relay shuts down the ignition motor generator and de-energises relay (5).

The D.C. circuit breaker, of the self-closing type, will now close, provided that the voltage across the rectifier terminals is correct and the D.C. bus bar voltage is not too high.

The closing coil is connected through an interlock on (2) and is also controlled by the adjustable voltage coils (7), (8) and (9);

(10) a series coil which opens the D.C. circuit breaker on overload. The operation of this coil is limited to three times, until reset by hand;

(11) a contactor which closes the loading resistance when the D.C. breaker opens;

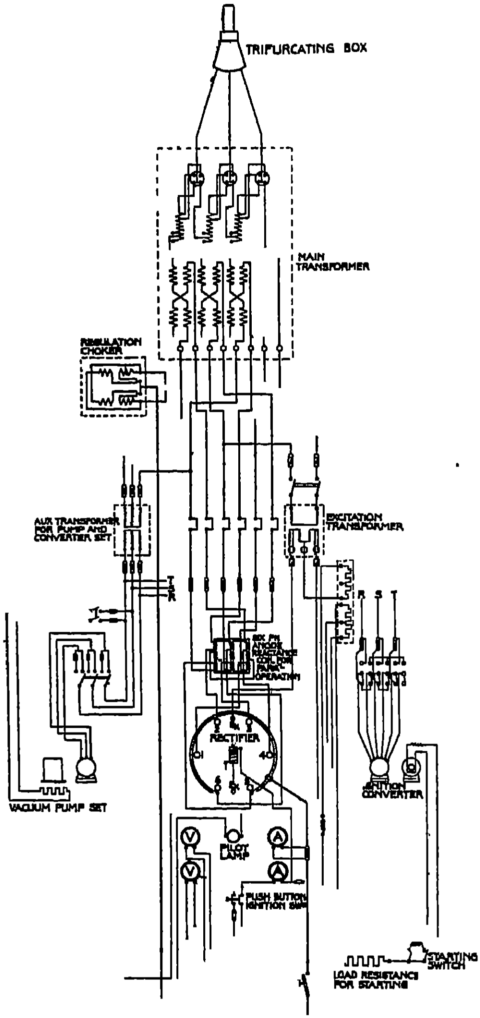

FIG. 188.—Diagram of connections for non-automatic working.

(12) a three-phase inverse time limit overload relay, the operation of which shuts down the set;

(13) a relay which is operated by the action of relay (12) and prevents the oil switch being closed until reset by hand.

For remote control the supply to relay (2) is provided for by means of one core of a pilot cable through a control switch, while the coil circuit of ignition relay (5) is looped through pilot wires to a switch at the control substation.

The lamps shown at the control substation clearly show

(i) when the oil switch closes or opens,
and (ii) when the ignition arc is struck.

The opening or closing of the D.C. circuit breaker is indicated by the low voltage lamp connected to two cores of a telephone cable.

The diagram of connections for a non-automatic substation is given in Fig. 188, and the method of operation in this case is as follows:—

The main threc-phase oil switch is closed, thereby energising the transformer, and the anodes on the rectifier circuit are thus made alive. The motor generator set is then started up and on pushing the button the solenoid of the ignition anode is energised, and the rectifier commences to function, although still on open-circuit on the load side. On account of the high open-circuit voltage it is necessary to switch in a small load to stabilise this voltage, but this resistance is switched out of circuit as soon as the main load comes on. The following protection is afforded:—

If the rectified voltage is not approximately equal to that on the D.C. bus bars (if such exists), the D.C. breaker will not close.

A failure of the E.H.T. supply causes a disconnection of the rectifier and all its auxiliaries.

When the E.H.T. supply is restored the relays operate in their proper sequence and reconnect the rectifier.

A short-circuit on the transformer will open the main oil switch.

A short-circuit in the rectifier due to loss of vacuum will also perform this function.

A heavy load or short-circuit on the D.C. load will open the

D.C. breaker, and the loading resistance will be switched in to maintain the correct voltage on the rectifier.

Operation and Attendance.—One of the advantages of the mercury vapour rectifier is the elimination of all synchronising devices, thereby greatly increasing the speed of operation. Under normal working conditions it is only necessary to close the main circuit breaker and press the push button. In the case of new plants the pump must also be operated at all loads, and for heavy loads also the pump should not be allowed to be disconnected from the container. In the case of new plants the sets can be started up in the space of five to ten minutes, whereas when a set has been running for a few months, a second or so only suffices to parallel the rectifier with the bus bars.

Occasional attendance is required at the station to lubricate the rotary plant, pump, etc., otherwise the rectifier itself will only require attention every year or so when the anodes have to be renewed.

CHAPTER XI.

MERCURY VAPOUR RECTIFIERS (*cont.*).

Glass Bulb Rectifiers.

General.—The glass bulb type was the pioneer of the mercury vapour rectifier, and was manufactured for many years in such sizes as would be capable of rectifying small powers only. Later developments enabled the difficulties attending the manufacture of a suitable container to be overcome, and so permitted larger units to be erected. With the advent of the metal case rectifier, however, which it was considered would have no rival in the glass bulb variety, improvements in design proceeded at such a rate that nowadays single units capable of rectifying 100 K.W. in a single glass bulb are available and are reliable in service.

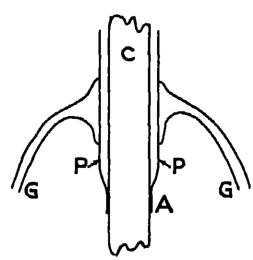

Fig. 189.—Type of seal for large currents.

Seal.—One obvious difficulty in the manufacture of such a unit is in the glass seal for the leading-in wires, which must be of a suitable diameter to carry the large currents required without cracking the seal on expansion. Many schemes have been put forward for improving the technique, and it would now appear that seals capable of passing currents of the order of 200 amperes are within the sphere of practical politics. One such seal is illustrated in Fig. 189 where C represents the copper rod, or leading-in wire, and P is a platinum tube slipped over the rod and sealed to the rod at A. The glass vessel G is then moulded on to the platinum sleeve with another joint at P. Glass and platinum having approximately the same coefficient

of expansion, the joint P is unaffected by the heat from the bulb or that conducted from the leading-in rods.

For some other types of seal, employed on thermionic rectifiers, but which are equally applicable to the mercury vapour rectifier, reference should be made to page 316. It certainly appears that modern methods set no limit to the current which can be led into an evacuated glass bulb.

Occluded Gas.—The same trouble is experienced with the glass bulb rectifier as with that in the metal case, so far as the obtaining of the vacuum is concerned, with the difference that the greater difficulty is met with in the former case during the processes of manufacture, as any subsequent loss of vacuum is out of the control of the user, any trouble at this stage resulting in a total failure of the bulb and necessitating its replacement. The gases which are occluded in the glass and in the metal anode and support wires must be eliminated if the bulb is to function satisfactorily, and careful evacuation to pressures of the order of 10^{-3} mm. is important. The best of bulbs cannot last for ever, and it is certain that final failure, excluding the possibility of mechanical breakdown, is due to a partial loss of vacuum. The cause of such a leakage is difficult to analyse, and may be due to one of three reasons, viz. (i) gradual evolution of gas occluded in the glass or metal parts, (ii) porosity of the glass, allowing air to percolate from the atmosphere into the bulb, and (iii) leakage past a faulty joint, the flaw being, possibly, of molecular dimensions, yet sufficient to allow air to leak slowly into the bulb. These defects are hard to trace, but will lead to certain failure in the end. Thus care in manufacture is essential to a long life.

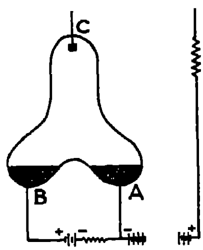

FIG. 190.—Diagram of simple starting device for glass bulb rectifier.

Starting Devices.—The method of striking the arc in a glass bulb rectifier is to bring the anode and cathode together, and after the arc has struck, separating them to the requisite distance.

Fig. 190 represents diagrammatically the practical adaptation of such a method, whereby the bottom of the glass bulb is provided with two reservoirs, each containing mercury. A cathode lead A and a starting anode lead B are brought out from the bulb, one from each of the two pools of mercury; the main anode C is led into the top of the tube. If the bulb is tilted until the mercury from B just makes contact with that in A, and is then adjusted into the vertical position, a break occurs in the liquid, a subsidiary arc is formed between A and B, and free electrons will be emitted from the cathode A. The electric field due to the difference of voltage between C and A will cause the electrons to be deflected to C and the vapour stream will be diverted to the main anode. The arc will maintain itself under these circumstances for one half cycle, or so long as the potential of C is positive to that of A. When C falls to zero potential, the arc will be extinguished unless some external precautions are taken to maintain it. The first condition then of continuous operation is, that the main mercury electrode must be a cathode; and for starting that this same electrode must be a cathode to the starting electrode B.

For small power outputs the most usual form of rectifier is the biphase variety which will operate from a single-phase circuit, but all forms of glass bulb rectifiers embody a tilting mechanism for starting purposes.

Automatic devices take the form of self-tilting mechanism, which is actuated as soon as the main arc ceases or when the main switch is thrown in on first starting up. The gear is simple in construction, and consists of a solenoid switch which operates the tilting of the bulb, and which is supplied from a small auxiliary transformer. On closing the main switch this solenoid is energised through a relay provided with a retaining contact and a differential coil, so that when a current flows in the bulb the solenoid retains the bulb in its normal position. As soon as the arc is extinguished the relay is again energised, provided that the supply is still connected; and the bulb is tilted once more. This cycle of operations usually persists until an arc is struck. Protective gear can be supplied with

this apparatus, and is so arranged that when the solenoid plunger is drawn down with too great a velocity (due to a heavy or short-circuit current) it trips the main circuit breakers and cuts out the rectifier completely. A further device is occasionally provided whereby, if the tilting mechanism is repeating the above cycle of operations indefinitely, owing to the arc not striking, the main supply is cut off.

Voltage.—In Chapter VIII. a simplified formula is given of the voltage relationships in a rectifier, viz. :—

Single-Phase—

$$\mathscr{E}' = 1{\cdot}11kE_M + e_a + c\mathscr{E}_s.$$

Multiphase—

$$\mathscr{E}' = \frac{E_M + e_a + c\mathscr{E}_s}{k\frac{\sqrt{2}m}{\pi}\sin\frac{\pi}{m}}$$

where $\mathscr{E}'$ is the voltage between each anode and the neutral point of the transformer, E_M is the rectified voltage, e_a is the arc drop, $\mathscr{E}_s$ is the drop across the inductance, m is the number of phases, and the constants $k = 0{\cdot}97$ and $c = 1{\cdot}2$ and are manufacturing constants.

Then for biphase circuits

$$\mathscr{E}_1' = 2\mathscr{E}_1$$

and for three-phase circuits

$$\mathscr{E}_1' = 1{\cdot}73\mathscr{E}_1, \text{ etc.}$$

With these corrections and assuming that the drop across the inductance is small and that the arc drop is 15 volts, these formulæ agree to a close approximation with those given by the manufacturers of the glass bulb rectifiers, viz. :—

Single-Phase—

$$\mathscr{E}_1' = (E_M + 15) \times 2{\cdot}35.$$

Three-Phase—

$$\mathscr{E}_1' = (E_M + 15) \times 1{\cdot}60,$$

where $\mathscr{E}_1'$ is the effective transformer voltage between each anode.

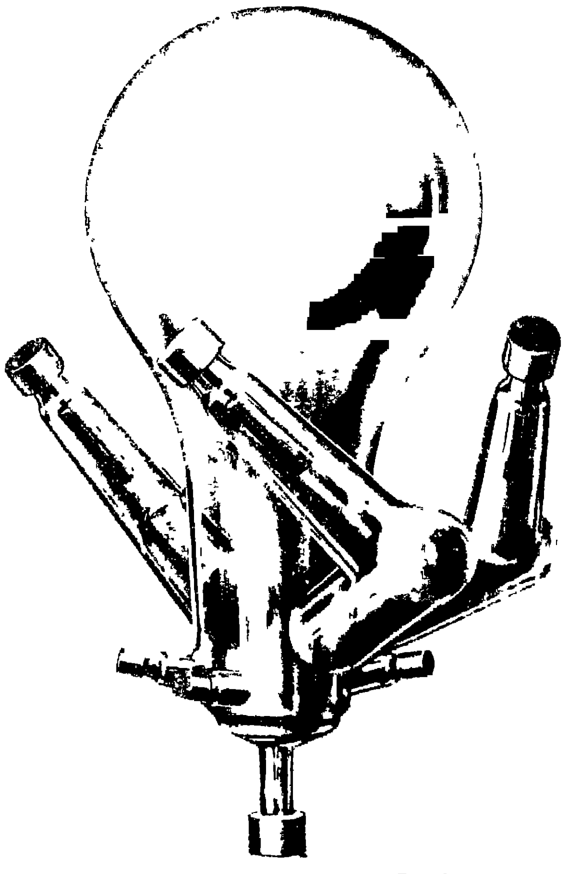

FIG. 191.—Three-phase 150 ampere bulb made by the Hewittic Electric Co.

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Type of Bulb.—Fig. 191 indicates the general shape of a bulb such as is supplied with the standard equipment for large power outputs. These bulbs are now manufactured in sizes which are capable of passing currents ranging from 60 to 200 amperes, and a hexaphase arrangement can thus rectify about 100 K.W. per unit. The bulbs can be designed to operate on a biphase, three- or six-phase system in one glass container, although it is preferable to limit the number of phases to three and employ two separate units of three phases each if a six-phase circuit is required on account of the smoother wave form obtained.

As regards the life of these bulbs, there appears to be little data available as to any definite length of life. They appear, in isolated cases to last a considerable time—often many thousands of hours, and it is possible that if statistics were available it might be found that the average life is limited rather by mechanical failure due to the constant jarring of the starting gear than by electrical or physical breakdown.

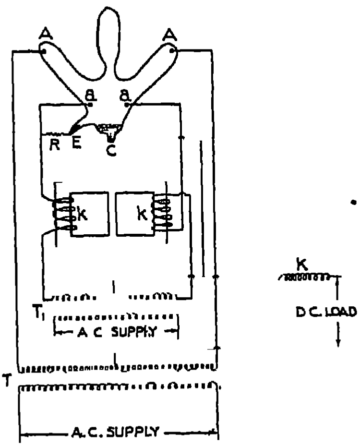

FIG. 192.—Diagram of exciting anodes.

Although the seal is satisfactory for the rated current of the rectifier it is dangerous to pass any overload for long periods. Thus the glass bulb rectifier is at a disadvantage in this respect as compared with its rival. Overloading should therefore be avoided, as if the maximum rating is exceeded the anodes may become excessively hot and either heat conduction may crack the seal, or alternatively the rectifier may conduct in both directions and heavy currents will pass.

Intermittent Loading.—When the load is intermittent the arc tends to become extinguished if the current falls to too low a value, in which case any increase in the load will

bring the tilting mechanism into operation and re-start the arc. This will result in a constant jarring of the bulb and also in sluggish picking up of the load. One device which provides for the separate excitation of the rectifier is illustrated in Fig. 192, where E is an auxiliary anode used in conjunction with the tilting device, and R a resistance inserted to limit the value of the starting current. For traction loads or on similar duty where intermittent loading is likely, the exciting anodes *aa* are provided (AA being the main anodes), and are fed from an auxiliary transformer T which can be disconnected when required, and which will only absorb about 100 watts.

Voltage Regulation.—The normal regulation of a glass bulb rectifier between no-load and full load is approximately plus or minus 10 per cent., but this does not represent the best that can be obtained, although inherently the bulb cannot be expected to give a much better performance. Over-all regulation can be reduced to plus or minus 1 or 2 per cent. by means of automatic gear (or manually operated if preferred) which by means of electrically operated remote control will vary the applied voltage by steps on the auto-transformer.

Parallel Operation.—Bulbs can be operated in parallel if so desired and if supplied from the same source. They will take a reasonably equal share of the load, and the only operation in synchronising is to throw in the main starting switch; but if parallel operation is required it should be definitely specified, as it is advisable that the bulbs should be paired to ensure that their characteristics are similar.

Equipment.—The Hewittic Electric Co. have developed the use of glass bulb rectifiers to such an extent that the arguments against the use of a fragile piece of apparatus in power engineering no longer apply: indeed, advantages accrue from this particular construction. The bulbs with all their automatic gear can be included in one cubicle and the appearance of a substation employing these rectifiers is that of switchgear alone.

Fig. 193 illustrates two of these cubicles in course of erection. Each one consists of a unit for an output of 150 amperes at 550 volts, three-phase, the two units together forming a hexaphase arrangement. The photograph is taken from the back of the

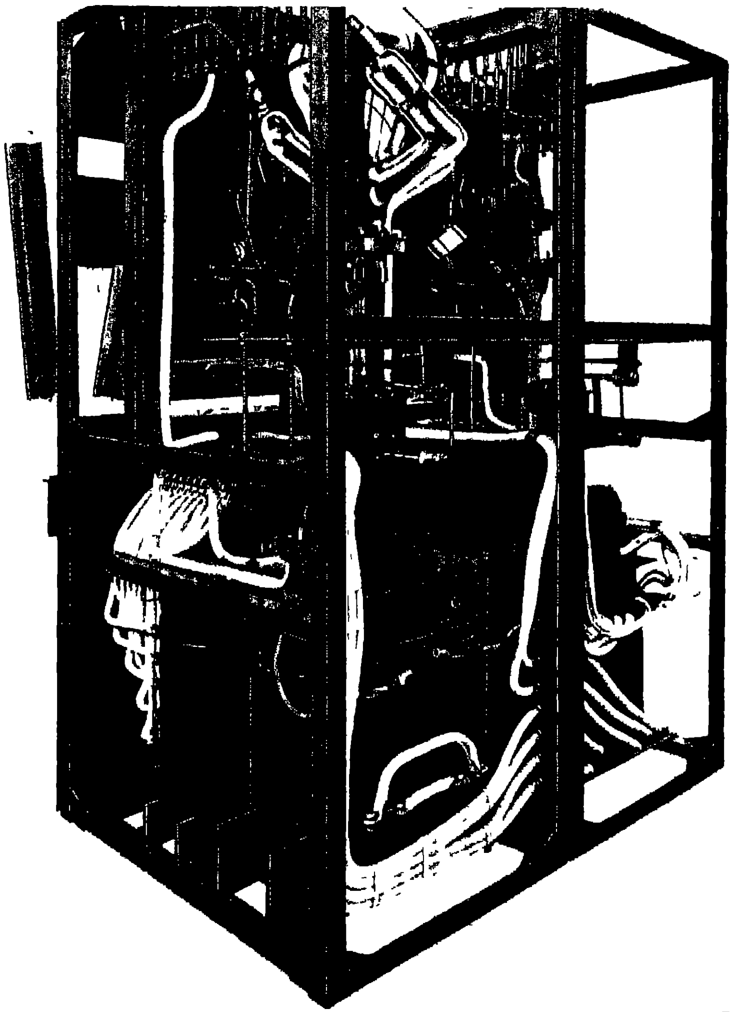

FIG. 193.—Rectifier cubicle, 150 amperes, 550 volts.

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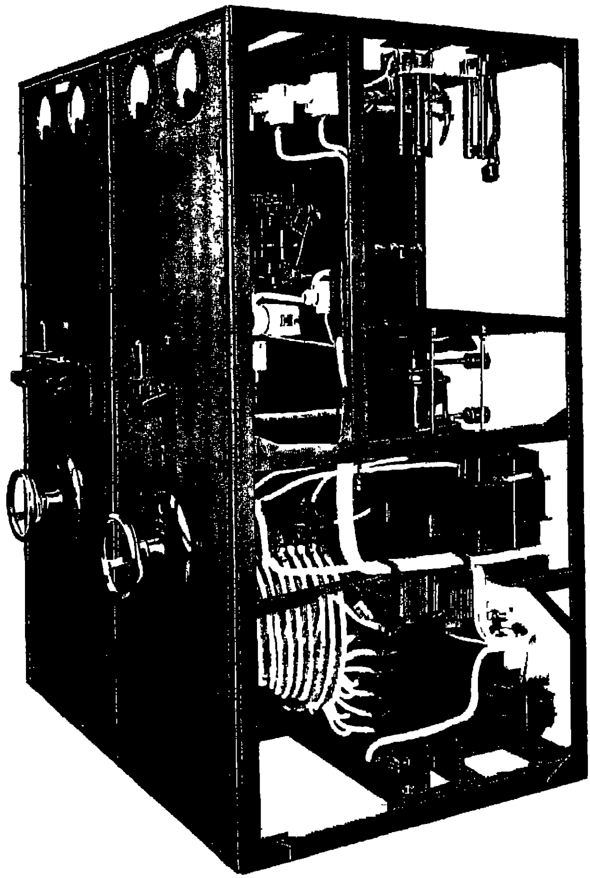

FIG. 194.—Rectifier cubicle, 100 amperes, 230 volts.

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cubicle which has its side and end plates removed, and shows the bulb in position.

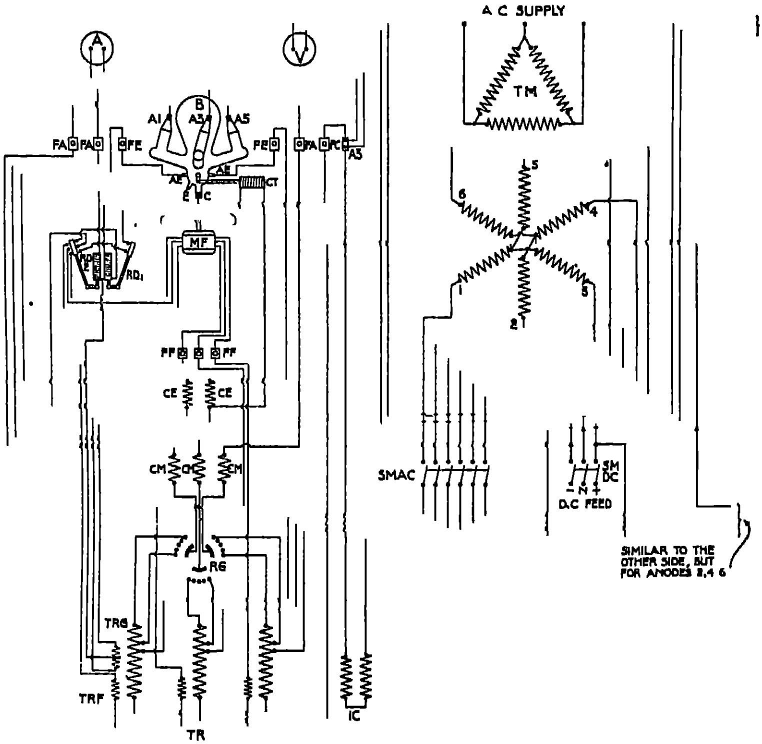

FIG. 196.—Diagram of connections of glass bulb rectifier.

A—Main anode electrode.
AE—Exciter anode electrode.
AM—A meter.
AS—A meter shunt.
B—Rectifier bulb.
C—Main cathode electrode.
CE—Exciter anode choke coil.
CM—Main anode choke coil.
CT—Tilting coil.
E—Auxiliary electrode.
FA—Main anode fuse.
FC—Main cathode fuse.
FE—Exciter anode fuse.
FF—Fan motor fuse.
IC—Cathode induction.
MF—Fan motor.
RD 1 and 2—Starting relays.
RG—Voltage regulator.
SMAC—Main switch AC.
SMDC—Main switch DC.
TM—Main transformer.
TR—Regulating transformer.
V—Voltmeter.

Fig. 194 is a similar photograph taken from the front, and is for a unit supplying 100 amperes at 230 volts.

This illustration shows that the switchgear on the panel is simple, consisting only of a main switch and a voltage regulator.

When provided with an automatic voltage regulator there is not much more complication. Fig. 195 is a photograph of a similar panel to that of Fig. 194 but provided with a motor-operated switch for the transformer tappings.

Diagram of Connections.—For the benefit of those who are interested in the actual operation of the automatic gear, a diagram of connections of the complete apparatus is given in Fig. 196, and the sequence of events is as follows :—

On closing the main switch SM . AC the transformer TR is energised and supplies current via its subsidiary secondaries as follows: TRF, via the fuses FF, starts up the fan motor for the cooling of the bulb. TRG supplies current via the tilting solenoid CT through the contacts of RD1 and RD2 back to TRG, the contacts of the relay RD being made in the upper position when no current flows in the solenoid coil. The tilting mechanism operates. The exciter terminal E is thus connected to the transformer TRG and the subsidiary arc is struck. Immediately this occurs the arc is diverted to the main anodes which are alive at the instant of closing the main switch.

When current flows through the exciter terminals relay RD is energised and RD1 breaks the tilting coil solenoid; at the same time RD2 disconnects RD1, which returns to its original position so as to be available for a through passage for the starting current in case of a failure to start.

The exciter anodes AE with their choke coils are provided in case of a reduction of the load or a stoppage of the current.

The above does not include a description of the automatic voltage regulator but, as will be seen from Figs. 195 and 196, this operates on tappings and does not present any unusual features.

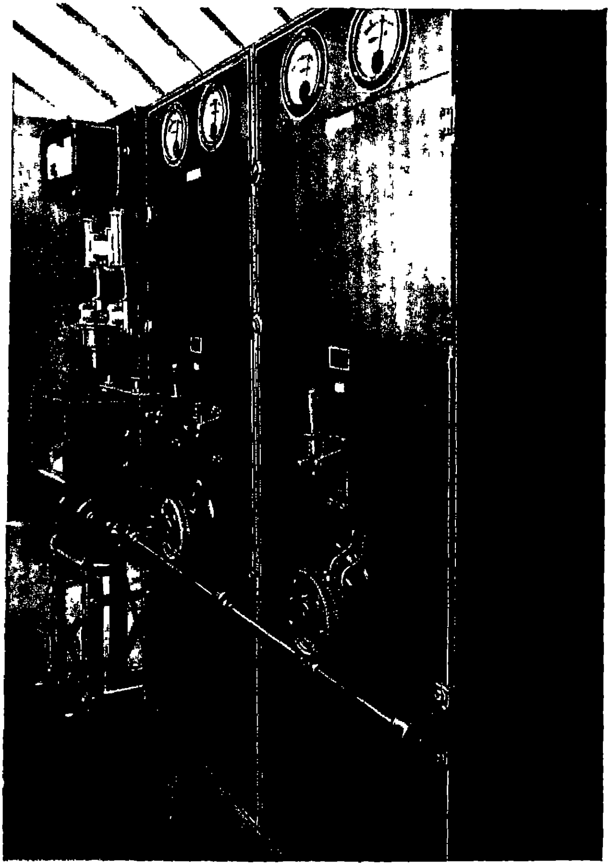

FIG. 195.—Rectifier cubicle with automatic voltage regulator.

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CHAPTER XII.

(VACUUM TUBES) THERMIONIC RECTIFIERS.

General.—The thermionic rectifier first used by Fleming for the rectification of small currents in a wireless receiving aerial, in addition to its evolution into the three electrode valve, has been developed into an effective device for rectifying alternating currents at voltages up to 120,000 volts for currents of a few milliamperes, and 20,000 volts for power outputs of several

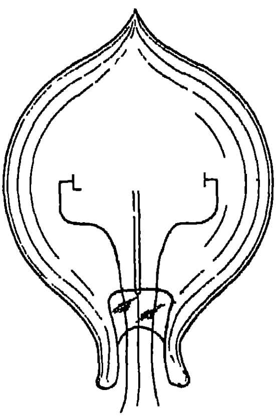

FIG. 197.—Simple thermionic rectifier (Fleming type).

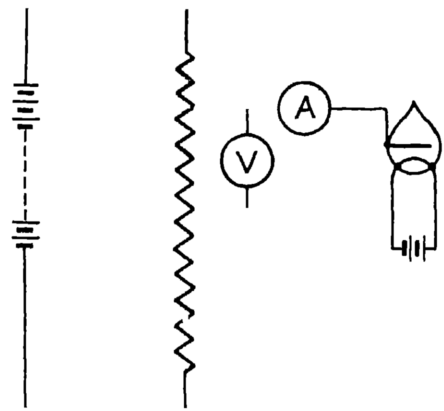

FIG. 198.—Test apparatus for thermionic rectifiers.

hundred kilowatts; it is with these aspects of the thermionic rectifier that this chapter is concerned.

The thermionic rectifier in its simplest form consists of a filament of tungsten wire that can be heated to incandescence by the passage of a current. The filament is surrounded by a metal anode, and enclosed in a glass bulb that has been evacuated to the highest attainable degree. Leads from the ends of the

filament and the anode are brought through a glass pinch; Fig. 197 illustrates a simple thermionic rectifier of this type which may be considered as a modern edition of the original Fleming valve.

In order to obtain experimental data an arrangement such as is shown in Fig. 198 can be employed; and curves can then be plotted for varying filament currents and anode to filament voltages.

A set of such curves for a particular rectifier is given in Fig. 199 where the lowest curve is for a filament current of 0·62 ampere, the next for 0·64 ampere, while the upper curve is for

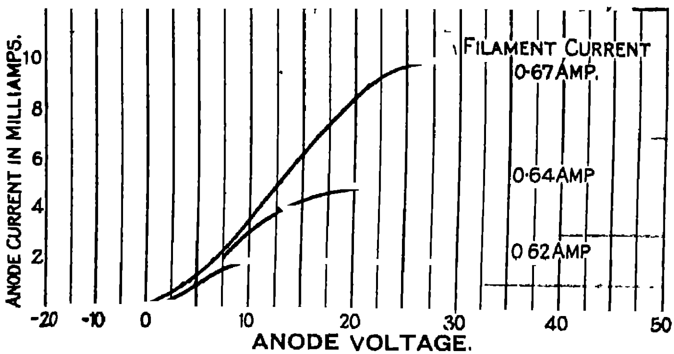

Fig. 199.—Characteristic curves of a thermionic rectifier.

0·67 ampere; and illustrates the general properties of the thermionic rectifier treated below.

Properties.—The first general property of the thermionic rectifier is that the anode current only flows when the anode is at a positive potential with regard to the filament, the reason being, as is explained more fully below, that the anode current is carried entirely by electrons, which are emitted from the filament. The anode being cold does not emit electrons, while owing to the high degree of vacuum attained there is no appreciable quantity of positive ions due to ionisation. The high vacuum rectifier thus differs from the gas discharge tube or the electrolytic rectifier in giving practically perfect rectification (page 6).

Secondly it is seen that if the filament current is maintained constant, and the anode voltage increased positively from zero, that the anode current increases for a time and finally assumes a steady value after which an increase of anode voltage has no effect.

This final anode current, the value of which, is usually termed the "saturated emission," or simply the "emission," is determined by, and is a measure of, the number of electrons emitted from the filament in unit time. The characteristic curves further show that this saturated emission increases when the filament current is increased. Thus for the rectifier considered, Table XXIII. can be prepared.

TABLE XXIII.

Filament Current Amperes.	Emission Milliamperes.
0·62	2·0
0·64	5·0
0·67	10·0

It will be seen that with a filament current of 0·67 ampere, the saturated emission is 10 milliamperes; yet it is necessary to apply a considerable positive voltage, in this case about 30 volts, to obtain approximately the full emission. The explanation of this is found in the mutual repulsion of the electrons—those that have left the surface of the filament tending to force back those that were on the point of leaving.

Thermionic Emission.—It is now well known and firmly established on a sound experimental and theoretical basis that heated metals emit electrons, and that the number of electrons emitted per unit time, at a definite temperature, is a definite physical property of the metal. A full account of the phenomenon will be found in Richardson's work on the subject.

From theoretical considerations it can be shown that the emission current per unit surface of a metal is given by the equation

$$i = a\sqrt{T}\epsilon^{-\frac{b}{T}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where a and b are constants of the metal, and T is the absolute temperature (degrees Kelvin).

Approximate values of a and b for tungsten, which is the only metal that will be considered, are

$$a = 2{\cdot}6 \times 10^7$$
$$b = 5{\cdot}25 \times 10^4.$$

For tungsten the emission in amperes per square centimetre for a range of temperatures is given in Table XXIV., in which also the watts radiated as thermal energy are given.

TABLE XXIV.

Degrees Kelvin (1).	Filament Watts per Sq. Cm. (2).	Emission Milliamperes per Sq. Cm. (3).
1000	0·9	$1{\cdot}2 \times 10^{-11}$
1500	6·9	6×10^{-4}
1800	16·4	8×10^{-1}
2000	26·9	4·2
2100	34·0	15·1
2200	43·0	48·8
2300	53·0	137·7
2400	65·0	364·8
2500	77·5	891·0
2600	90·0	2044·0

As would be expected from the experimental form of the equation, the emission current increases rapidly with temperature, and it might be expected that since the emission increases at a far greater rate than the energy emission, the highest possible temperatures (such for instance as 2800 degrees C.) would be employed in practice. The rate of evaporation of tungsten, however, also increases rapidly with temperature; and a cathode at 2800 degrees readily volatilises. It is therefore necessary to effect a compromise in the following way: the temperature of a tungsten cathode is such as to produce a reasonable emission, but yet not sufficiently high as to result in too short a life due to volatilisation of the filament. Thus for any particular purpose there is an optimum temperature, but its actual value depends on many circumstances; for cathodes consisting of tungsten

filaments of about one millimetre in diameter, an approximate figure for the emission is about 0·5 ampere per square centimetre, and about 5 amperes emission per kilowatt.

It will be noted that only tungsten has been considered, this is because, although several other cathode materials are employed, as for instance, coatings of the oxides of barium and strontium, and also thoriated tungsten, their use appears to be restricted at present to comparatively low voltages, and they do not appear to have been satisfactorily developed for use on high voltage circuits of the order of thousands of volts.

The nearly perfect unilateral conductivity of the valve *in vacuo* is explained, but if there is any appreciable quantity of gas present, there may be gas discharges which will mar the unidirectional nature of the current, and may be so violent as to destroy the electrode. The removal of the gas from a valve is thus the most important process in the manufacture.

The Space Charge.—It has been seen on page 293, that a considerable potential difference between the anode and the cathode may be necessary to produce the full emission current to the anode, due to the fact that the electrons in transit, being negatively charged bodies, produce negative volume electrification tending to force back electrons emitted from the filament. The theory, which is of practical importance, has been fully worked out by Langmuir.

Consider the case that admits of the simplest treatment, which was originally considered by Child and Sir J. J. Thomson, namely, that of a plane cathode in front of which is situated a parallel plane anode. Assume an axis of co-ordinates perpendicular to the plane, and the origin at the cathode; and let V be the voltage at any point. Further suppose that the electrons have negligible initial velocities; then their velocity v will be given by the equation

$$\tfrac{1}{2}mv^2 = Ve$$

where m is the mass, and e is the charge of the electron.

Assume that n electrons are passing from unit area of the cathode per unit time, so that ne is the anode current per unit

area, then the electron density at any point will be at the rate of

$$n/v$$

and the volume of electrification at any point will be

$$-\frac{ne}{v} = -\frac{i}{v}.$$

The value of V at any point must obey Poisson's Law, viz.,

$$\frac{\partial^2 V}{\partial x^2} = -4\pi\rho$$

where ρ is the volume electrification, which has already been shown to be

$$-\frac{i}{v}.$$

Thus

$$\frac{\partial^2 V}{\partial x^2} = \frac{4\pi i}{v}$$

$$= 2\pi i\sqrt{\frac{2m}{eV}}.$$

This equation can be readily integrated. The boundary condition at the surface of the cathode is seen to be

$$\frac{dV}{dx} = 0$$

for the case under consideration when the current is determined solely by the space charge, for it is apparent that if $\frac{dV}{dx}$ were positive at the surface of the cathode, then more electrons would leave the surface, and thus the current i would increase, and similarly if $\frac{dV}{dx}$ were negative i would decrease. Applying this boundary condition

$$\left(\frac{dV}{dx}\right)^2 = 8\pi i\sqrt{\frac{2mV}{e}}$$

and
$$i = \frac{\sqrt{2}}{9\pi}\sqrt{\frac{e}{m}}\,\frac{V^{\frac{3}{2}}}{x^2}$$

which is of the form

$$i = K\frac{V^{\frac{3}{2}}}{x^2}$$

where K is a universal constant. Employing, as units, amperes, volts and centimetres, this equation becomes

$i = 2{\cdot}33 \times 10^{-6}\dfrac{V^{\frac{3}{2}}}{x^2}$ amperes per square centimetre of cathode area.

It has been assumed in the foregoing that the initial velocities of the electrons are negligible: this is not strictly true, although they may be neglected for voltages of 50 or more, yet they become important at the lower voltages.

A fuller theory of the space charge, in which account has been taken of the initial velocities has been given by T. C. Fry.

A more important case than that above, is that of the cylindrical rectifier, which has also been treated analytically by Langmuir. If the cathode diameter is d_0 and it surrounded by a concentric cylindrical anode of diameter d, then the anode current per centimetre of length can be shown to be given by the equation

$$i = 2{\cdot}92 \times 10^{-5}\frac{V^{\frac{3}{2}}}{d\beta^2},$$

the units again being in amperes, volts and centimetres. β^2 is a complicated function of d/d_0, which has been tabulated by Langmuir and Blodgett, and whose values are plotted in Fig. 200.

It will be seen that for most cases of filament rectifiers d/d_0 is large and β^2 approximates to unity, for all practical calculations, but for a case considered later (page 322) where d/d_0 is itself nearly unity, then β^2 is very small, and plays an important part.

It is thus seen that in both the plane and cylindrical type of rectifier, for any voltage applied to the anode, there is a certain limiting value of the anode current which cannot be exceeded, however great the electron emission from the filament. This limitation is termed the Space Charge Limitation, and the current limit is the Space Charge Limited Anode Current at that particular voltage.

The 3/2s law explains the form of the lower portion of the characteristic shown in Fig. 199, where the anode current is less than the emission; as soon as the anode voltage has reached a value where the space charge limited current is equal to the emission, then by further increasing the anode voltage the anode volts cannot increase the anode current, and the curve becomes flattened.

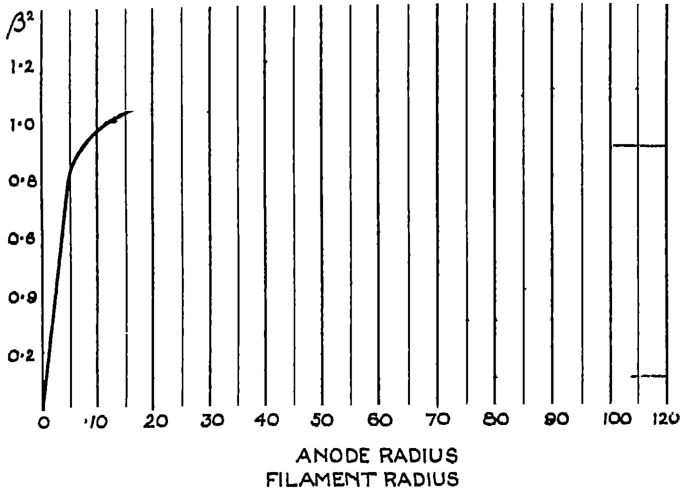

FIG. 200.—Values of β^2.

Theoretical Analysis.—The theoretical analysis of a thermionic valve circuit is not as simple a matter as that of the mercury vapour rectifier, but it is possible to investigate the various current and voltage relationships if certain assumptions are made.

Take the case of a rectifier supplying a single-phase circuit as shown in Fig. 201.

If e_2 is the voltage across the load and e_1 is the drop in the rectifier and the supply voltage is $E \sin \theta$, then

$$E \sin \theta - e_1 = e_2 \quad . \quad . \quad . \quad . \quad (3)$$

Also e_1 is given by

$$i = Ke_1^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $$K = 14{\cdot}65 \times 10^{-6} \cdot \frac{l}{r}.$$

Further, the load condition is

$$e_2 = iR \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and hence

$$i^2 = K^2(E \sin\theta - Ri)^3.$$

Thus the main rectified current will consist of a sinusoidal wave of the single-phase type and the voltage will be of closely similar shape and in phase with it.

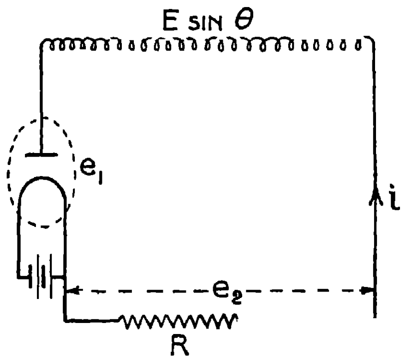

FIG. 201.—Simple valve rectifier circuit.

To calculate the actual losses in the bulb it is necessary to refer to the three equations (3), (4), and (5). By substitution a cubic equation connecting e_1 and θ is deduced of the form

$$e_1^3 - \frac{e_1^2}{R^2K^2} + \frac{2Ee_1 \sin\theta}{R^2K^2} - \frac{E^2 \sin^2\theta}{R^2K^2} = 0$$

from which the drop can be calculated for various angles θ.

Efficiency of Rectifier.—To take a practical example consider a high voltage rectifier which has a loop filament which must be reduced to its equivalent straight filament to conform to the calculations. To accomplish this, assume the filament length is halved, the watts consumption and superficial area of the filament remaining constant, which also determines that

the temperature of the filament remains the same. These conditions provide for the following alterations in dimensions :—

Corrected area	=	original area multiplied by 4
,, diameter	=	,, diameter ,, ,, 2
,, current	=	,, current ,, ,, $2\sqrt{2}$
,, voltage	=	,, voltage divided by $2\sqrt{2}$

and with the constants for the tube given the figures in Table XXV. are arrived at :—

TABLE XXV.

	Actual dimensions with loop filament.	Corrected dimensions for straight filament.
Anode radius	27 mm.	27 mm.
Anode length	70 mm.	70 mm.
Filament voltage	12·5	4·43
Filament current	24	67·5
Filament watts	300	300
Filament diameter	0·63 mm.	1·26 mm.
Filament sectional area	0·31 sq. mm.	1·24 sq. mm.
Filament effective length	150 mm.	75 mm.
Filament superficial area	3 0 sq. cms.	3·0 sq. cms.

From this table

$$K = 14{\cdot}65 \times 10^{-6} \times 70/27 = 0{\cdot}38 \times 10^{-4}$$

and assuming the load $R = 5 \times 10^6$ ohms

and $E = 20{,}000$ volts

the cubic equation reduces to

$$e_1^3 - 0{\cdot}3 \times 10^{-4} e_1^2 + 1{\cdot}1 e_1 \sin\theta - 11{,}000 \sin^2\theta = 0$$

and the solution of this equation may be taken to be of the form

$$e_1^3 - 11{,}000 \sin^2\theta = 0,$$

for the maximum value of e_1, as the error in so doing is not more than 2 per cent.

The current i will have a sinusoidal form to a very near approximation, viz.,

$$i = \frac{20{,}000}{5 \times 10^6} \sin\theta$$

$$= 0{\cdot}004 \sin\theta \text{ amperes.}$$

The power absorbed in the tube due to the anode drop is therefore approximately

$$\frac{1}{\pi}\int_0^{\pi} \sqrt[3]{1{\cdot}1 \times 10^4 \sin^2\theta}(4 \sin \theta) \, . \, d\theta$$
$$= 37 \text{ milliwatts}$$

and the filament consumption is 300 watts.

The efficiency is calculated as follows:—

neglecting the loss due to the anode drop, the input is

$$(20{,}000 \sin \theta)_{\text{mean}} \times (0{\cdot}004 \sin \theta)_{\text{mean}} + 300$$
$$= 8{\cdot}3 + 300 = 308{\cdot}3 \text{ watts}$$

and the output is 8·3 watts. The efficiency is therefore 2·7 per cent. and is very low on account of the high relative value of the filament watts. This is not a representative case, but has been given to show the effect of working with a valve not suited to the requirements.

The efficiency can be improved by using a valve with a less consumption in the filament, but it may also be increased by taking a larger current from the rectifier. In the second example assume that the load R is 100,000 ohms. The equation for e_1 then becomes

$$e_1^3 - 0{\cdot}07\, e_1^2 + 2900\, e_1 \sin \theta - 2{\cdot}8 \times 10^7 \sin^2 \theta = 0,$$

and again the middle terms may be neglected whence

$$e_1 = \sqrt[3]{2{\cdot}8 \times 10^7 \sin^2\theta} = 310(\sin \theta)^{\frac{2}{3}}.$$

This curve is shown in Fig. 202: the mean value of e_1 in this case being 98 volts over one complete cycle.

The current curve is now seen to be

$$i = 0{\cdot}2 \sin \theta$$

whose mean value is 0·064 ampere. The loss due to the anode drop is therefore approximately

$$0{\cdot}064 \times 98 = 6 \text{ watts}.$$

The input in this case is

$$0{\cdot}064 \times 6400 + 300 + 6 = 713,$$

and as the output is 410 watts the efficiency is 57 per cent.

It will be seen, therefore, that the efficiency of such a system increases rapidly with increasing load but can only reach high values when the input is large compared with the filament watts.

Referring to Table XXV. (page 300) the watts input to the filament are 300 and the watts per unit area of the filament are therefore 100. This corresponds to a filament temperature of approximately 2700° K. and an emission of 1·52 amperes per unit filament area, or a total of 4·56 amperes. This is obviously too high a figure, and there is therefore some inaccurate assumption in the use of the table. The error is due to the neglect of the cooling effect of the leading-in wires and supports on the ends of the filament; and the connection

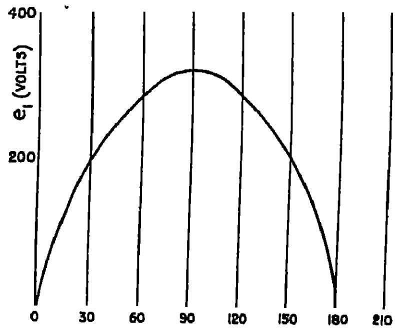

FIG. 202.—Volt drop in rectifier.

between Columns (1) and (2) of the table must be used with caution, as the whole of the watts supplied to the filament are not used to raise its temperature equally along its length. It is important, however, to be able to obtain a relation between the diameter of a filament, the current passing through it, and its temperature. The equations connecting these quantities have been carefully investigated by Langmuir and the curve below is obtained from the resulting data. The current is given in Fig. 203 for a cylinder of tungsten one centimetre in length and one centimetre in diameter; and as there is a fundamental relation between the current and the diameter for any one temperature of the form

$$I \text{ varies as } d^{\frac{3}{2}}$$

the equivalent current in a cylinder 0·63 mm. in diameter can be ascertained as follows:—

$$I = 24\left(\frac{10}{0\cdot63}\right)^{\frac{3}{2}} = 1510 \text{ amperes.}$$

From the curve in Fig. 203 this corresponds to a temperature of 2470° K.; the difference between this result and the one obtained above of 2700° K. represents the cooling effect of the electrodes, etc.

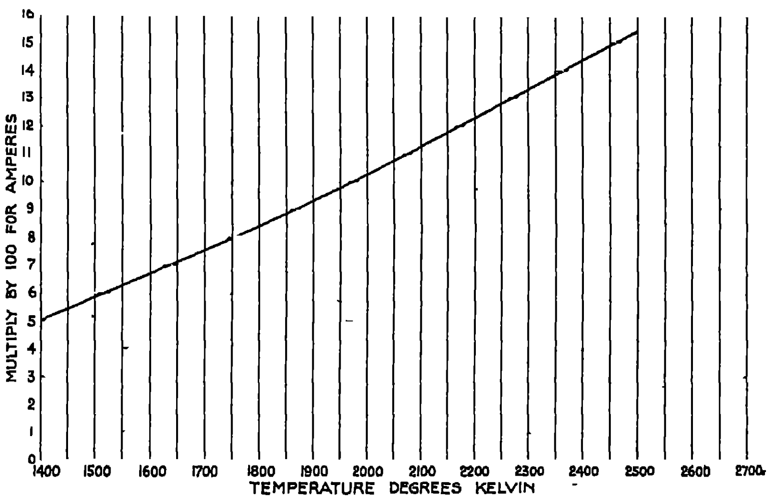

FIG. 208.—Ampere-temperature characteristics for tungsten wire one centimetre long and one centimetre in diameter.

From Table XXIV. this temperature corresponds to an emission of 720 milliamperes per square centimetre or a total emission of 2·160 amperes. This agrees with what would be expected for the saturation current for this particular rectifier, and shows that the valve was working at about $\frac{1}{10}$ saturation point in the second of the two examples.

Assuming that it is possible to run this rectifier up to its saturation current then the current curve would be represented by

$$i = 2\cdot16 \sin \theta.$$

The maximum voltage drop across the bulb, assuming that Child's equation holds at the saturation point, is given by

$$i = 0{\cdot}38 \times 10^{-4}\, e_1^{\frac{3}{2}}$$

or

$$e_1 = 850 \text{ volts maximum value.}$$

The watts absorbed by the rectifier are

$$\frac{1}{\pi}\int_0^{\pi}(850 \sin\theta)(2{\cdot}16 \sin\theta)d\theta \text{ approximately}$$

$$= 920 \text{ watts,}$$

and the input is therefore

$$6400 \times 0{\cdot}69 + 300 = 4700 \text{ watts}$$

The output is

$$6400 \times 0{\cdot}69 - 920 = 3530 \text{ watts,}$$

which represents an overall efficiency of 75 per cent., and is the ideal maximum figure obtainable with this particular rectifier.

The above examples have been worked out without due regard to the wave form; and to be more correct the calculations should have been made from the equations on page 299. It is rarely necessary, however, at the present time to go to this trouble, and these examples have been given to illustrate the use of the approximate solution.

Effect of Reactance in Circuit.—So far only a simple circuit of a rectifier supplying current directly to a load has been considered; and the resulting current wave will consist of a number of loops (either single or biphase) in which the current drops to a zero value every half cycle. It is possible, however, to apply the device employed in the case of mercury vapour and mechanical rectifiers of inserting a condensive or inductive reactance in the rectified circuit to smooth out the large current variations. This impedance may consist of either a condenser or an inductance or both, with the difference in this case that, as high voltages are usually employed, an inductance would necessarily be of large dimensions; and it is therefore common practice to employ a capacity in shunt across the rectifier terminals, as shown in Fig. 204.

With the same notation as before six equations may be evolved connecting the various quantities during the period that the current is flowing through the valve, viz.,

$$
\begin{aligned}
E \sin \theta &= e_1 + e_3 \\
e_2 &= e_3 \\
e_3 &= \frac{1}{C}\int i_3 d\theta \\
e_2 &= Ri_2 \\
i_1 &= Ke_1^{\frac{3}{2}} \\
i_1 &= i_2 + i_3.
\end{aligned}
$$

These equations result in a differential equation which is difficult, if not impossible, of solution, and, moreover, which

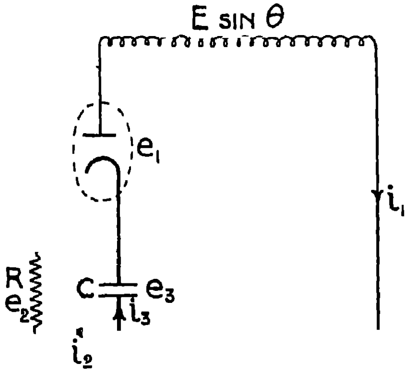

FIG. 204.—Circuit with capacity for smoothing current wave.

cannot be simplified without making erroneous assumptions Some other method has to be evolved for solving the problem, and an ingenious approximation is due to Prof. Fortescue.

Fig. 205 indicates the general shape of the various current and voltage curves where e_3 and e_3' represent the voltage of the condenser during charge and discharge respectively.

The assumption made in the use of this method is that the valve current i_1 rises to a saturation value and remains there for a considerable portion of the cycle. This is not strictly true because in most instances saturation current is not even

approached before the valve is overloaded; but it may be taken to represent a mean value of the anode current within a margin of accuracy.

The first point to notice is that no current will be passed by the valve until the supply voltage is equal to the counter E.M.F. of the condenser, or in other words, if θ_a is the angle at which current begins to flow

$$\theta_a = \sin^{-1}\frac{e_3}{E}$$

where E is the voltage of supply (maximum value).

Two further assumptions are necessary, viz. (i) that the time taken for i_1 to reach its maximum value is negligible, and (ii) that this current is zero at a time equivalent to θ_a electrical degrees before the end of the half cycle; this is the

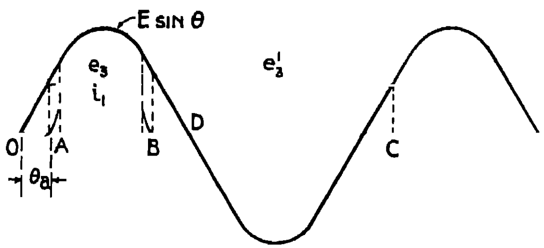

FIG. 205.—Supply and capacity voltages in a thermionic rectifier.

same as saying that $AO = BD$. This latter assumption entails a constant value for e_3 which in practice must vary in amount depending on the circuit constants. With these assumptions it may be said that the anode current is flowing for a time equivalent to

$$\frac{\pi - 2\theta_a}{2\pi} \cdot T$$

seconds per cycle, where T is the time of one complete cycle. Hence the quantity of electricity stored in the condenser during the period AB is

$$(i_1 - i_2)\frac{\pi - 2\theta_a}{2\pi} \cdot T \text{ coulombs}$$

where i_1 and i_2 are the valve and load currents respectively.

But the quantity of electricity discharged by the condenser,

during the period BC must be equal to this amount. Hence if it is assumed that $i_2' = i_3' = i_3$, where dashes represent the discharge period, and that these are now the mean instead of the instantaneous values of the currents, the condenser is discharging for a period represented by

$$\frac{T}{2\pi}(\pi + 2\theta_a)$$

and therefore

$$\frac{i_1 - i_2}{i_2} = \frac{\pi + 2\theta_a}{\pi - 2\theta_a}$$

whence

$$\frac{i_1}{i_2} = \frac{2\pi}{\pi - 2\theta_a} \qquad . \quad . \quad . \quad . \quad (6)$$

This with the equation

$$\theta_a = \sin^{-1}\frac{e_3}{E} = \sin^{-1}\frac{e_2}{E} \qquad . \quad . \quad . \quad (7)$$

will enable the various currents required to be calculated from the given data.

Suppose that the variation of the rectified voltage permissible is be_3 then the change of charge on the condenser is, by substituting for i_1

$$\frac{\pi + 2\theta_a}{2\pi} . Ti_2$$

and the capacity required to produce this variation is

$$C = \frac{i_2}{e_2} . \frac{\pi + 2\theta_a}{2\pi} . \frac{1}{bf} \qquad . \quad . \quad . \quad (8)$$

where f is the frequency of supply.

Finally there is the load equation

$$Ri_2 = e_2 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In the four equations (6), (7), (8), and (9) there are nine quantities, of which five must be chosen before the equations can be solved; b, f, E and R are supply and circuit constants, i_2 is the output current and is fixed by the conditions of load, and the four remaining quantities i_1, e_2, θ_a, and C can be ascertained.

It is instructive to apply this method to the example above

where the load consists of a non-inductive resistance of 10^5 ohms, and $i_2 = 0{\cdot}064$ ampere, $b = 0{\cdot}1$, $f = 50$ cycles and $E = 20{,}000$ volts. From the equation it is found that $\theta_a = 0{\cdot}331$ radian, $C = 1{\cdot}21$ microfarads and $i_1 = 0{\cdot}161$ ampere.

Thus the overall efficiency of rectification in this case amounts to 40 per cent., whereas seeing that the loss in the condenser is negligible, there is a considerable divergence in the results. In the general use to which these rectifiers are put it is rarely possible to work to any great accuracy, as would be the case with large power rectifiers, and it is a simple matter in practice to make any corrections that are necessary when the calculations are completed, or to correct for these assumptions by the insertion of a resistance in the load circuit. Furthermore, although in the above example a 10 per cent. variation in the current and voltage is specified, yet this figure can be caused to vary within wide limits by varying the capacity.

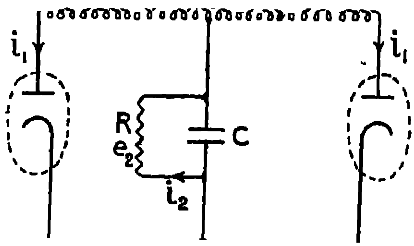

FIG. 206.—Smoothing condenser in a biphase system.

If a biphase arrangement is used as shown in Fig. 206, the corresponding formula for the capacity is

$$C = \frac{i_2}{e_2} \cdot \frac{\theta_a}{\pi} \cdot \frac{1}{bf},$$

which, by filling in the values above, is seen to be $0{\cdot}84$ microfarad, showing a large saving in the size of the capacity required for the same voltage fluctuation. It should be remembered here that whilst the voltage is halved the current is increased in the ratio of 1 to $\sqrt{2}$.

The application of this particular method has been described

by Prof. Fortescue in a paper before the Physical Society where a number of corrections are applied and curves are plotted for differing circuit constants. The chief point to recollect is that a valve rarely works at a saturation value, as the temperature would in most cases be so high as to cause destruction of the tube.

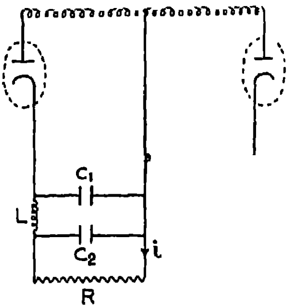

FIG. 207.—Wave filter for smoothing current wave.

Wave Filter.—If the use of a single condenser is insufficient it is possible to employ a form of wave filter similar to that used in telephone circuits to eliminate undesirable harmonics; and this method has been adapted to thermionic rectifier circuits with considerable success by A. W. Hull.

Take the case of a circuit diagram as shown in Fig. 207, where the filter circuit consists of two condensers C_1 and C_2 with an inductance of L henries between them feeding current to the load R. If e_1 is the voltage across C_1 then

$$i = C_1\frac{de_1}{dt}$$

and as the current i may be considered to be constant (the variation being stipulated by the method to be small)

$$-\delta e_1 = \frac{iT}{C_1}$$

where T is the time of one complete cycle to a near approximation, and δe_1 is the voltage fluctuation across the terminals of C_1. It is apparent that the fluctuation of voltage across C_1 is in the same ratio to the fluctuation across C_2 as that of the impedances of the two circuits, and if δe_2 is the fluctuation across C_2,

$$\frac{\delta e_2}{\delta e_1} = \frac{\dfrac{1}{pC_2}}{\dfrac{1}{pC_2} - Lp} = \frac{1}{1 - LC_2p^2}$$

but $$\delta e_1 = \frac{iT}{C_1} = \frac{2\pi i}{pC_1}$$

and therefore $$\delta e_2 = \frac{2\pi i}{pC_1(1 - p^2LC_2)}.$$

The best apportionment of these capacities may be ascertained by writing

$$C = C_1 + C_2$$

and differentiating δe_2 with regard to C_2 and equating to zero, which results in

$$p\{1 - Lp^2(C - C_1)\} + Lp^3C_1 = 0$$

or $$C_1 = \tfrac{1}{2}\left(C - \frac{1}{Lp^2}\right),$$

and $$C_2 = \tfrac{1}{2}\left(C + \frac{1}{Lp^2}\right),$$

which gives

$$\delta e_2 = -\frac{8\pi i}{p^3L\left(C - \frac{1}{p^2L}\right)^2}.$$

Thus if the periodicity is high

$$C_1 = C_2 = \tfrac{1}{2}C.$$

As e_2 is the voltage across the load

$$Ri = e_2$$

and

$$\frac{\delta e_2}{e_2} = -\frac{8\pi}{Rp^3L\left(C - \frac{1}{p^2L}\right)^2}$$

which provides an equation for the percentage fluctuation required.

This method is only applicable in the case of high frequencies if the use of capacities of reasonable size is a necessary condition. For instance, with a load of 10^5 ohms and an inductance of 500 henries, the following table indicates the percentage voltage fluctuation :—

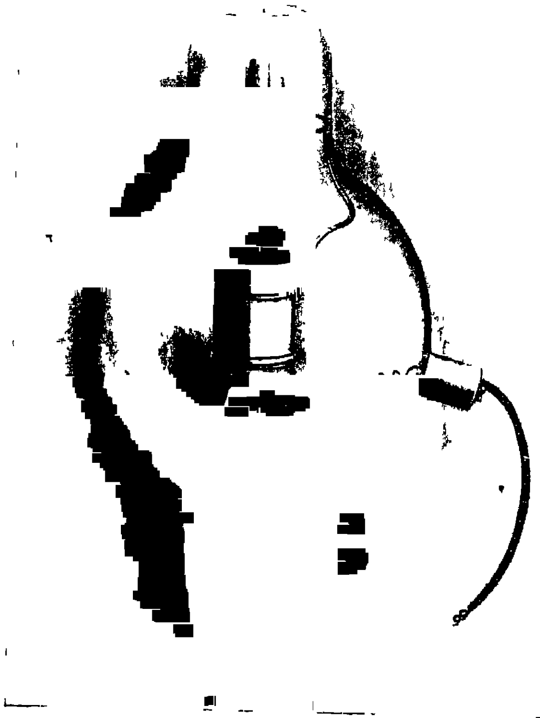

FIG. 208.—U.1 rectifier.

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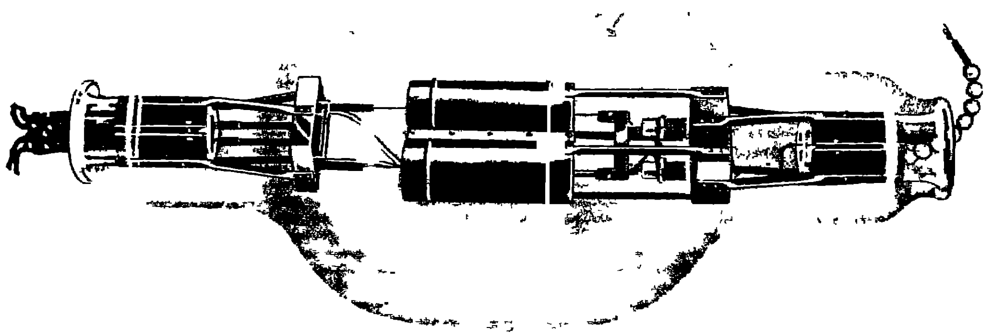

FIG. 209.—M.R.9 rectifier.

[To face page 311.

1 microfarad	50 cycles,	fluctuation	1·7 per cent.
0·01 „	100 „	„	7·3 „
0·01 „	1000 „	„	$7{\cdot}3 \times 10^{-3}$ „

This shows that at the higher frequencies a very small condenser will have a large effect on the undulatoriness; but at the lower frequencies a condenser on a big scale is required, unless the load resistance is high. This can only be possible where small currents are required unless again the voltage of supply is chosen to suit, which would entail high voltage transformation with its consequent expense. Thus, unless the case under consideration is just suited, the cost of smoothing the current wave by these means is likely to be high, and the secret of a cheap installation is (if practicable) so to choose the frequency of supply that it is in the neighbourhood of 2000 cycles. This is sometimes not difficult in modern plants, and with the improved types of high frequency generators a reduction in the size of the transformation plant may often be effected at the same time. This point is evidenced later.

Representative Rectifiers. — Four types of commercial rectifier are shown in Figs. 208, 209, 210, and 211. The first the U. 1 rectifier, Fig. 208, is an early type developed during the war; the others are more modern in design. For example, Fig. 209, the M.R. 9 is a large type of glass rectifier, and the C A.R. 2, Fig. 210, a large power rectifier with a water-cooled anode. The E.H T. 3 type, Fig. 211, is a high voltage rectifier for small currents.

U. 1 Rectifier.—The U. 1 rectifier is suitable for voltages up to 2500 volts, and has the following characteristics:—

Filament volts	12·0
Filament amperes	4·0
Dissipation watts	150·0
Total emission m a	150·0
Overall length mm.	220
Diameter of bulb mm.	120
Maximum voltage rms.	500

Fig. 212 gives the characteristic curves.

M.R. 9 Rectifier.—This is a more modern design for use up

to voltages of 10,000 volts, and differs in several important respects from the U. 1 type. In the first place the anode is of

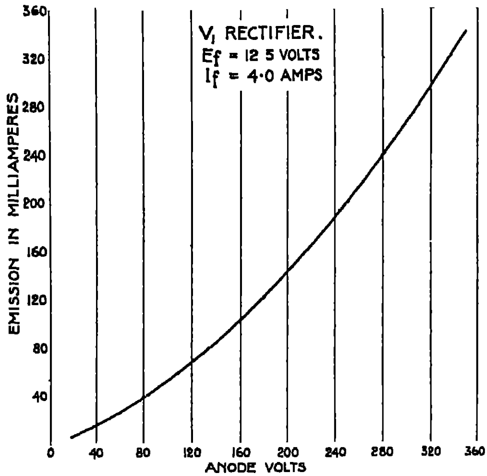

FIG. 212.—Characteristic curves for a U. 1 rectifier.

molybdenum, having a much higher melting-point than nickel, so that it is not so easily damaged on short-circuit.

The filament is in two distinct parallel portions each of which is axially mounted in a nearly closed cylinder; a cross-section of the anode is shown in Fig. 213.

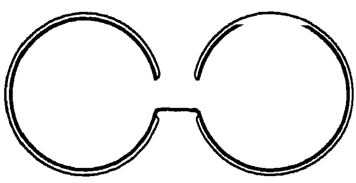

FIG. 213.—Cross-section of a M.R. 9 rectifier anode.

The slits in the anode are necessary to enable the anode to be passed over the filament system in assembly. This construction has several advantages over that of the U. 1—firstly, since each filament is axially placed in relation to the cylinder, i.e. in a position of electrostatic equilibrium, it is free from the action of electrostatic forces, which may be large, and might result in destructive effects on the filament at voltages over 10,000, were it not so placed. Secondly, each filament is situated in a cylinder of small radius, and there is no mutual interference of the fields of the respective filaments,

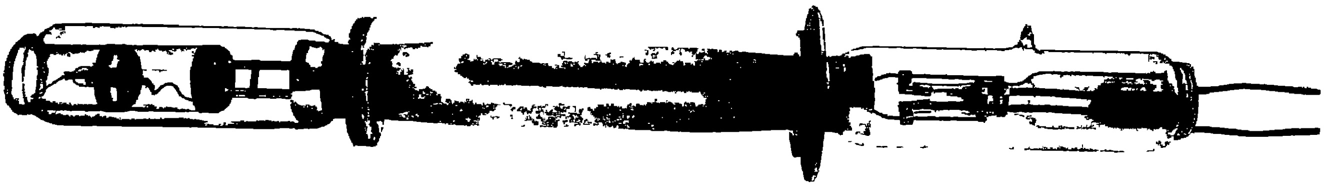

FIG. 210.—C.A.R.2 rectifier (water-cooled anode).

[*To face page* 312.

Fig. 211.—E H.T.3 rectifier.

[See page 312.

as is the case with the U. 1. Thus the anode volts for any particular anode current are considerably reduced.

An approximate characteristic curve is shown in Fig. 214.

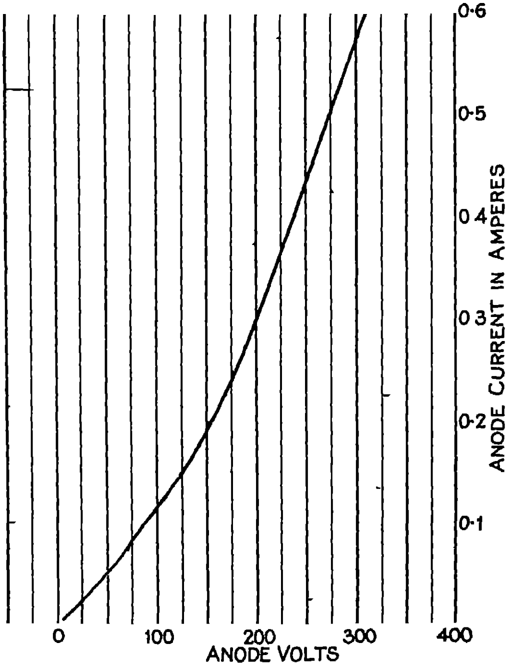

FIG. 214.—Characteristic curves of M.R. 9 rectifier.

The general particulars of the rectifier are as follows:—

Filament volts . . .	20
Filament amperes . .	24
Dissipation	3 K.W. approximately
Total emission . . .	1 ampere approximately
Volts to saturate emission .	450
Maximum voltage rms.	12,000

C.A.R. 2 Rectifier.—This rectifier is illustrated in Fig. 210 and differs from the preceding types, in which the anodes are completely enclosed in the evacuated bulb of glass, and on that account can only dissipate energy by thermal radiation, in having an anode that is external, and is cooled by a flow of

water through a jacket (not shown in the photograph). It is considerably larger in bulk, as will be seen from the following particulars:—

Filament volts	20
Filament amperes	50
Total emission amperes	5
Volts to saturate emission	530
Overall length mm.	750
Maximum voltage rms.	12,000

The anode consists of a copper tube, which is centrally indented along two-thirds of its length, so that its cross-section is as shown in Fig. 215.

The filament consists of two lengths of tungsten about 1 millimetre in diameter, and twenty centimetres long, con-

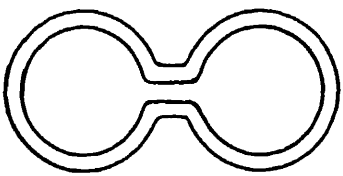

FIG. 215.—Cross-section of a C.A.R 2 rectifier anode.

FIG. 216.—Filament system of C.A.R. 2 rectifier.

nected at their extremity by a cross bar as illustrated in Fig. 216.

To each end of the anode is joined a wide glass tube, which at one end carries the filament leads, and at the other the guiding and tensioning arrangements for the filament cross bar.

Each leg of the filament is mounted in one of the cylindrical chambers of the anode. This design is obviously an adaptation of the M.R. 9 principle to an external anode construction, the connecting space between the two chambers of the anode serving the same purpose as the slits in the anode of the M.R. 9.

Owing to the small diameter and great length of the anode the voltage drop in the rectifier is quite small.

Fig. 217 gives the characteristics of the C.A.R. 2 rectifier, and

it is seen that to obtain the full emission approximately 500 volts drop occurs between the anode and the cathode.

E.H.T. 3 Rectifier.—The E.H.T. 3 rectifier is illustrated in Fig. 197, and has the characteristics shown in Fig. 221.

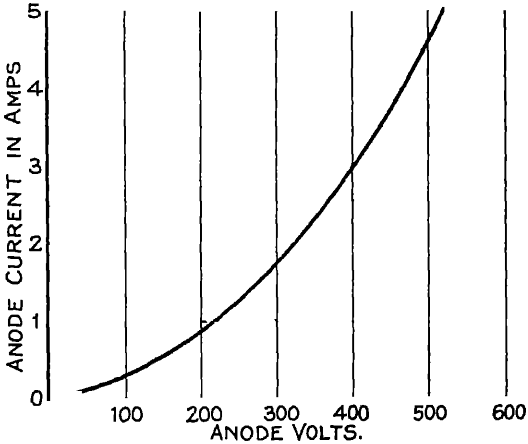

Fig. 217.—Characteristics of the C.A.R. 2 rectifier.

This rectifier has the following divisions:—

Filament volts	8·0
Filament amperes . . .	8·0
Total emission milliamperes .	200
Impedance, ohms . . .	750
Overall length mm. . . .	530
Diameter of bulb mm. . .	180
Maximum voltage peak . .	150 K.V. (Reverse)

and is chiefly used for such purposes as cable testing (page 509) and in any case where a high voltage is required with a relatively low current output.

Seals.—It will be noted that the sizes of certain types of thermionic rectifier have increased to a considerable extent; and that filament currents of the order of 100 amperes and even higher can be employed. This improvement is the direct outcome of the newer types of glass seals which are now practicable on a commercial scale. Reference to the large water-cooled rectifier in Fig. 224 will show that there are four main seals,

viz. two joining the two bulbs to the anode, and two for the filament leads, the two latter being of the later type.

To illustrate the possibilities of the seals Fig. 218 shows a freak reduction joint from large bore tube to comparatively smaller tube. The glass is joined directly to invar cylinders in a form of glass lathe, and no difficulty is experienced in their manufacture or maintenance.

The filament seals are illustrated in greater detail in Fig. 219 which also indicates the thick metal connections to the tungsten filament. The actual details of construction are shown in Fig. 220.

T is a platinum thimble on to which the glass joint G is made. The main connection L, made of large stranded copper wire soldered solid at the end, is inserted into the thimble to the point P. A clamp C is then placed over the thimble and into it is inserted the main filament lead and support R. On tighten-

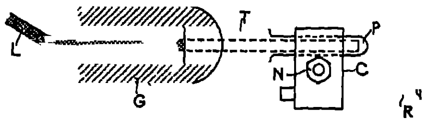

FIG. 220.—Details of filament seal.

ing the nut and bolt N the thimble is squeezed until good electrical contact is made. Thus electrical connection is ensured through the walls of the thimble, which expands only in a longitudinal direction with heat and does not in any way tend to crack the glass seal.

It will be apparent that such arrangements as these will enable big improvements to be made in thermionic rectifiers, and possibly also in mercury vapour rectifiers, as there appears to be no limit to the safe current that can be carried, if the seal is made sufficiently large.

Manufacture and Evacuation.—In the process of manufacture, after assembly, the valve is sealed on to a first grade vacuum pump system, and is then baked in an oven up to as high a temperature as the glass will stand without collapsing, in order to remove gas from the glass. The anode is subsequently "bombarded" to remove all occluded gas. It has already been

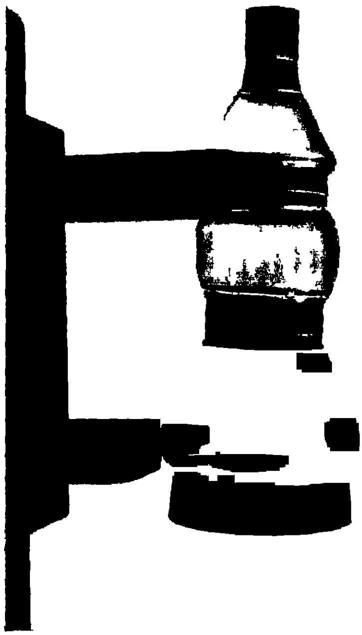

FIG. 218.—Glass to metal seals.

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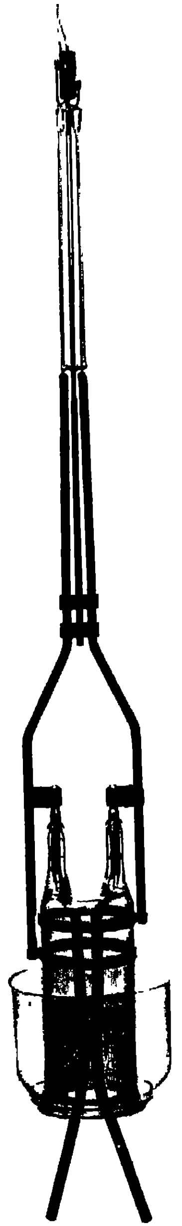

FIG. 219.—Filament seals.

[*To face page* 317.

seen that if a positive potential is applied between the anode and the cathode of a rectifier, electrons leaving the cathode attain a velocity proportional to the square root of the voltage; and owing to their mass they possess kinetic energy. When an electron strikes the anode it is brought to a state of comparative rest, and its kinetic energy appears as heat in the material of

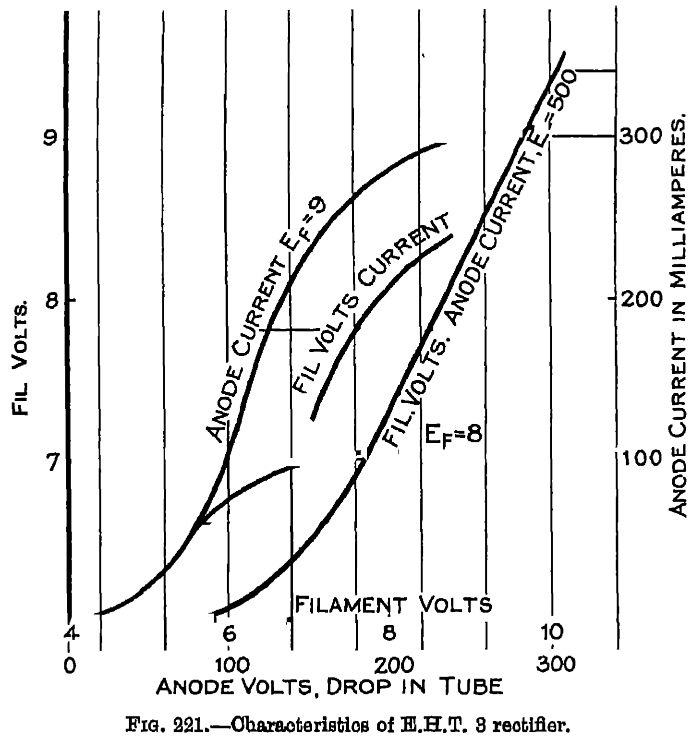

FIG. 221.—Characteristics of E.H.T. 3 rectifier.

the anode. The anode is thus heated by being hammered or bombarded by a stream of electrons reaching it.

If the anode current is I amperes, and the anode voltage V, then the anode is receiving energy at the rate of IV watts.

The process of bombardment consists in heating the anode in this way by applying a voltage between the anode and the cathode with the filament alight, and adjusting either the

filament current or the anode voltage such that the anode is heated to as high a temperature as is practicable.

After these two processes devoted solely to the removal of all gas from the system, glass and electrodes, the valve is sealed off from the pump.

Magnetron Effect.—The largest rectifier so far described, the C.A.R. 2, is designed for a filament current of 50 amperes. In attempting to construct larger rectifiers than this, and with bigger emissions, it would be necessary to increase the cathode area, either by lengthening the filament, or by increasing its diameter, or by a combination of a variation of both of these parameters. There is a definite limit to the first, because a long filament may easily have such an emission, that the emission current (which has to flow from the valve along the

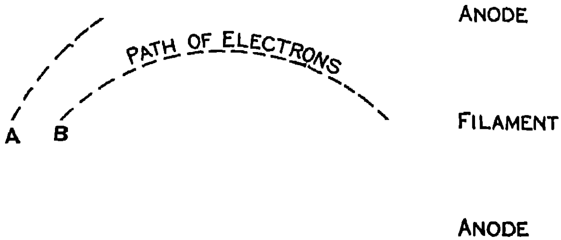

FIG. 222.—Magnetron effect.

filament) will burn out the ends of the filament; so that a long filament must be cut up into a number of shorter filament systems connected in parallel.

There is also a limit to the diameter of the filament, due to the deflection of electrons by the magnetic field of the filament heating current. This effect was first considered practically by Hull in the "Journal of the Institution of the American Electrical Engineers," and was also treated by Richardson in the "Philosophical Magazine." It is due to the tendency of moving electrons to perform spiral orbits in a magnetic field.

Fig. 222 represents a longitudinal section of a cylindrical rectifier. The lines of force due to the filament current will be circles about the filament, and in the section shown will be perpendicular to the plane of the paper.

An electron leaving the filament instead of moving direct to the anode will be deflected as shown by the dotted line A, and if the filament current is large enough, the electron may not reach

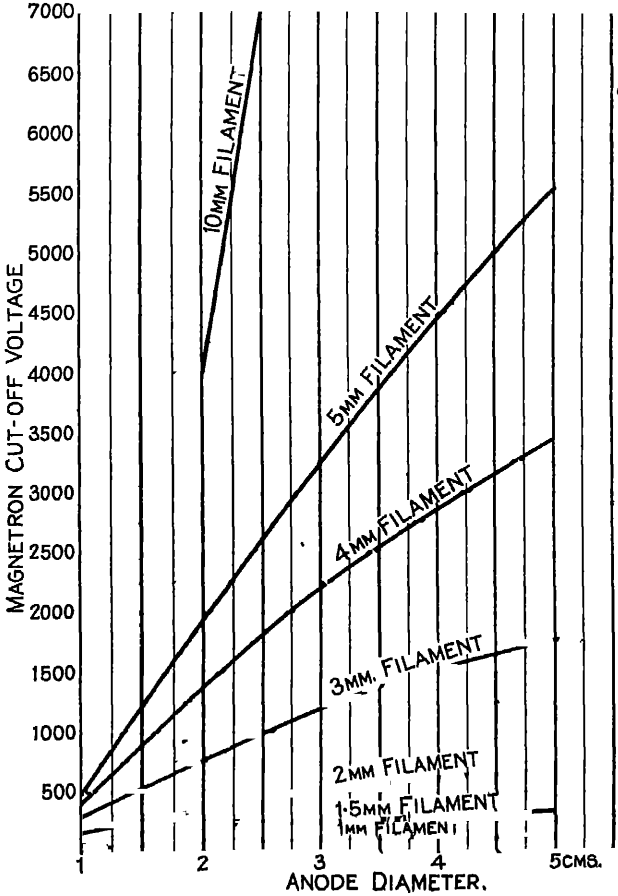

FIG. 228.—Critical voltage of the magnetron effect.

the anode at all, but its direction may be reversed; in which case it will return to the filament as shown by the dotted line B. Thus for any filament current there will be a lower limit of the anode voltage below which no electrons will reach the anode.

Hull has given the following equation for determining this critical voltage:—

$$V_c = 0{\cdot}0188\ I^3 \left\{\log \frac{d}{d_c}\right\}^2$$

where V_c is the critical voltage for a filament current I, d and d_c being the anode and cathode diameters respectively. If the

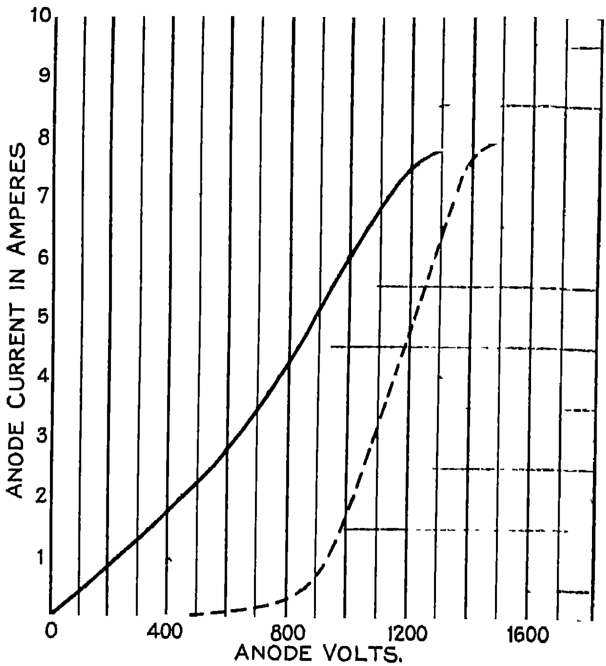

FIG. 225.—C.A.R. 3 Rectifier characteristics showing magnetron effect.

cathode is a tungsten filament operating at 2500 degrees Kelvin, and heated by direct current, this equation becomes

$$V_c = 44100\ d^3 \left(\log_{10} \frac{d}{d_c}\right)^2.$$

A series of curves calculated from this equation are given in Fig. 223, and it is seen that though the effect is quite small in the rectifiers previously described, with filaments of several millimetres diameter, it rapidly increases, and the

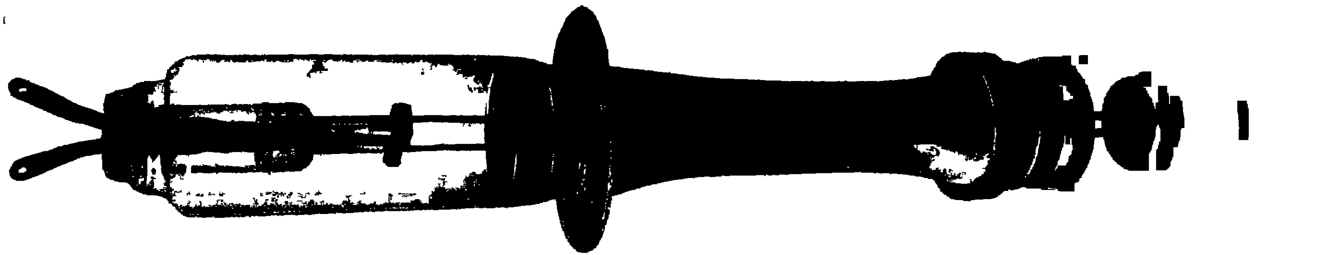

FIG. 224 *a*.—Large water-cooled rectifier. C A R. 3 type.

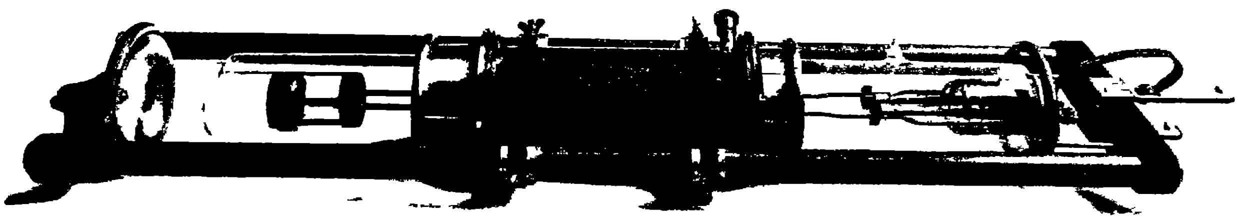

FIG. 224 *b*.—Large water-cooled rectifier in holder C.A.R 3 type.

[*To face page* 321.

critical voltage reaches several hundreds or thousands of volts. There appears to be a limit to the size of filament somewhere in the neighbourhood of 80 amperes, such that filaments larger than this cannot be satisfactorily used without serious loss in efficiency.

As a practicable example of the phenomenon, Fig. 224 shows an experimental rectifier constructed in the Research Laboratories of the General Electric Company, Wembley, of a

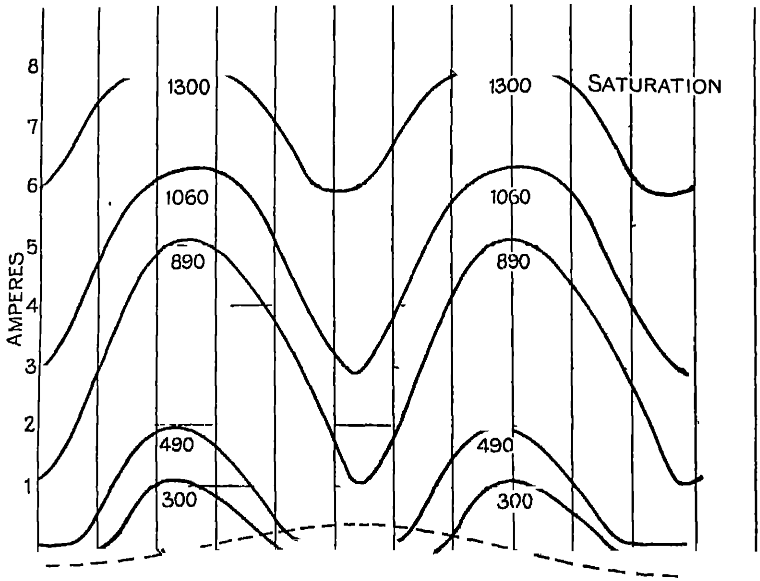

FIG. 226. – Current curves showing magnetron effect.

general design similar to that of the C.A.R. 2, but somewhat larger, and having a filament of approximately 1·8 mm. in diameter taking 125 amperes; Fig. 224 *a* shows the rectifier without its water jacket, whilst Fig. 224 *b* illustrates the same rectifier fitted with its jacket and a holder for ease of mounting, as shown in Fig. 240. The diameter of each chamber of the anode is 3·3 centimetres, so that from the curves of Fig. 228 it will be seen that a considerable magnetron effect is to be expected.

Fig. 226 illustrates curves taken from oscillograms of the anode current at a number of different steady D.C. voltages, with the filament heated from a 50 cycle supply; the dotted line shows the phase relation of the filament current to the anode current. It is seen that with 300 volts on the anode the anode current is entirely cut off for a large part of the cycle; and that even with an anode potential of 1300 volts, the magnetic field of the filament current, when at its maximum value, can still appreciably diminish the anode current.

From the curves of Fig. 226, and further data, the two curves of Fig. 225 have been obtained; the thick line shows the characteristic of the rectifier when there is zero filament current, while the dotted line indicates the characteristic, when the filament current is at its maximum value of

$$125 \times \sqrt{2} = 177 \text{ amperes.}$$

Further data on the Magnetron Effect are given on page 491.

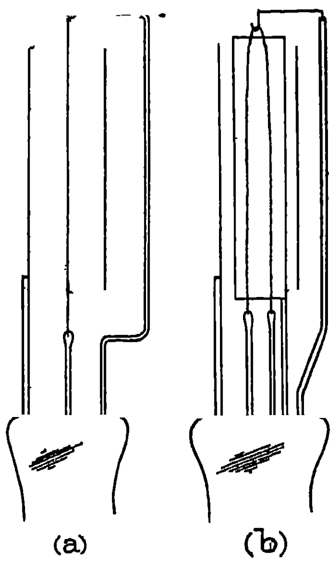

FIG 227.—Rectifiers to demonstrate the effect of β^2.

The Significance of β^2.—It has been pointed out on page 297 that β^2 is usually approximately unity in value, and therefore is of little import. If, however, the ratio of the diameter of the cathode to that of the anode, is in the neighbourhood of unity, the factor β^2 becomes important.

Fig. 227 illustrates two experimental rectifiers, constructed to demonstrate the importance of β^2.

One rectifier, viz. that in Fig. 227 *a*, has a cylindrical anode whose diameter is 1·15 cm., and whose length is 4 cm.; whilst the cathode is a filament of tungsten 0·009 cm. in

diameter, so that in this case d/d_c is large. In the companion rectifier (Fig. 227 *b*), the anode is of the same dimensions, but the cathode instead of consisting of a tungsten filament is a nickel tube 1 cm. in diameter, coated with a mixture of the oxides of barium and strontium. It is heated to approximately 900 degrees Kelvin by means of thermal radiation from a heavy

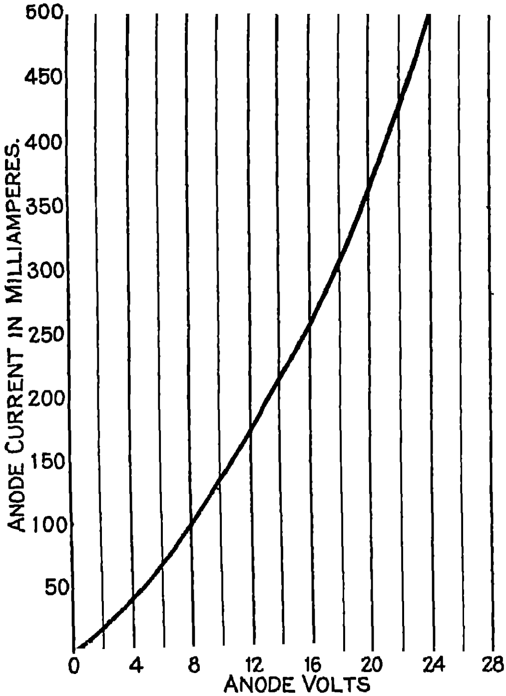

FIG. 228.—Characteristic curves of two special rectifiers as shown in Fig. 227.

40 watt tungsten filament situated internally. This filament is solely to supply heat and is insulated from the cathode and anode, when the measurements are made. In this case the ratio d/d_c is 1·15, the corresponding value of β^2 is 0·017, and therefore it would be expected that since the anodes are of the same dimensions the anode current for the same voltage would be 1/0·017 = 60 times as great as with the filament rectifier.

The observed characteristic curves of the two rectifiers, shown in Fig. 228, demonstrate that this is substantially true.

It must be observed that the construction of the two rectifiers is not sufficiently accurate for this to be a complete check on the computation of β^2, it does, however, furnish a proof that Langmuir's theory is substantially correct.

The remarkable characteristic of the rectifier illustrated in Fig. 227*b*, quite unattainable by any filament type of rectifier, suggests that the general significance of β^2 should be considered; though it does not appear at present, that rectifiers similar to that shown in Fig. 227*b* can be constructed on a large scale for high voltages.

Rectifier Design.—First consider the following problem: Given an anode of diameter of 5 cm. and length 10 cm., and having a cathode whose emission is 0·5 ampere per square centimetre, it is required to determine the total emission, the anode voltage to saturate this emission, and the kilowatts to be dissipated by the anode when saturating the emission with varying cathode diameter.

Using the values of β^2 given by Langmuir and Blodgett, the following set of data can be determined as shown in Table XXVI., and the results are plotted in Fig. 229:—

TABLE XXVI.

d_c Cm.	Emission Current Amps.	$\frac{d}{d_c}$.	β^2.	Anode Voltage.	Kilowatts Dissipated in Anode.
0·1	1·5	50	1·0936	928·0	1·385
0·2	3	25	1·0812	1455	4·365
0·5	7·5	10	·9782	2508	18·810
1	15	5	·7666	3384	50·780
1·5	22·5	3·333	·5748	3659	82·32
2	30	2·5	·4121	3552	106·56
2·5	37·5	2·0	·2798	3181	119·3
3	45	1·667	·174	2619	117·85
4	60	1·25	·042	1280	78·8
4·5	67·5	1·111	·0094	490·4	33·1
4·8	72·0	1·042	·00162	158·5	11·41

The result is remarkable as it is seen that if a limitation is imposed, as must be the case, either on the voltage drop that

can be allowed, or on the kilowatt loading that the rectifier can dissipate, and if full use is to be made of the available emission, then the cathode must either be less than a certain diameter or greater than another larger diameter. For example, if a 1000 volt drop is the permissible maximum, then the cathode

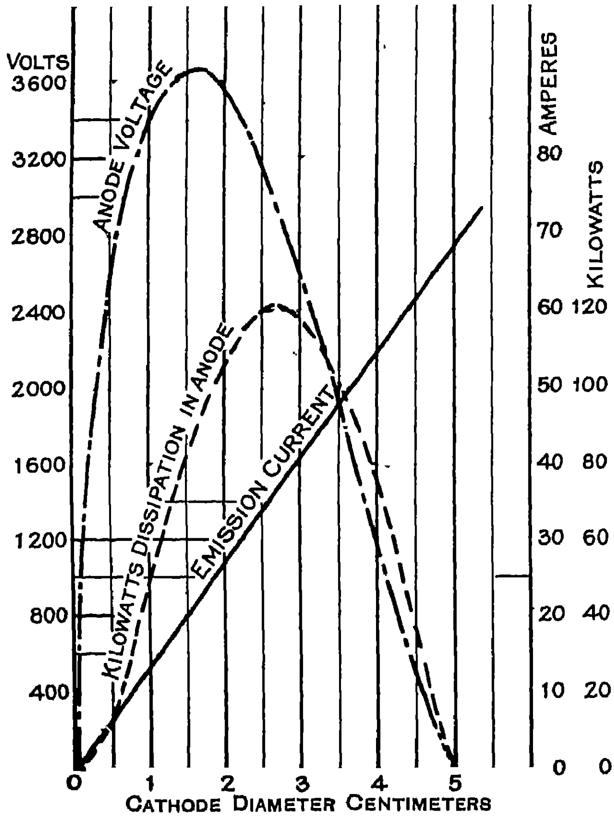

FIG. 229.—Design data for a rectifier.

diameter must in this case be either less than 1 mm., or it must be greater than 4·1 cm.

This suggests the consideration of general alternative designs for a rectifier to some definite specification. Take, for example, a rectifier which is to have an emission of 100 amperes, and an anode voltage of 1000 volts to saturate this emission; assume

again the figure of 0·5 ampere per square centimetre of the cathode area. This determines at once the area of the cathode as 200 sq. cm , which can be obtained in practice, either by means of a long fine cylindrical cathode or short thick ones ; once the diameter is settled, the length is determined, and further, as the emission and voltage drop are fixed, the anode diameter can be ascertained.

This has been done, and the results for various cathode diameters are given in Table XXVII. The watts dissipated per square centimetre, on the assumption of a 35 kilowatts

TABLE XXVII.

ALTERNATIVE DIMENSIONS OF RECTIFIERS.

Diam. of Filament. Cm. d_c	Length of Fil. Cm.	Diam. of Anode. Cm. d	Spacing. Cm.	Volume of Anode. Cm.^{3}
·025	2400	22	11	918,000
·05	1200	11	5·48	114,000
·1	600	5·5	2·7	14,170
·2	300	2·75	1·28	1,782
·35	170	2	·825	584
·5	120	1·8	·65	305·5
·75	80	1·8	·52	208·5
1	60	2	·5	188·5
2	30	2·8	·4	185
3	20	3·8	·4	227
5	12	5·8	·4	317
10	6	10·75	·375	545

	Area of Anode. Cm.^{2}	Watts Per Cm.^{2} Dissipation at Anode.	$\frac{\text{Area of Anode}}{\text{Area of Filament}}$.
025	166,000	·2108	881
·05	41,500	·8436	220
·1	10,380	3·372	55·2
·2	2,594	13·49	13·8
35	1,069	32·74	5·07
·5	679	51·56	3·6
·75	453	77·26	2·4
1	377	92·80	2·0
2	264	132·5	1·4
3	239	146·4	1·27
5	219	159·75	1·16
10	202·6	172·7	1·07

Dissipation = 35 kilowatts.*

* This 35 K.W. is based on 20 K.W. spent in cathode heating, and 15 K.W. in bombardment when used in a hexaphase system.

dissipation in the anode, under normal running conditions, are also given.

Some of the results are plotted in the curves in Fig. 230.

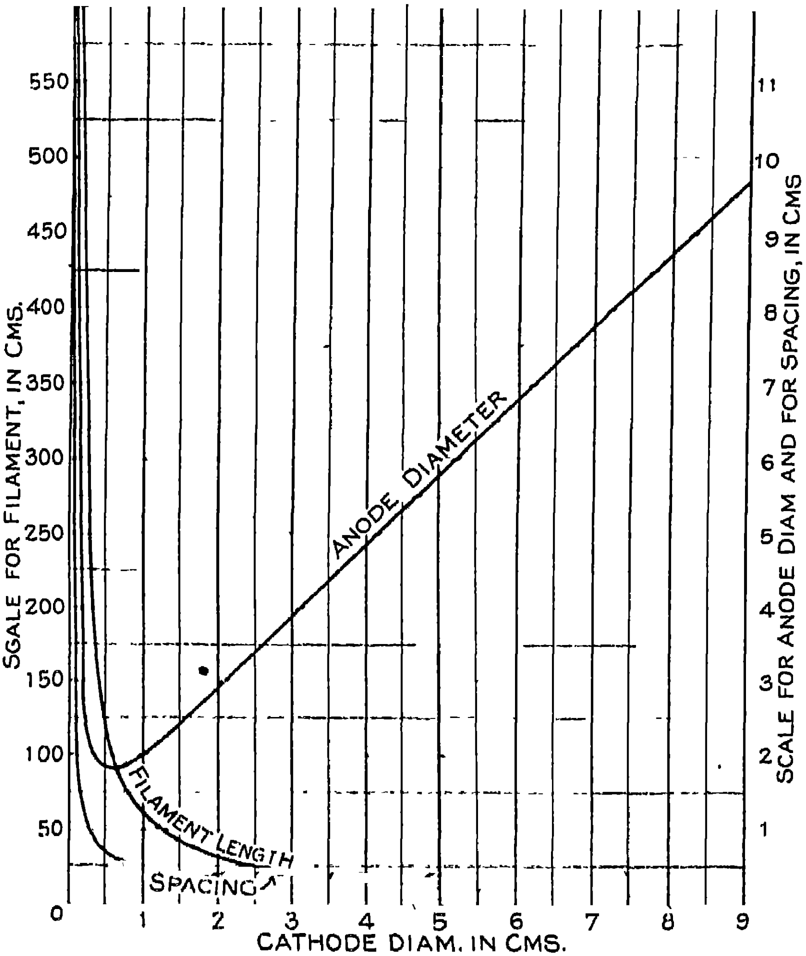

FIG. 230.—Design data for rectifiers.

Arising from the foregoing, the more striking points are:—

(i) As the cathode diameter is increased the volume of the anode decreases with great rapidity to a minimum, after which it increases.

(ii) The anode diameter decreases in the same way to a minimum and then increases.

Fig. 231 illustrates the first point indicating the comparative sizes of the anodes.

(*a*) Represents the anodes of the 0·1 cm. cathode diameter design, split into 25 sections;

(*b*) Represents the 0·5 cm. cathode design, split up into 5 sections; while

(*c*) Represents the 5 cm. design.

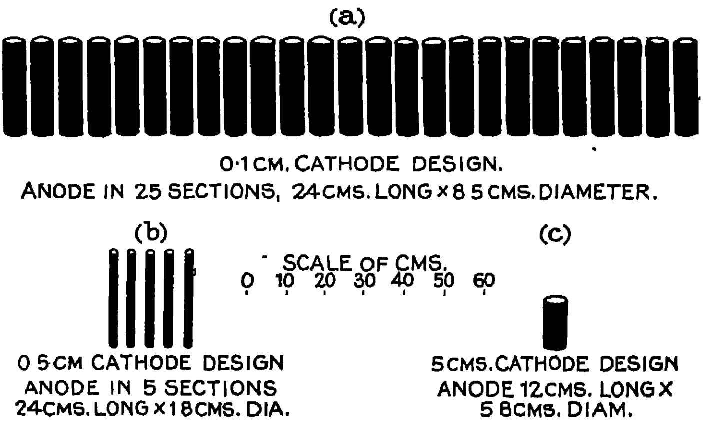

FIG. 231.—Relative anode sizes.

Though the designs based on a larger cathode diameter do not appear to be at present available, there is little doubt that advance in this direction will be necessary, if the thermionic rectifier is to cope with outputs greater than a few hundred kilowatts, in order to avoid the necessity for large banks of small individual rectifiers with cathode diameters of the order of a millimetre or so.

Operating Notes.—There are several operating notes which are of interest:—

(*a*) *Regulation of Filament Heating Supply.*
(*b*) *Irregular Heating of Filament.*
(*c*) *Short-circuit Protection.*

(a) Regulation of Filament Heating Supply.

As pointed out on page 294, evaporation of the tungsten filament is appreciable at the temperatures employed normally, and indeed is, apart from accident, the determining factor in the life of the filament. Evaporation increases rapidly with temperature, and, in fact, such a small increase as 2¼ per cent. in the terminal voltage is sufficient to double the evaporation and halve the life; an increase of 5 per cent. reduces the life to one quarter of its rated value.

Thus it is essential, in order to obtain a good life from a tungsten filament, to pay the greatest attention to its heating supply, and to employ instruments of the highest grade, in measuring the filament voltage. In passing, it may be worthy of mention that the first grade instruments according to the Standard Specification of the Engineering Standards Association are by no means good enough for work of this character. Meters of Laboratory Standard accuracy should always be employed where at all possible.

The adjustment of the filament supply should be always made from the point of view of the voltage across the filament, rather than the current taken by it. The reason for this course is obvious; the filament will become thinner as time passes, and if a constant current is maintained, the temperature rises, and with it the rate of evaporation of the filament.

Table XXVIII. gives the calculated lives of a 0·035 centimetre filament run at constant supply volts, and at constant current. The figures are calculated on the basis of Langmuir's evaporation figures, and the assumption that the life ends when the diameter has been decreased 10 per cent. by evaporation.

The results are remarkable, and moreover agree generally with observations. Thus the filament should be operated at a constant voltage. It is true that the voltage may have to be increased slightly from time to time, to raise the emission to the requisite value, but the filament runs no risk of being grossly maltreated, as may be the case if it is operated from a source of constant current.

TABLE XXVIII.

Initial Temperature Degrees K.	Constant Current.		Constant Volts.		Life at Constant Temperature (Hours).
	Life (Hours)	Final Temperature Degrees K.	Life (Hours).	Final Temperature Degrees K.	
2550	260	2760	1150	2505	770
2440	1400	2700	7500	2400	5700

Further, in the case of a bank of rectifiers, since there are always slight differences between individual rectifiers of the same make, a constant bus-bar voltage should be maintained with small individual adjustment for each rectifier.

(*b*) *Irregular Heating of Filament.*

An important feature in the design and operation of thermionic rectifiers, is the uneven heating of the filament due to the emission current. The general effect is that the negative end of the filament is hotter than the positive end. From the point of view of design this limits the length of filament that can be used. In operation where the filament is heated by alternating current from the same source as the current to be rectified, it is desirable, from time to time, to reverse the filament leads. In a big installation the most satisfactory solution is to run the filament from a supply of slightly different frequency from that of the current to be rectified ; and for large 50 ampere filaments, a difference of frequency of 5 cycles is sufficient. The polarity of the rectifier filament is then changed five times a second with respect to the current to be rectified, and both ends of the filament are overheated in turn, while the thermal capacity of the filament is sufficient to prevent the temperature following the heat changes, and it maintains an approximately constant temperature along its length.

(*c*) *Short-circuit Protection.*

In normal operation, the voltage across the rectifier when it is passing current, is only a few hundred or perhaps one thousand

volts. If, however, the load is short-circuited, the full voltage of the supply is applied between the anode and the cathode, the anode is bombarded by electrons that have acquired a velocity equivalent to the full supply potential, and unless the supply circuit breakers operate rapidly, either the anode is melted, if of nickel, or if of molybdenum, it attains such a temperature that the electron emission is sufficient to destroy the filament with the back bombardment on the other half cycle; the more efficient the rectifier, the worse the effect.

As a protection against this danger, a small circuit breaker may be placed in series with each anode, so adjusted as to open the circuit if the anode current increases unduly. An alternative

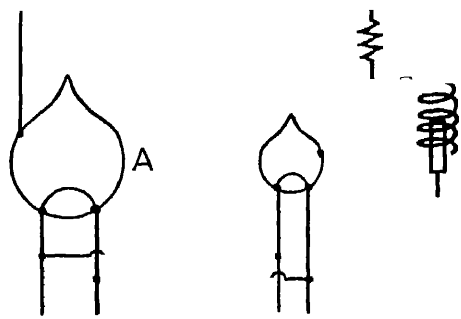

FIG. 232.—Rectifier protection.

scheme which appears to operate satisfactorily is indicated in Fig. 232.

In parallel with the main rectifier or bank of rectifiers A is connected an auxiliary rectifier, having in its anode circuit the coil of a solenoid and a series resistance; in normal operation, the resistance limits the anode current to a small fraction of the total emission of the filament, insufficient to operate the solenoid. In the event of a short-circuit, the voltage across the auxiliary rectifier rises so as to supply sufficient current to operate the solenoid which can be arranged to trip the main circuit breaker.

Efficiency and Comparison with the Mercury Vapour Rectifiers.—There is a very marked difference between the two

types of rectifier. While the mercury rectifier is essentially a heavy current, low voltage device, the thermionic rectifier is essentially a high voltage device, and cannot be used with great efficiency unless the rectified voltage is of the order of several thousands of volts. A simple example will illustrate this point —supposing a 5 ampere rectified current is being obtained at 10,000 volts, by means of a hexaphase system, employing six C.A.R. 2 rectifiers; then the output will be approximately 50 kilowatts, and it would be expected that a drop of approximately 500 volts would be entailed in the rectifiers, so that 2·5 kilowatts will be lost in heating the rectifier anodes. In addition, 6 kilowatts are required to heat the six rectifier filaments. Thus the total losses are of the order of 8·5 kilowatts, and the efficiency is 85 per cent. If the rectified current of 5 amperes were supplied at 20,000 volts, the output would be doubled, while the losses would be the same, and the efficiency would rise to 92 per cent. At 2000 volts supply, however, the efficiency would only be 54 per cent.

It will be noticed that with modern types of rectifier, the main loss lies in the filament heating system, and thus the thermionic rectifier can only hope to become a low voltage device through the development of more efficient cathode materials than tungsten, capable of withstanding high voltages.

Protection from Surges.—A further point in the safeguarding of these circuits is the advisability of including wherever possible, a high resistance in the load side to absorb the energy from any surges which are liable to occur. The device described on page 136, consisting of a glycerine and copper sulphate resistance, has been found to provide excellent protection. This type of resistance has a high heat capacity which is useful in dissipating a considerable amount of energy lasting for a short period, although it cannot carry large currents of the order of 30 milliamperes for long. These tube resistances have been effective in preserving the lives of valves which would have flashed over if not so protected.

CHAPTER XIII.

INSTALLATION OF THERMIONIC RECTIFIERS.

THE use of thermionic rectifiers of high power has increased rapidly during the past two or three years, and it is not too much to predict that in the future, installations of this nature will become a commercial practicability for relatively large powers.

One of the chief difficulties has been mentioned in the previous chapter, on page 324. Another disadvantage is that the thermionic rectifier is essentially a high voltage device, on account of the relatively large voltage drop across the tube, and the loss due to the filament heating. Thus if economy can be effected in the cost of high voltage transformers, the difficulties will be reduced; but if at the same time a high unidirectional voltage can be obtained from a lower voltage transformer then a further economy will be effected.

Several devices are available which will help in this direction, the chief of which are now described.

Bridge Connection of Rectifier.—Single and biphase circuits are open to the objection that the transformer is not able to supply in the first case its full current, and in the second its full voltage. Various devices are available to overcome this objection, and the one most commonly used is that of the Wheatstone Bridge or Graetz method which is illustrated in Fig. 233.

The valves are placed in the positions of the four resistances in the bridge arms, two valves being in opposition to the others, and the load R takes the place of the galvanometer. When current is flowing from the transformer secondary in the direction of the single-headed arrows, the valves 1 and 4 function; whereas during the other half cycle, when current is reversed

and flows in the direction of the double-headed arrows, valves 2 and 3 will allow current to pass. Thus the full transformer voltage will be attained across the load less twice the drop in the rectifiers; but the losses are correspondingly greater as four valves have to be employed, and the loss from this cause is four times as great as in the single-phase case, and twice as great as in a biphase arrangement. Nevertheless, this is usually of minor importance where high voltage work is concerned, as above 10,000 volts the increase in cost of the transformers due to the doubling of the voltage is considerable.

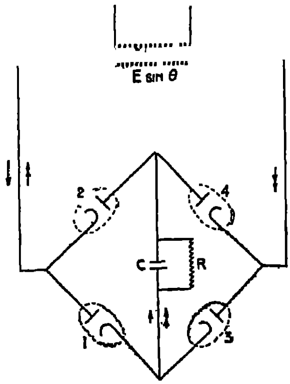

Fig. 233.—Bridge of Graetz connection for rectifiers.

Inverted Valve Connection.—If the cost of transformers is of moment it is possible to obtain double the voltage of the transformer across the load by other methods.

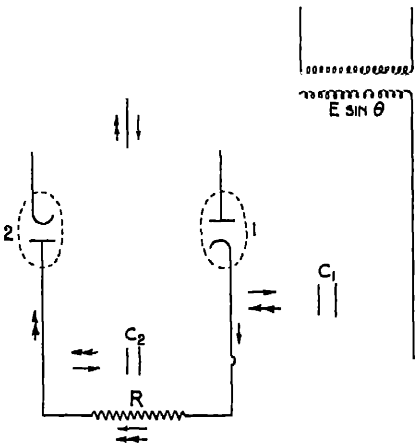

Fig. 234.—Inverted valve and condenser arrangement.

Fig. 234 shows an arrangement whereby two condensers C_1 and C_2 are connected to two valves, the one being reversed to

the other. The mode of operation is as follows: when valve 1 is passing current, as shown by the single-headed arrows, condenser C_1 is being charged and C_2 is discharging through the load. During the next half cycle when valve 2 passes current C_2 is being charged and C_1 discharges itself through the load;

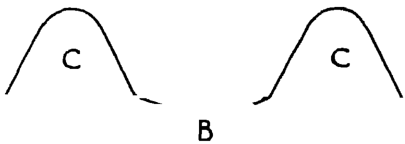

FIG. 235.—Unsymmetrical wave form.

and thus the load is subject to the double voltage of both the valve and the condenser between each half cycle. In this arrangement a wave form is obtained with a double frequency undulation and a voltage double that of the transformer secondary. This circuit is subject to calculation by the method of Prof. Fortescue (page 305).

Inverted Valves for Measuring Unsymmetrical Waves. A device which is useful in thermionic rectifier circuits where measurements are to be made of any reverse current which is being passed by the rectifier is worthy of attention. Assume that the wave form is of the shape shown in Fig. 235, where the big loops CC represent the main rectified current and the loop B the back current it is desired to measure. The circuit should be connected as shown in Fig. 236, where 1 and 2 are two valves large enough to pass the main load current, and connected in opposition. A_1 and A_2 are two moving coil, and a_1 and a_2 two moving iron ammeters. If the current represented by the loops CC in Fig. 235 is in the direction of the arrow, valve 1 will enable A_1 to read this current, and valve 2 will enable ammeter A_2 to measure the back current B. The ratio of the readings of A and a will give the form

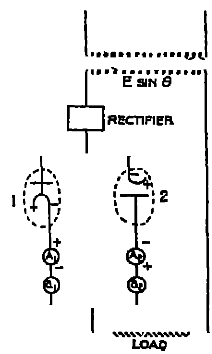

FIG. 236.—Diagram of connections for measuring back currents.

factor of each wave from which some idea of its shape may be obtained. If the filaments are incorrectly connected a small circulating current will be observed which vitiates the accuracy of the results, and the polarity of the filament batteries is therefore important, and has been indicated in the sketch.

Maximum Voltage between Anode and Cathode.—In the case of a biphase system as in Fig. 206 when the left-hand valve is conducting and at the maximum current value, the potential of the anode is $+E$, if the voltage wave is of the form $E \sin \theta$; but this entails a potential on the right-hand valve of $-E$ which is non-conducting at this instant. As the left-hand valve is passing current the drop in voltage across the valve is small and thus the voltage on the filament of

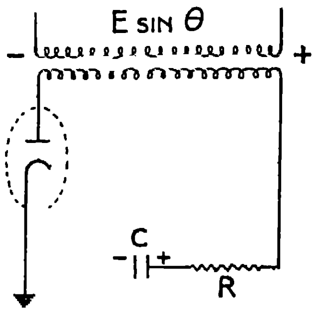

FIG. 237.—Single-phase circuit with condenser.

the right-hand valve is also approximately $+E$. The voltage across the valve which is not passing current is thus $2E$ or twice the rated maximum voltage of the transformer ($= 2{\cdot}82\mathscr{E}$ if $\mathscr{E}$ is the effective voltage). The rectifier must therefore be so designed as to withstand a voltage equal to at least three times the normal supply voltage, otherwise a discharge may take place with disastrous results to the tube.

This state of affairs normally applies to a biphase arrangement, as in the case of single-phase circuits the valve will drop the voltage sufficiently, but under certain conditions it may apply equally to single-phase circuits. In the diagram in Fig. 237 suppose that the system of cabling, the switchgear, etc., has a high resistance leak to earth, then the insulation will be charged

up to a maximum potential of $1{\cdot}4E$ during the conducting period; and when the valve is not passing current, it may happen that this charge does not leak away sufficiently rapidly that its potential is appreciably lessened during the period of the non-conducting half cycle. The circuit is thus analogous to the insertion of a condenser of capacity C and if the rate of leak is slow enough the voltage of $2{\cdot}8E$ across the valve may be attained in a single-phase circuit. Assume an insulation resistance of ω ohms, then a condenser of capacity C will discharge a voltage $1{\cdot}4E$ through such a resistance at an initial rate of

$$\frac{1{\cdot}4E}{\omega C} \text{ volts per second.}$$

If f is the frequency of supply, in one half cycle the voltage lost will be approximately

$$\frac{1{\cdot}4E}{2f\omega C}.$$

If the valve is to be strained to the upper limit of $2{\cdot}8E$ this quantity must be small compared with the voltage of charge or $1{\cdot}4E$, and thus

$$\frac{1}{2fC} \text{ must be less than } \omega.$$

If $f = 50$ cycles per second, and $C = 1$ microfarad, ω can be as low as 10,000 ohms; but if C is only 0·001 microfarad, ω must be at least 10 megohms. This calculation shows the necessity for caution in the use of rectifiers on high voltage circuits as regards the equipment used, especially if any high tension switching devices are employed. It is always safer on single-phase circuits to use a rectifier with a factor of safety of three. The interesting point is also raised that where the system has a small self-capacity, as is usually the case, the insulation resistance should be higher than would otherwise be necessary.

This question of the electrical stressing of valves may be developed to include the case of multiphase circuits. Assume a system of m phases, where the voltage waves will take the form

$$E \sin \theta,$$

or generally $$E \sin\left(\theta + \frac{2\pi k}{m}\right),$$

where k is any integer 0, 1, 2, etc., up to $(m - 1)$.

If the first valve of a series of m valves corresponding to the m phases is non-conducting at the time when the value of θ in the expression $E \sin \theta$ does not lie between $\pm \frac{\pi}{m}$ it will have a voltage across it of

$$E\left\{\sin\theta - \sin\left(\theta + \frac{2\pi k}{m}\right)\right\}$$

it being assumed that the valve whose voltage is

$$E \sin\left(\theta + \frac{2\pi k}{m}\right)$$

is conducting at this instant.

This voltage difference will be a maximum when

$$E\frac{d}{d\theta}\left\{\sin\theta - \sin\left(\theta + \frac{2\pi k}{m}\right)\right\} = 0$$

or when

$$\tan\theta = \frac{\cos\frac{2\pi k}{m} - 1}{\sin\frac{2\pi k}{m}}.$$

The maximum voltage across the valve is then

$$- E\sqrt{2\left(1 - \cos\frac{2\pi k}{m}\right)}.$$

Now the maximum value this expression can have is $-2E$, the negative sign indicating that the voltage is in the reverse direction across the first valve and the value of k is then $m/2$. Thus in this instance also the peak voltage across the valve is 2·8 times the effective voltage of supply, and, as in the other case, the valve must be designed accordingly.

It will be noted that care is required in the installation of rectifiers, and at the same time economy may be effected, by the use of special connections, and transformer designs. The real economy in transformer voltage is especially apparent if the load is tapped across the rectifier terminals (i.e. the anode

and one of the filament leads), in which case practically double the transformer voltage will be obtained. It is not to be expected that large powers can be utilised, but currents of from 20 to 30 milliamperes can be obtained; or in any case the equipment will be of sufficient capacity for cable testing and the like.

Transformer Capacity.—A further economy can be effected in the terminal voltage required, by using a hexaphase system where the rectified voltage equals the peak value of the supply voltage (page 26). On the other hand, however, if a

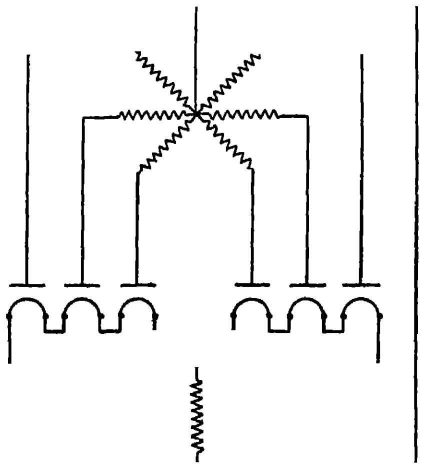

FIG. 238.—Simple hexaphase system.

straight hexaphase system is employed as shown in Fig. 238, a loss of overall efficiency will result from the uneconomical utilisation of the secondary windings, and as has been demonstrated on page 228, the transformer secondary and primary capacity must be calculated from the equations

$$P_s = \pi \frac{(E_M + e_1)I_M}{\sqrt{2m}\,\sin\frac{\pi}{m}},$$

$$P_p = \frac{P}{\eta \cos \phi},$$

where E_M and I_M are the rectified voltage and current respectively, e_1 is the drop in the tube, m is the number of phases, P_s is the output, and equals $E_M I_M$, η is the transformer efficiency, and $\cos \phi$ is the power factor of the system.

By using two three-phase transformers, so connected electrically that their secondaries lead one another by 60 degrees, as shown on page 223, the utilisation factor of the secondaries is greatly improved.

As an example of the effect of this apparent paradox, in a hexaphase system, the rating of the secondaries must be 70 per cent. greater than the rectified power; and if the rectified voltage is 25,000 volts, the peak supply voltage will be 25,000 volts, and the effective voltage of supply, i.e. the voltage from an outer terminal of the transformer to the neutral point, would be only 17,600 volts, whereas in a biphase arrangement this figure would be increased to 28,000 volts.

Thus although the current rating of the transformer secondary is high, the voltage rating is less, and it may happen that the extra cost of the increased copper is more than counterbalanced by the saving in insulation; while at the same time a smoother wave form is obtained.

Hexaphase Rectifier System.—As an example of the type of installation possible using thermionic rectifiers, a description is given of the plant recently installed at the Research Laboratories of the General Electric Company, Wembley. This equipment must only be considered as an experimental plant for special purposes, but as it is probably the first of its kind in this country, it is worthy of description. The requirements were, a hexaphase supply, i.e. a reasonably uniform direct current wave form, without the aid of a filter circuit, and capable of supplying a load of 5 amperes at 30,000 volts.

It was also to be possible to split this system into three separate circuits with independent control of each biphase current at the same voltage. The voltage control was to be uniformly graded, so that an induction regulator was required; and furthermore complete protection was a necessary condition, as the equipment would be operated by unskilled labour. The

supplies were to be led into cages, where experimental work would be in progress, and entrance to these cages was also to be protected.

A distinction would appear to be essential in installations of this nature, between the protection of apparatus and that afforded to manual operators. The second is, of course, mandatory, and the first optional, depending chiefly as it does, on its relative cost, and the ease and cost of replacement in case of breakdown. A study of the diagram of connections reproduced in Fig. 239 will show that as complete protection as possible has been provided under both counts.

The main rectifier frame is designed for three banks each consisting of six water-cooled rectifiers, large type (Fig. 224), i.e. in all eighteen, which allows a bigger load output than that for which the installation is designed.

The various main conditions of supply are six in number:—

(i) Hexaphase 30,000 volts 5 amperes.

(ii) Hexaphase 15,000 volts 10 amperes.

(iii) Separated working 30,000 volts 5 amperes on each phase.

(iv) Separated working 15,000 volts 10 amperes on each phase.

(v) Separated working 30,000 volts 5 amperes on one phase, and 15,000 volts 10 amperes on the other two phases.

(vi) Separated working 30,000 volts 5 amperes on two phases, and 15,000 volts 10 amperes on one phase.

The method by which the various voltages and currents are obtained is generally by cross connection of the main transformers T_1, T_2, and T_3 and the switches S_1 and S_2. The changes on the transformer tappings are made by special links; and the switches S_1 and S_2, the function of which will be explained later, are held in one position or the other by crutches of a special design, without the use of which, contact cannot be made. Correlation of these links with the protective gear is made by means of a special box, containing six compartments, so arranged that only one compartment can be opened at one and the same time, and each compartment refers to one of the six conditions

of working above mentioned. Each compartment contains its links, the use of all of which each in its correct position must be ensured before the main contactor will close. Finally, every link in a closed compartment must be present, otherwise the push button circuit of the main three-phase contactor cannot be made.

The load has been shown on the diagram to be three valves representing one of the four conditions of separated working; and it will be apparent that when operating under hexaphase conditions, valves 2 and 3 must be removed to make room for the use of the one position, viz. phase 1, which is the only hexaphase position. Thus switches S_1 and S_2 transfer the contacts on the entrance gate, and on the valve filaments V_2 and V_3 to the one phase 1 only, and the protection on the other two phases is relaxed.

At the same time the grids of valves 1, 2, and 3 can be supplied either with a small negative potential from 400 watt 4000 volt transformers T_4, T_5, and T_6, or with positive or negative direct current from a 1000 volt 10 kilowatt motor generator set *MG*, the change being accomplished by switch S_3. This motor generator set, which consists of two 500 volt machines connected either in series or parallel, can, by manipulating switch S_4, be earthed on either the positive or negative side.

The induction regulator *IR* is operated by the small direct current motor *M*, controlled by a push button P_1, the speed of which, and thus the rate of rise of voltage of the main E.H.T. transformer, is adjusted by the field rheostat R_1. This regulator can be inserted in any or all of the main transformer circuits by means of the switches S_6, S_7, and S_8, a link being combined with these switches, ensuring that they are all in one and the same position when working under hexaphase conditions. The same precaution applies to the tapping switches S_9, S_{10}, and S_{11}, controlling the steps of voltage on the main three-phase transformer T_7.

The main control is by means of the contactor C_1 with its push buttons P_2 and P_3 ("on" and "off"). The individual control of each phase is by contactors C_2, C_3, and C_4 with their

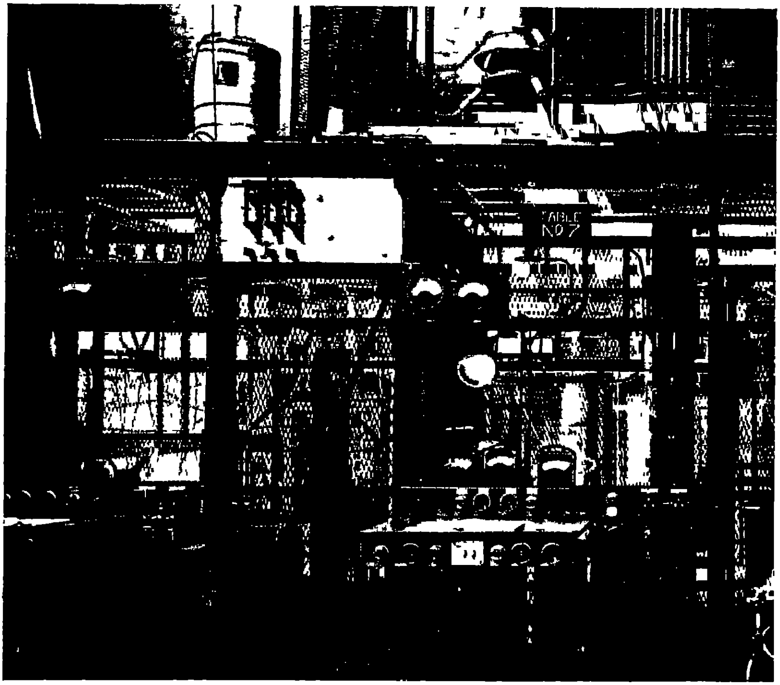

FIG. 240 *a*.—Hexaphase rectifier installation at the Research Laboratories of the General Electric Co., Wembley. Front view, showing filament, controls, etc.

[*To face page* 342.

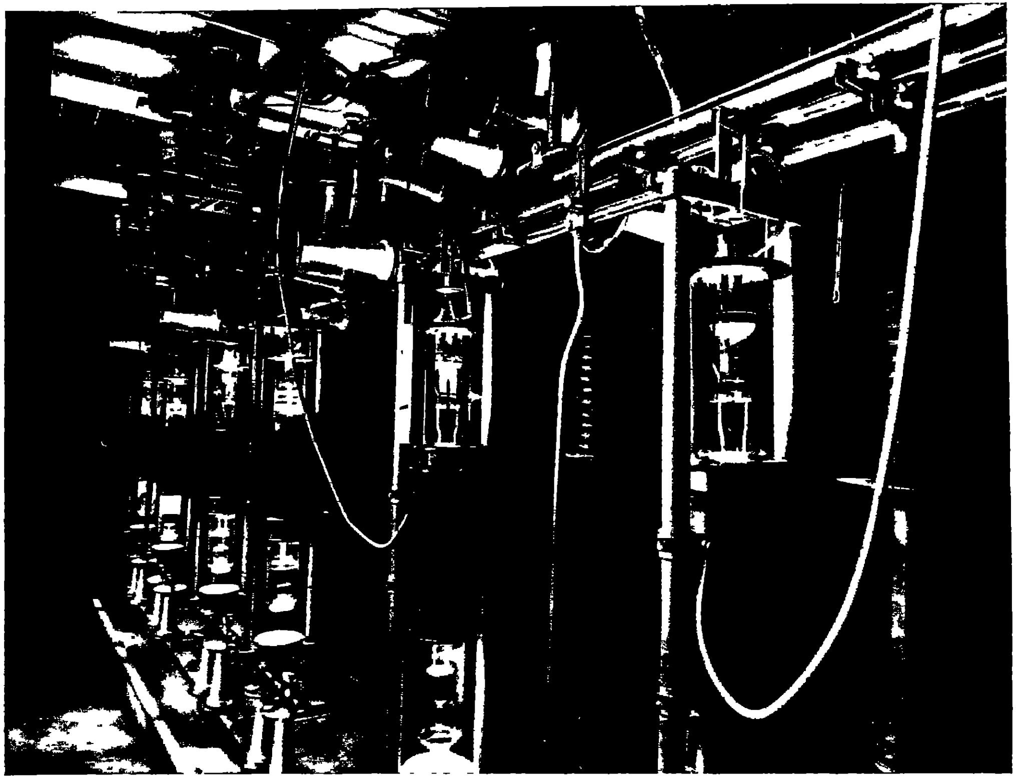

FIG. 240 *b*.—Hexaphase rectifier installation at the Research Laboratories of the General Electric Co., showing mounting of rectifiers and coiled pipes for voltage reduction of cooling water.

[*To face page* 343.

push buttons P_4, P_5, and P_6, and here again under hexaphase working, switch S_1 transfers the operation of contactors C_3 and C_4 to push button C_2. In these contactors it will be noted that resistances are included, so that the momentary rush of current on switching in the circuit is controlled.

One special feature of the system is the provision of the spark gaps, and resistances shown at X (6 in number). These devices fulfil the dual purpose of protection for the operator and apparatus. They are mechanically operated by the door of entry into the danger area, and are so arranged that on the first attempt to open the door, the earthed switch arm makes contact with one of the spark gaps connected in series with a liquid resistance (page 136), and thus any E.H.T. will show its presence, as a series of short sharp discharges which will not damage the apparatus. The door has then to be slightly closed before it can be finally opened, when the whole system is solidly earthed.

The water supply to the anodes of the rectifiers is from a cooling tank system, provided with a centrifugal pump, and with protective gear in the shape of water-mercury switches, which cut off the main current in case of a failure of the water system. The electrically charged water is passed in the case of each rectifier through a heavy rubber tube, wound spirally on a paxolin cylinder, the water from each pipe running finally back through an earthed drain to the tanks. The reason for the tanks is that by using the water over again, the rectifiers run no danger of becoming furred.

Photographs of the installation are reproduced in Fig. 240.

Future Developments.—The above system represents a relatively high power, low voltage equipment; when an E.H.T. installation is required of the order of 120 kilowatts it will be found that the possible rectified power is greatly reduced, and that at the present time the higher the voltage the lower the power available. This is due to the fact that so far rectifiers have not been developed which will cope with large currents at high voltages. There is no fundamental reason why the difficulties of design should not be overcome, although, as has been seen in Chapter XII., limitations must be imposed in certain directions.

Also the higher the voltage the higher the efficiency, so that there is every incentive to the production of high voltage devices. Comparing the relative designs of a 30,000 volt, high current, and a 120,000 volt, low current tube (see Figs. 224 and 211) it will be noted that marked improvements have been made in the past two or three years, and it is not too much to expect that in a corresponding period in the future, the high voltage, high power tube will be available.

Production of Direct Current Without the Aid of a Filter. —In Chapter XII a standard type of filter circuit has been

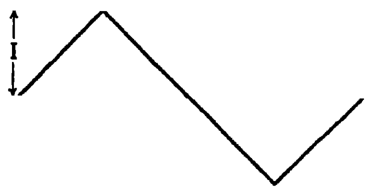

FIG. 241.—Triangular wave form.

described, but it is possible, by means of an ingenious device, described by Maxstadt, to eliminate such a complication.

This method depends for its success on the particular wave

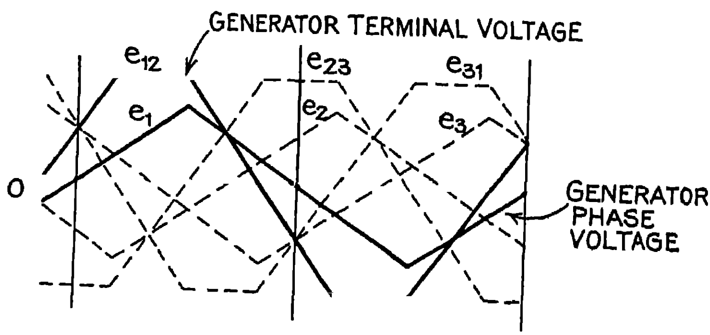

FIG. 242.

form of the supply. This supply cannot be obtained, on account of its specialised nature, from the mains, and a separate motor generator has to be inserted into the system.

The triangular wave form of Fig. 241 has been shown on

page 33 to consist of a fundamental and odd harmonics, according to the equation

$$i = \frac{8I}{\pi^2}\left\{\sin\theta - \frac{1}{3^2}\sin 3\theta + \frac{1}{5^2}\sin 5\theta - \ldots \infty\right\}.$$

If a three-phase generator can be so constructed as to produce a current of a similar wave form in each phase, it will be

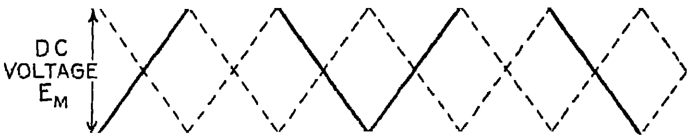

FIG. 243.

found that whether the windings are connected in star or delta, the resulting current wave will have all the third harmonics or multiples of third harmonics, eliminated.

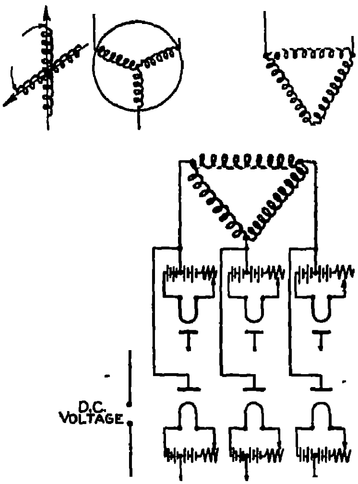

FIG. 244.—Connection in a circuit with no filter.

This point is exemplified in Fig. 242 where it is seen that the resulting generator voltage consists of a wave formation similar to that of Fig. 22.

The equation to this formation is

$$i = \frac{6\sqrt{3}\,I}{\pi^2}\left\{\sin\theta - \frac{1}{5^2}\sin 5\theta + \frac{1}{7^2}\sin 7\theta - \quad . \; . \; . \quad \infty\right\},$$

and if this supply is rectified by means of thermionic rectifiers, the resulting rectified wave form is unidirectional with practically no pulsation, as is evidenced in Fig. 243.

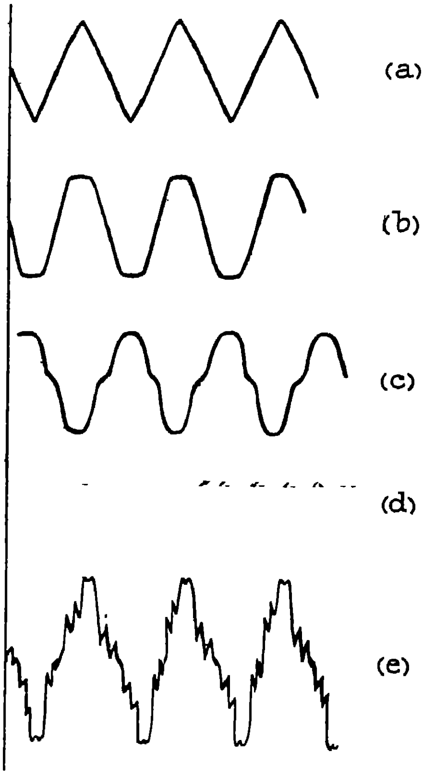

FIG. 245.—Oscillograms of non-filter device.

The arrangement of the rectifiers is indicated in Fig. 244.

The chief difficulty in the device is in the design of a suitable generator to give the requisite wave form; but apparently this was overcome by correctly spacing the rotor conductors irregularly, and by specially shaping the pole pieces.

That the equipment functions satisfactorily is evidenced by the oscillograms reproduced in Fig. 245, where curve (*a*) is the voltage in one leg of the generator, (*b*) the generator terminal voltage, (*c*) the voltage impressed on the transformers, (*d*) the resultant rectified current on a non-inductive load, and (*e*) is the exciting current of the transformer, and shows the distortion to be expected.

Although the ideally smooth curve was not obtained, the method is of such promise, and of such fundamental soundness as to warrant further experimentation. This is essentially the case when one considers the expense of high voltage condensers for filter circuits and the losses entailed.

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CHAPTER XIV.

GAS FILLED TUBES.

General.—In Chapter XII. on Thermionic Rectifiers it was stated that in the early days of manufacture where high voltage circuits were employed, erratic conditions obtained owing to the lack of a high state of evacuation in the bulb, and consequent conduction by means of ionisation by collision, resulting in the bombardment of the cathode by positively charged atoms instead of true thermionic emission. In certain forms of rectifier, however, the tube is deliberately filled with gas, and three of such types will be considered in this chapter.

NEON TUBE RECTIFIER.

Theory of Operation.—In this type of rectifier, which is of the low voltage type and is illustrated in Fig. 246, the electrodes consist of a metal cylinder containing a coaxially-placed metal rod of small diameter. There is no hot cathode such as a filament which is heated by a separate supply, and the bulb is filled with a gas (in this case neon) to a small pressure of about ·01 mm. of mercury. It is seen in Chapter VII. that the negative glow is the chief contributor to the luminosity, and that as the pressure in the bulb decreases this glow increases in thickness after which if the pressure be further decreased it will gradually disappear. In the neon lamp and also in the rectifier the pressure is reduced until the negative glow consists of a thin luminous layer on the cathode about $\frac{1}{8}$ inch thick, and the bulb is then sealed off. Neon is chosen to fill the bulb because the

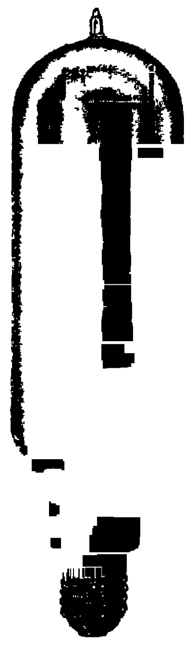

Fig. 246.—Single-phase neon rectifier.

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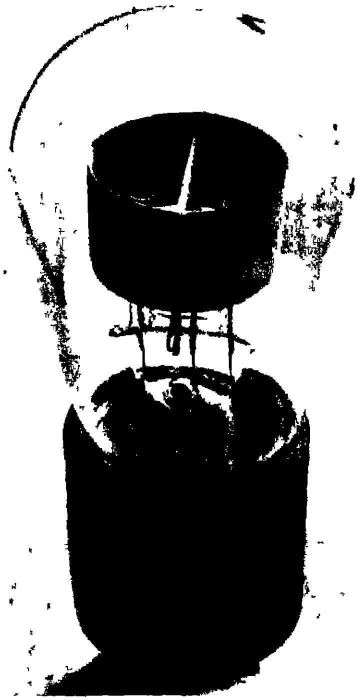

Fig. 248.—G.R.1 rectifier (G.E.C.).

[*To face page* 351.

cathode fall is low for metals in this gas and hence the starting voltage, or voltage at which the glow first appears, is small.

On the application of a voltage between the electrodes which is alternating in character, current will flow in each half cycle in opposite directions, and the cylinder and rod will alternately become anode and cathode. On account, however, of the much greater area of the cylinder, the greater current will flow when it is the cathode, and hence a rectifying effect will be obtained, although it is to be noted that perfect rectification is not possible. The rectification ratio, as the ratio of the mean current in one direction to that in the other may be termed, will thus be seen to be approximately the ratio of the respective electrode areas.

In all glow lamps of this description the volt-ampere characteristics consist of a curve such that after the glow has started a very small increase in the voltage implies a great decrease in resistance. A stabilising resistance is therefore inserted in the circuit which has the effect of flattening the curvature and rendering the device inherently stable. This entails a slightly less voltage between the outer terminals of the tube owing to the drop in this resistance, and the rectification ratio is thus slightly less than that of the respective areas.

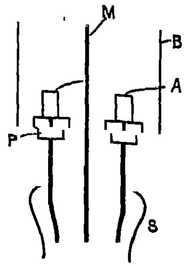

FIG. 247.—Sketch of G.R. 1 rectifier.

A commercial type of neon rectifier for use as a battery eliminator on wireless sets is described on page 475. The neon unit is of the biphase type as shown in the sketch in Fig. 247.

The metal box B is divided by a partition M, and through the base of the box the two anodes AA project insulated by two porcelain bushes PP, or in the later models by air alone. The three leads are then taken through the seal S to the rectifier cap.

The appearance of the rectifier is illustrated in Fig. 248.

Current-Voltage Characteristics.—The general current-voltage characteristic of a neon rectifier is given by the empirical relation

$$i = \frac{e - e_g}{\frac{1}{AM} + R} \quad . \quad . \quad . \quad . \quad (1)$$

where e is the supply voltage, i the current in amperes (both instantaneous values), A the surface area of the cathode which is assumed to be completely covered with the glow, M the slope of the characteristic per unit cathode area, R the series resistance, and e_g the point at which the characteristic would cut the axis, and which is here called the starting voltage.

Thus the quantity A will have two values, viz. A_L when the cylinder is the anode and which will give rise to the larger mean current value, and A_s when the rod is the anode. The values of these constants for a particular design of rectifier are as follows :—

e_g = 150 to 170 volts.
$1/A_LM$ = 180 to 160 for a supply voltage of 200 to 240 R.M.S.
$1/A_SM = 1 \times 10^4$ to 3×10^4.
R = 500 ohms.

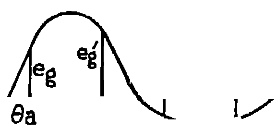

FIG. 249.—Voltage wave in neon rectifier.

If an alternating voltage $E \sin \theta$ be applied across the tube, a glow will not appear until the voltage e_g is reached and the glow will disappear again when the voltage drops to e'_g which is slightly different in value from e_g. No great discrepancy will be apparent in ordinary calculations if these two figures are taken to be identical.

Fig. 249 represents the state of affairs, current flowing in the positive direction from a time equivalent to θ_a till $\pi - \theta_a$ and a lesser negative current from $\pi + \theta_a$ to $2\pi - \theta_a$ where the angle θ_a is given by

$$\theta_a = \sin^{-1}\frac{e_g}{E}.$$

Mean Value of Current.—The mean value of the current

can be ascertained by employing equation (1) for the current-voltage relation, from which

$$I_M = \frac{1}{\frac{1}{A_LM} + R} \cdot \frac{1}{2\pi}\int_{\theta_a}^{\pi-\theta_a}(E \sin \theta - e_g)d\theta$$

$$= \frac{1}{\frac{1}{A_LM} + R}\left\{\frac{E \cos \theta_a}{\pi} - e_g\left(\frac{1}{2} - \frac{\theta_a}{\pi}\right)\right\}$$

$$= \frac{\delta}{\frac{1}{A_LM} + R}.$$

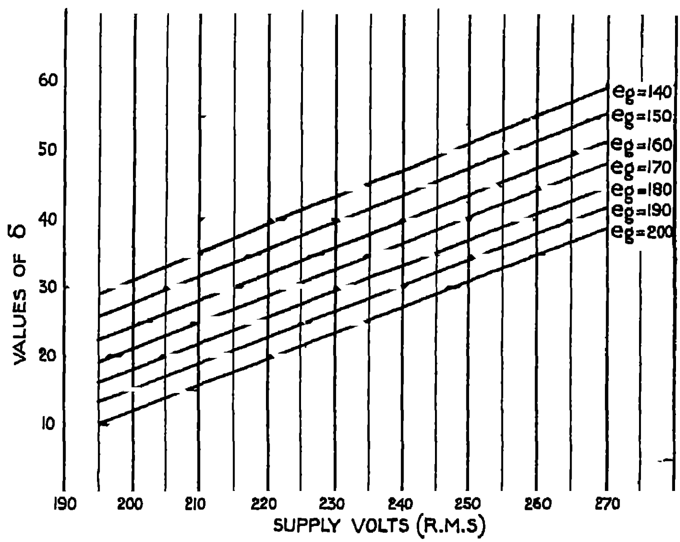

FIG. 250.—Curves of $\mathscr{E}$ and δ.

A similar expression can be obtained for the period π to 2π when the smaller reverse current passes, and the mean value of this negative current is

$$I_M' = \frac{1}{\frac{1}{A_SM} + R}\left\{\frac{E \cos \theta_a}{\pi} - e_g\left(\frac{1}{2} - \frac{\theta_a}{\pi}\right)\right\}$$

$$= \frac{\delta}{\frac{1}{A_SM} + R}.$$

The net current (mean value) available for rectification purposes is

$$I_M - I_M' = \delta\left\{\frac{1}{\frac{1}{A_LM} + R} - \frac{1}{\frac{1}{A_SM} + R}\right\}$$

and values for δ with varying R.M.S. supply voltages and differing values of e_g are plotted in Fig. 250.

Effective Value of Current.—The R.M.S. current is given by the general expression

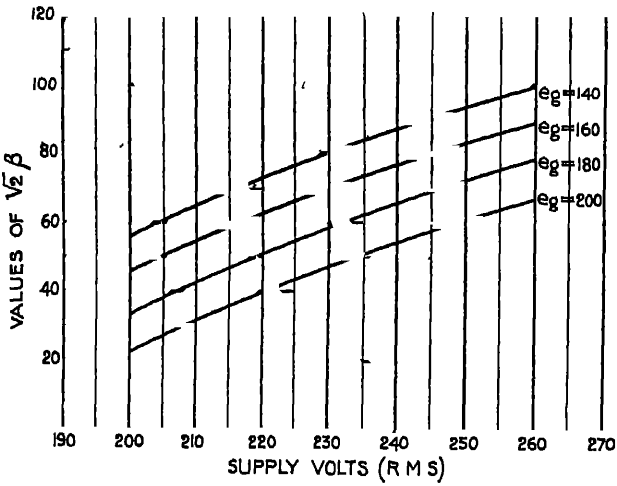

FIG. 251.—Curves of $\mathscr{E}$ and $\sqrt{2}\beta$.

$$\mathscr{I}^2 = \frac{1}{\left(\frac{1}{A_LM} + R\right)^2} \cdot \frac{1}{2\pi}\int_{\theta_a}^{\pi-\theta_a}(E \sin \theta - e_g)^2 d\theta,$$

or when both the large and small loops are considered,

$$\mathscr{I} = \sqrt{2}\beta\sqrt{\frac{1}{\left(\frac{1}{A_LM}+R\right)^2} + \frac{1}{\left(\frac{1}{A_SM}+R\right)^2}}.$$

As the second term in the square root is usually small, it may be neglected, and

$$\mathscr{I} = \frac{\sqrt{2}\beta}{\frac{1}{A_LM} + R}$$

where $\beta^2 = \left\{\frac{E^2}{4}\left(\frac{1}{2} - \frac{\theta_a}{\pi} + \frac{\sin 2\theta_a}{2\pi}\right) - \frac{Ee_g \cos \theta_a}{\pi} + e_g{}^2\left(\frac{1}{2} - \frac{\theta_a}{2\pi}\right)\right\}$.

Values of $\sqrt{2}\beta$ plotted in a similar fashion are given in Fig. 251.

The form factor of the wave is thus

$$\frac{\sqrt{2}\beta}{\delta}$$

and is usually in the neighbourhood of 1·9, to 2·0 indicating a wave which has a pronounced crest (see page 28).

Power.—The power taken by the rectified circuit cannot be assumed to be the product of the effective current and voltage, but it may be calculated by integrating the product

$$\frac{1}{\frac{1}{A_LM}+R} \cdot \frac{1}{2\pi}\int_{\theta_a}^{\pi-\theta_a}(E \sin \theta)(E \sin \theta - e_g)d\theta,$$

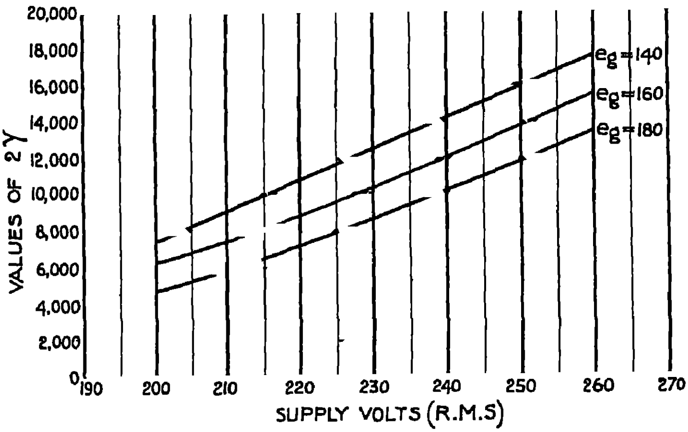

Fig. 252.—Curves of $\mathscr{E}$ and 2γ.

which results in the true power

$$P = \frac{2\gamma}{\frac{1}{A_LM} + R}.$$

neglecting terms containing $\frac{1}{A_sM}$, and where

$$\gamma = \left\{\frac{E^2}{4}\left(\frac{1}{2} - \frac{\theta_a}{\pi} + \frac{\sin 2\theta_a}{\pi}\right) - \frac{Ee_g \cos \theta_a}{2\pi}\right\}.$$

Values of 2γ are plotted in Fig. 252, and it is curious that this equation for P may be written in the form

$$P = \left(\frac{1}{A_L M} + R\right) \mathscr{I}^2 + e_g I_M$$
$$= 670 \ \mathscr{I}^2 + 160 \ I_M$$

for the above case where the load resistance is 500 ohms. Hence by inserting a moving iron and a moving coil ammeter in circuit the power can be calculated without the use of a voltmeter, and moreover the true power is measured by such means.

Use on Battery Circuits.—If this device is used in conjunction with any load which supplies a counter E.M.F. e it must be remembered that for e_g must be substituted $(e_g + e)$ in the first half cycle and $(e_g - e)$ in the second.

The following Table XXIX. has been calculated for a typical rectifier, and checks out with experimental results with reasonable accuracy :—

TABLE XXIX.

R.M.S. Voltage.	No Counter E.M.F. $e = 0$. $R = 0$. Amperes (I_M).	10 Cell Battery. $e = 20$. $R = 10$. Amperes (I_M).	25 Cell Battery. $e = 50$. $R = 10$. Amperes (I_M).
200	0·144	0·103	0·049
210	0·167	0·125	0·068
220	0·187	0·149	0·089
230	0·212	0 169	0·111
240	0·235	0·192	0·132
250	0·258	0·215	0·158

where $1/A_L M = 170$, $1/A_S M = 10^4$, and $e_g = 160$.

Finally, the following curves in Fig. 253 indicate the wave form to be expected and show that the rectification factor is considerable.

Starting Voltage.—In the above calculations it has been assumed that the current-voltage characteristic is linear or, in other words, that M is constant. In practice this is not the

case and the curvature of the characteristic must be taken into account if accurate results are required. In the neon lamp this error is not serious because the value of R is usually large compared with $1/AM$; but if greater accuracy is required it is necessary to conduct a series of experiments with the actual tubes which will be used, as the gas pressure in the bulb has a large effect on the characteristics. Thus if any one tube is tested on a direct current circuit its starting voltage e_g and the going-out voltage e'_g will be obtainable, from which the value of θ_a can be ascertained, and the value of $1/AM$ calculated.

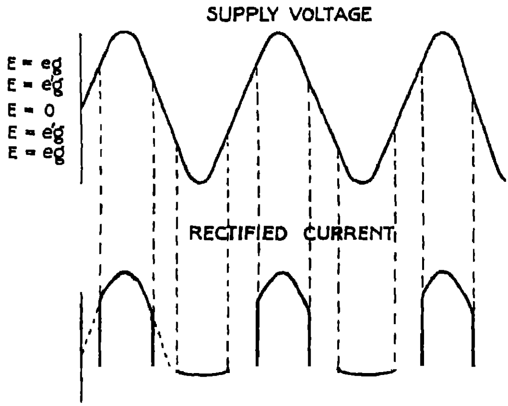

FIG. 258.—Wave form of neon rectifier.

Usually the difference between e_g and e'_g is only about 10 to 20 volts, but if it is desired to differentiate between the two voltages, the limits of the integrals will be corrected as follows:—

$$\int_{\theta_a}^{\pi-\theta_a} \quad \text{becoming} \quad \int_{\theta_a}^{\pi-\theta_b} \quad \text{etc.}$$

where $\theta_a = \sin^{-1}\dfrac{e_g}{E}$ and $\theta_b = \sin^{-1}\dfrac{e_g'}{E}$.

One of the difficulties experienced with these rectifiers is the increase in the value of the starting voltage. Schulze has shown that it may attain the values given in Table XXX.

TABLE XXX.

Time in Hours	Starting Voltage.
0	192
71	206
130	260
200	330
440	343
700	343

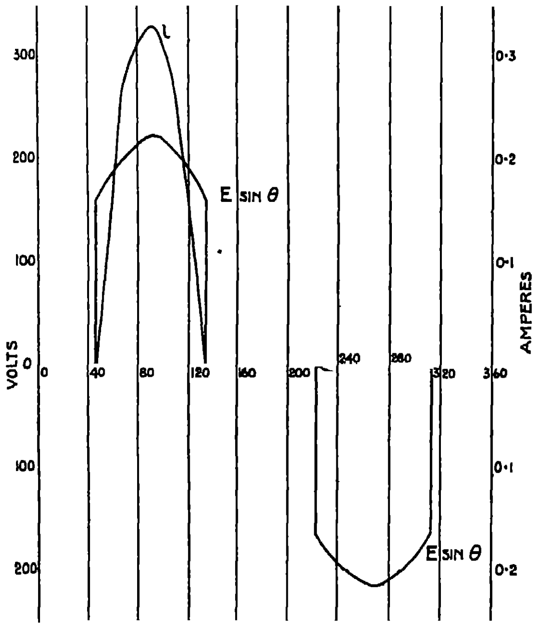

FIG. 254.—Voltage and current waves of a neon rectifier.

Numerical Example.—It is instructive to carry out an harmonic analysis of the wave form, and in the following example numerical values for the constants are assumed to be

$$e_g = e'_g = 160 \text{ volts.}$$
$$1/A_L M = 170.$$
$$1/A_S M = 10^4.$$
$$E = 220 \text{ volts.}$$
$$R = 10 \text{ ohms,}$$

whence $\theta_a = \sin^{-1}\dfrac{160}{220} = 46{\cdot}5°.$

Fig. 254 shows the voltage and current waves across the load, and if the current wave is analysed by the method of Chapter II, page 55, the following series is obtained :—

$$i = 0{\cdot}420 + 0{\cdot}105 \sin\theta - 0{\cdot}0848 \cos 2\theta - 0{\cdot}056 \sin 3\theta + 0{\cdot}042 \cos 4\theta + 0{\cdot}013 \sin 5\theta + 0{\cdot}001 \cos 6\theta + 0{\cdot}007 \sin 7\theta \ldots$$

The form factor is $\dfrac{\sqrt{2}\beta}{\delta} = 2{\cdot}16.$

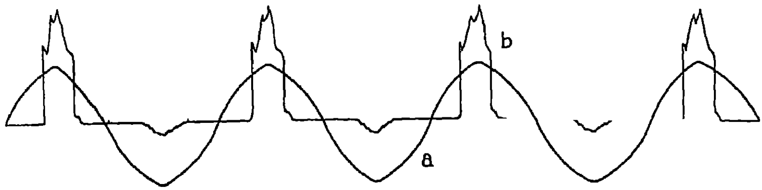

FIG. 255.—Oscillogram of current wave from neon rectifier.

An actual oscillogram results in a wave form almost identical with that of Fig. 254 where the current is led to a non-inductive load. The oscillogram is reproduced in Fig. 255.

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LOW VOLTAGE ARC TUBES.

Description.—This rectifier differs essentially from any yet considered; it consists of a glass bulb which is filled with an inert gas, usually argon, to a pressure of about 3 to 8 mm., and a filament F which is situated near a graphite anode A which is connected to one terminal B, the filament being supplied with current through an ordinary E.S. lampholder as shown in Fig. 256.

Theory of Operation.—In the case of a thermionic rectifier the production of a high vacuum ensures stability and consistency of results owing to the absence of ionisation by collision; but, on the other hand, a greater current may be carried for the same anode voltage if gas molecules are introduced into the bulb in sufficient quantities. The presence of positively charged atoms, which are the product of ionisation by collision, will partially neutralise the space charge, and reduce the requisite anode voltage for a given current. When the blue glow appears which evidences the presence of gas, rapid disintegration of the cathode or hot electrode always takes place. What has been said is subject to certain important qualifications and it is not at all possible to generalise on the effect of the presence of gas; for instance, oxygen, whether in the free state or in combination

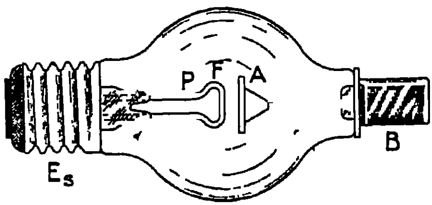


FIG. 256.—Single-phase rectifier bulb.

with other gases has the effect of cutting down the emission of electrons from the cathode—on the other hand, argon in the pure state, and generally the inert gases, have the effect of curtailing the disintegration of the cathode, whilst the addition of some impurity may have the opposite effect. Manufacturing a rectifier to work in a bulb with a gas pressure of the order of 1 to 10 mm. provides a problem difficult of solution, which in the shape of the Tungar rectifier was arrived at after many years of experimentation by the General Electric Ltd. of America, and a similar form the "Rectigon," which was evolved by the Westinghouse Co. of America, and the Philips, developed in Holland.

Gas Pressure in the Bulb.—The bombardment of the cathode by the positively charged atoms may be of such intensity, and the atoms have such a momentum, as to dis-

integrate the cathode surface which suffers rapid erosion in consequence; but if the gas pressure is increased there comes a time when their velocity is impeded and their momenta at the time of impact with the cathode so far reduced as to render them innocuous so far as disintegration is concerned; but here again care must be taken to choose exactly the correct gas pressure, because as the gas pressure is increased the number of molecules present is increased, and therefore although the velocity component of the momentum is reduced the mass component is increased. There is therefore an optimum point on the gas pressure curve where disintegration is a minimum and the anode current a maximum, and it is the aim of the designer of the rectifier to fill the bulbs to that pressure

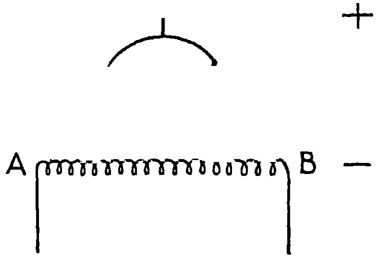


Fig. 257.—Anode and cathode in rectifier.

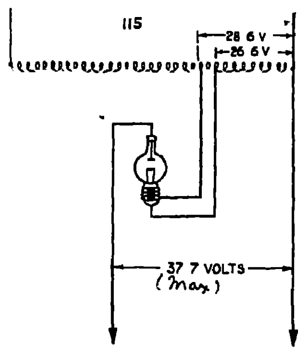


Fig. 258. — Connections for single-phase rectifier.

Dr. Langmuir has experimented with various gases in such tubes and their effect on the electron emission from the filament, and he found that argon was the only gas amongst those tried, including hydrogen, water vapour, nitrogen, etc., where the emission was not reduced by the presence of gas molecules.

Starting of Rectifier.—The rectifier is self-starting when the filament is heated by the passage of current, as free interchange of electrons can take place between neighbouring atoms, some of which are attracted to the anode by the electric field. The low voltage arc, by which conduction takes place, is thus formed.

Potential Gradient along the Filament.—As this form of rectifier is one which supplies a relatively large current at a low anode voltage, it will be apparent (Fig. 257) that owing to the potential gradient along the filament from A to B due to its resistance, there will be a varying emission from the filament to the graphite anode. Thus the filament will appear to be more brightly illuminated at A than at B, an effect which is mentioned on page 330 in the case of vacuum tubes. When the anode current exceeds a certain value (usually about one ampere) a vivid glow becomes apparent due to ionisation by collision (Chapter VII., page 164), and if the filament current is discontinued at this point the rectifier will continue to function

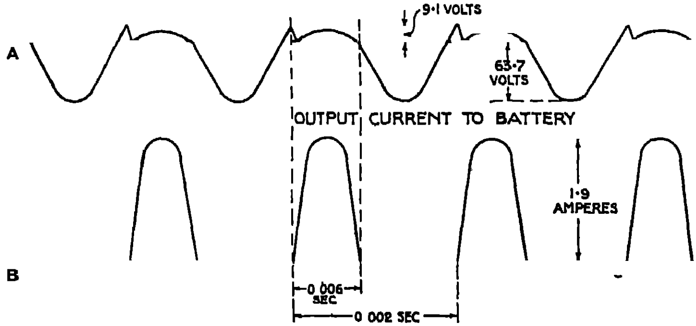


FIG. 259.—Current and voltage waves from rectifier.

correctly as the bombardment of the filament by the positive ions is of sufficient intensity to retain the filament at a high temperature. If the rectifier is required to be self-starting, however, the filament must be kept glowing.

Transformers.—With the equipment is supplied an auto-transformer connected as shown in Fig. 258. The transformer has two tappings at two volt intervals to supply the filament current, and the main supply is taken from one end of the transformer to the 26·6 volt tapping. The winding must be therefore carefully graded to carry the various currents continuously. With this tapping the maximum rectified voltage will be $\sqrt{2} \times 26{\cdot}6 = 37{\cdot}7$ volts.

Life of Bulb.—As regards the life of the bulbs, the manufacturers claim that the expected life averages 1500 to 2000 hours, but that in isolated cases rectifiers have lasted for from 6000 to 7000 hours.

Obviously the life will depend on a number of factors, and on manufacturing and working conditions. The upper limit of 2000 hours is a normal figure to expect and might with reason form the basis of an estimate.

Current and Voltage Waves.—The general shape of the current and voltage waves obtained is shown in Fig. 259, where B represents the rectified current wave charging a 24 volt

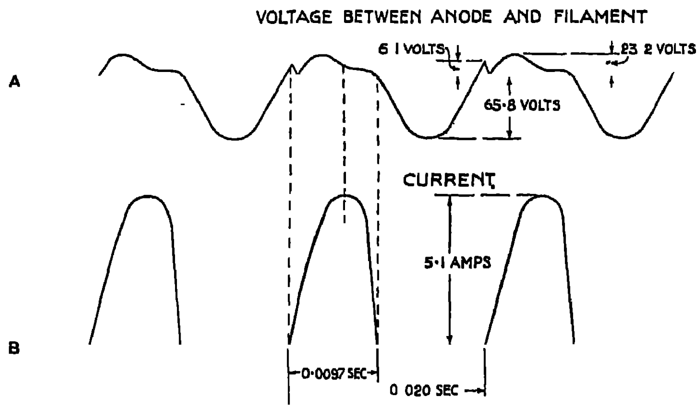


FIG. 260.—Oscillograms of rectifier with no filament current.

battery, and A the voltage wave across the rectifier terminals. During the first half cycle no current is flowing and the full voltage is obtained across the bulb; in the second half cycle there is a big drop owing to the resistance of the bulb.

Similar oscillograms for a rectifier working with the filament disconnected are given in Fig. 260.

In Fig. 261 oscillograms are given of the arc voltage and current of one of the two anodes of the Philips' rectifier, showing the drop of voltage due to the load current, and its sudden fall when the arc is struck. This characteristic of the rectifier renders its complete mathematical analysis difficult, as the

striking voltage is not so definite a point of discontinuity as in the other forms of rectifier.

One of these rectifiers (the Philips) is illustrated in Fig. 262, and consists of a bulb of the double anode type, thus providing biphase rectification, a transformer and a ballast resistance. The diagram of connections of the Philips' rectifier is given in

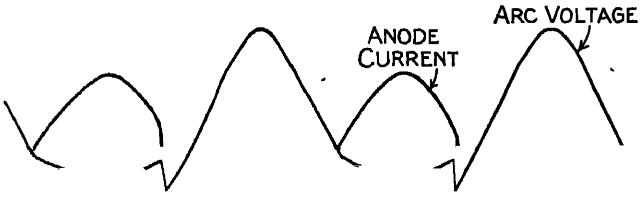


FIG. 261.—Oscillograms of 1·3 ampere Philips' rectifier.

Fig. 263, and it will be seen that the resistance is placed at the neutral point of the transformer. This resistance forms one of the chief features of the rectifier, and consists of an iron wire in an atmosphere of hydrogen. Thus in Fig. 262, one of the bulbs is the rectifying element, and the other the resistance. The essential reason for the inclusion of this resistance is the

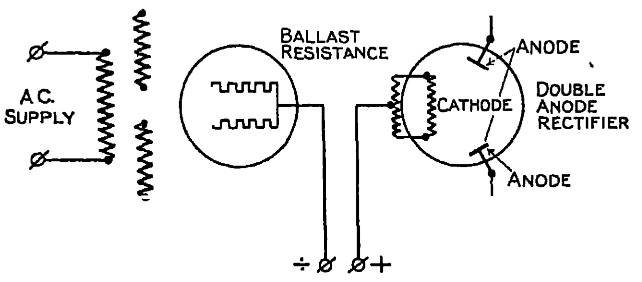


FIG. 263.—Diagram of connection of Philips' rectifier.

difference which exists between the ionisation voltage (16 volts) and the working voltage (6-7 volts), and its function is to stabilise the discharge under the two conditions. It also has the additional advantage of rendering the operation of the apparatus possible on wide variations of supply pressure and frequency. At the same time the characteristic curve of this

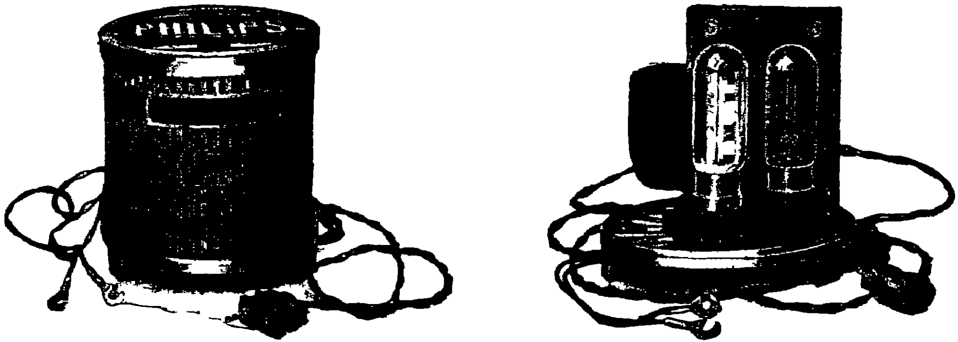


FIG. 262.—Philips' rectifier.

[*To face page* 364.

resistance, which is given in Fig. 264, shows that within certain limits, it will automatically regulate the rectified current to practically a constant value.

Theoretical Analysis.—No direct method is given of obtaining the actual relationship of voltage to current but test curves of the voltage drop show that the rectifier acts approximately as a simple resistance between certain limits (page 366). Assuming that this is true and that an inductance is inserted in the

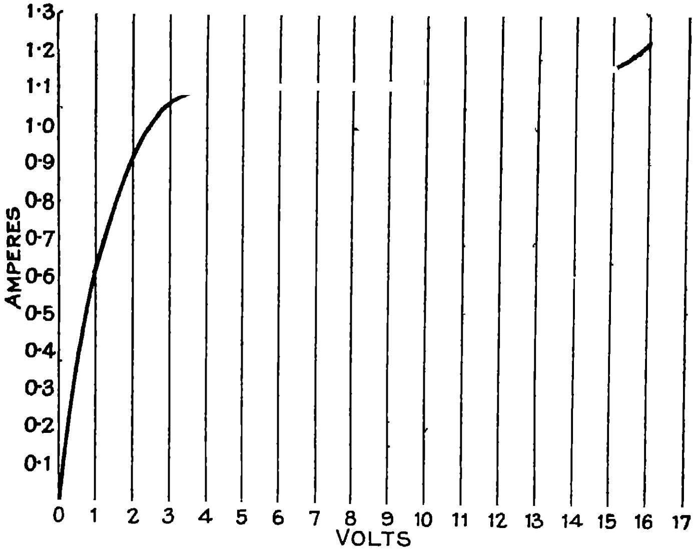


FIG. 264.—Characteristics of iron wire resistance of Philips' rectifier.

rectified circuit, and further that the rectifier is only working between these limits, the equation connecting the various quantities is

$$E \sin \theta = ri + x\frac{di}{d\theta} + Ri + R'i \qquad . \quad . \quad (2)$$

for the circuit illustrated in Fig. 265, where R' is the resistance of the tube. This equation can be written in the form

$$\frac{di}{d\theta} + \frac{r_2 i}{x} = \frac{E}{x}\sin \theta \qquad . \quad . \quad . \quad . \quad (3)$$

where $$r_2 = r + R + R'.$$

The solution of (3) is of the form

$$i = A\epsilon^{-\frac{r_2}{x}\theta} + \frac{E}{Z_1}\sin(\theta - \phi),$$

where $$Z_1 = (r + R + R' - jx)$$

and $$\tan\phi = \frac{x}{r_2}.$$

The terminal conditions of $i = 0$ when $\theta = 0$ give a value for A of

$$A = \frac{E}{Z_1}\sin\phi,$$

whence

$$i = \frac{E}{Z_1}\left\{\epsilon^{-\frac{r_2}{x}\theta}\sin\phi + \sin(\theta - \phi)\right\}. \qquad . \qquad . \qquad (4)$$

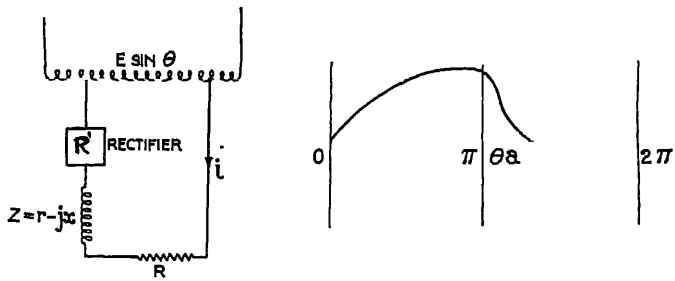


FIG. 265 *a* and *b* —Rectifier with inductance in circuit.

The angle of overlap θ_a may be obtained by putting in the other terminal conditions

$$i = 0 \text{ when } \theta = \pi + \theta_a,$$

whence an equation for θ_a may be obtained—

$$\sin\phi \,.\, \epsilon^{-\frac{r_2}{x}(\pi + \theta_a)} = \sin(\phi - \theta_a)$$

from which θ_a may be calculated.

The wave form expected is illustrated in Fig. 266 *b*.

The actual effect of inductance on the wave form is indicated in Fig. 266 where (*a*) is a copy of an oscillogram of current and voltage on a non-inductive load with a biphase arrangement, and (*b*) a three-phase circuit on no load, and (*c*) a similar circuit on a non-inductive full load.

Resistance of Tube.—The resistance of the rectifier does not, however, strictly obey Ohm's Law, and in effect varies with the load current.

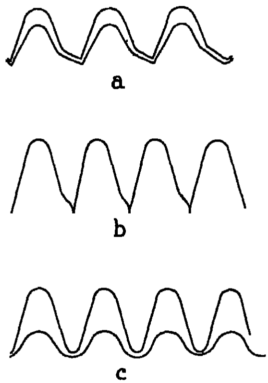


FIG. 266 *a*, *b*, and *c*.—Wave forms of current in rectifier.

The resistance-current curve at different loads is shown in Fig. 267, and the approximate temperature of the filament is indicated by the stated colour of the filament.

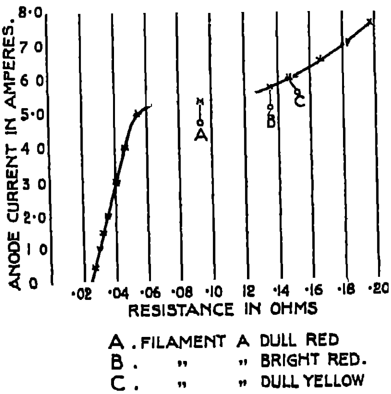


FIG. 267.—Resistance load curves of a rectifier.

The characteristic curves with the filament connected and also disconnected from the supply are illustrated in Fig. 268 in curves A and B respectively, and it will be seen that after a

certain limit the current may increase with a constant voltage drop across the tube. The case is thus analogous at this point to that of the mercury vapour rectifier, and equation (3) can be written in the form

$$E \sin \theta = ri + x\frac{di}{d\theta} + Ri + e_a$$

and if

$$r + R = r_3$$

$$\frac{di}{d\theta} + \frac{r_3}{x}i = \frac{E}{x}\sin\theta - e_a \quad . \quad . \quad . \quad (5)$$

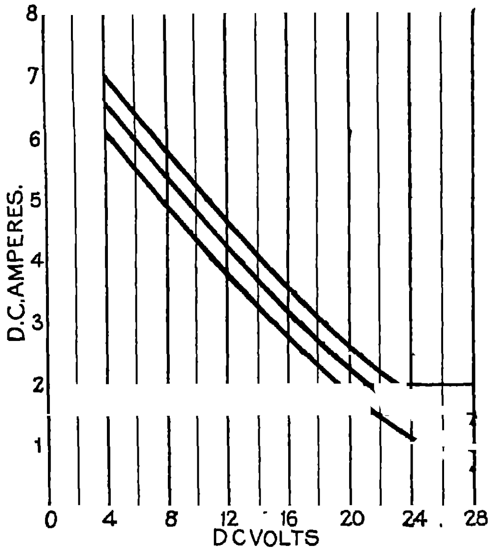


FIG. 268.—Voltage regulation of 7·5-15 volt tube.

The solution of equation (5) is

$$i = A\epsilon^{-\frac{r_3}{x}\theta} + \frac{E}{Z_3}\sin(\theta - \psi)\frac{e_a}{r_3} \quad . \quad . \quad . \quad (6)$$

where $$Z_3 = (r + R - jx) \text{ and } \tan\psi = \frac{x}{r_3}$$

and the terminal conditions may be arrived at as in the case of the previous example.

Polyphase Working.—These rectifiers adapt themselves to biphase or polyphase systems in the same way as other types of rectifier plants; and a biphase system will have a diagram of connections as shown in Fig. 269, oscillograms of which have been given in Figs. 266 *a*, *b* and *c*.

If desired an impedance Z may be inserted to smooth the rectified current wave, and if this is done with the volt-ampere characteristic assumption above, the circuit may be analysed in exactly the same way as that for the mercury vapour rectifier (see Chapter VIII., page 203), with the reservation that the volt drop across the tube may comply either with equations (4) or (6) according to the loading.

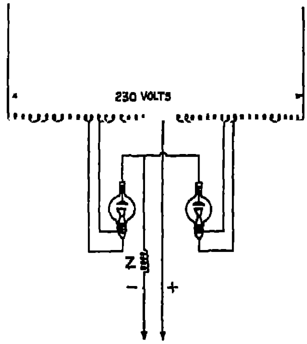


FIG. 269.—Tungar rectifier—biphase working.

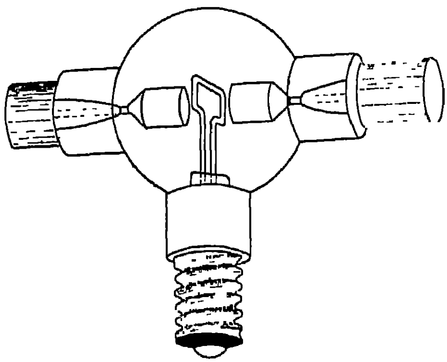
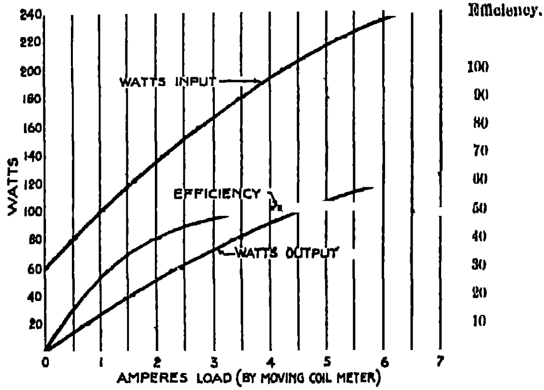


FIG. 270.—Tungar rectifier—double anode bulb.

The biphase arrangement may be simplified by using one bulb containing two anodes such as is illustrated in Fig. 270, and the method of operation is identical with that depicted in Fig. 269 above.

Characteristic curves are given in Fig. 271 *a* and *b* for the overall performance on single or biphase working, where the ordinates represent the rectified load. As the voltage on the load side

bears an approximately constant relation to the peak voltage of the auto-transformer, the value for this D.C. voltage may for all practical purposes be taken as between 37 and 38, the amount of the voltage drop varying from 7 to 15 volts.



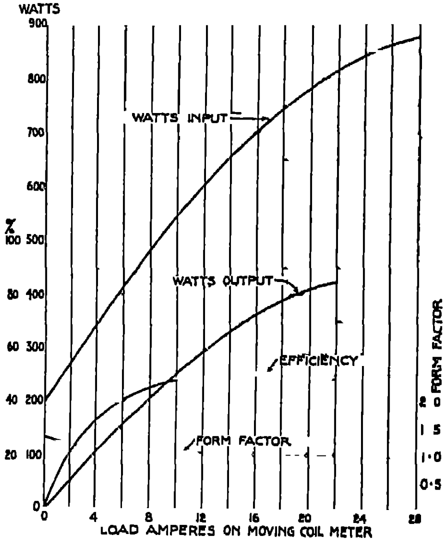


FIG. 271 *a* and *b*.—Characteristics of rectifier.

Effect of Frequency.—To show the effect of frequency, curves are given in Fig. 272 which indicate the variation in output with supplies of 60 and 100 cycles. The difference is

not due so much to the rectifier as to the transformer, but it is seen that the balance is in favour of working with a low supply frequency where possible.

Battery Charging.—It is interesting to note how the rectifier will behave when used on a circuit with a counter E.M.F. such as a battery charging load, where the number of cells used in series may be varied from 5 to 8. In this case the impedance Z is assumed to be zero, as there is little necessity for any wave smoothing. The charging current will then be a simple sinusoidal function, which commences when the supply voltage less

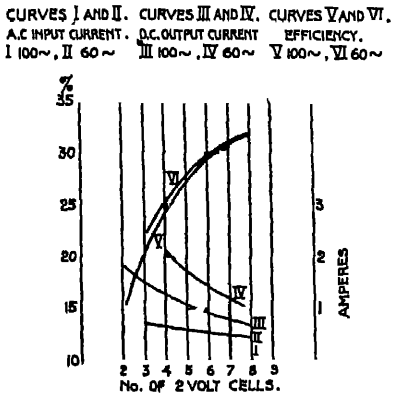


FIG. 272.—Battery charger.

A.C. volts 230. D.C. amperes 1-2.
D.C. volts 7·5-15. Cycles 60 and 100.

the voltage drop in the bulb, exceeds the counter E.M.F. of the battery. Thus the angle of cut-off is given by

$$\sin \theta_a = \frac{e}{E},$$

assuming that the volt drop in the tube is constant. Also

$$E \sin \theta - e_o - e_a - iR = 0,$$

where e_a is the drop in the tube, e_o is the counter E.M.F. and R is the external resistance. Write

$$e = e_a + e_o.$$

Therefore
$$i = \frac{E \sin \theta - e}{R}$$

and
$$I_M = \frac{1}{2\pi}\int_{\theta_a}^{\pi-\theta_a} \frac{E \sin \theta - e}{R} d\theta,$$

whence
$$I_M = \frac{1}{2\pi R}\{2E \cos \theta_a - \pi e + 2e\theta_a\} \quad . \quad . \quad (7)$$

Now $e_o = 2{\cdot}2\ n$ approximately if n is the number of cells, and $E = 37{\cdot}7$ and $e_a = 11$ volts.

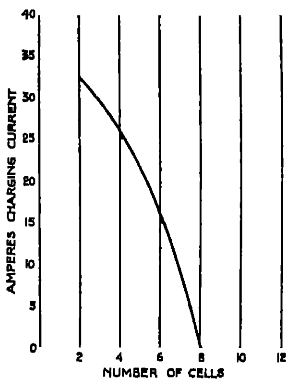


FIG. 273.—Number of cells v. charging current.

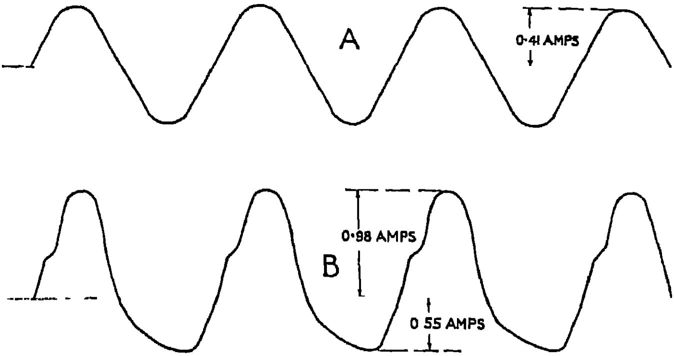
This curve, viz. equation (7), is plotted in Fig. 273, from which it will be seen that the maximum number of cells which can be charged is 8. The curve is plotted for a total external resistance of 1 ohm. If this resistance is higher the current will be proportionately reduced, but equation (7) shows that even if this is the case only the same number of cells can be charged. Normally this current of 30 amperes shown would not be permitted, the rheostat included being adjusted to suit the best charging current for the capacity of the cells.

The above figures are only to be used in the case of the low voltage rectifier. Another type making use of a different transformer ratio is available for higher voltages up to 75, in which case the figure of 37·7 should be substituted by 90 volts.

Primary Current.—The input current to the rectifier is naturally unsymmetrical in form. This is indicated in the oscillograms reproduced in Fig. 274 where curve A represents the transformer primary current on no-load and B with a battery on charge at 0·26 amperes and 24 volts.

Certain of these types of rectifier have been developed rapidly in the past few years, and are now manufactured in

such sizes as will enable comparatively large batteries to be charged efficiently. At the same time the voltage has been increased so that accumulators up to 15 cells can be charged in series entailing a voltage of at least 42 volts across the battery. A current capacity of about 12 amperes can also be obtained.



A . BATTERY DISCONNECTED = 115 VOLTS . 50 ~
B . ,, ON CHARGE . LOAD = 0·26 AMPS AT 24 VOLTS .

FIG. 274.—Current wave on primary of transformer.

With convenient transformer connections, a three-phase rectifier can be constructed, and with middle tappings this can be converted into a hexaphase arrangement.

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"S" TUBES.

Theory of Operation.—A new type of rectifier on the principle of the gas tube, and which does not make use of thermionic emission, as both electrodes are cold, has been developed by Messrs. Bush and Smith.

This tube consists of a glass vessel filled to a pressure of about 0·1 mm. of mercury with some inert gas; and between the electrodes of which in some cases a magnetic field exists.

In a gas tube with no hot cathode, conduction takes place by ionisation by collision, but in order that an electron should ionise a neutral molecule it must be in a position to collide with one during its passage from the cathode to the anode. The chance of such a collision depends (i) on the number of molecules

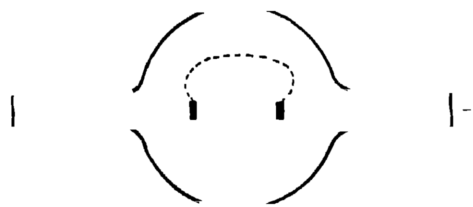

FIG. 275.—Large bulb with electrodes close together.

present, or in other words on the pressure of the gas in the tube, and (ii) on the potential gradient between the electrodes, as the velocity of the electrons must be sufficient to produce ionisation (Chapter VII., page 164). If the electrodes are spaced a distance apart which is less than the mean free path (which represents the average distance through which an electron moves before collision takes place) ionisation cannot take place and very poor conduction will result under normal conditions.

This is borne out by experiments on a bulb with electrodes 1 mm. apart and filled with helium to a pressure of 2 mm. A voltage of 10,000 failed to pass a measurable current. When the electrodes were withdrawn to a spacing of 2 to 3 centimetres, the tube broke down at about 300 volts.

If the bulb is big compared with the size of the electrodes, as in Fig. 275, such that although ionisation by collision does

not take place between the plates, yet the path of stray electrons near the edge of the electrodes follows the dotted line, a few molecules will be ionised on account of the increased length of travel. If now the size of the bulb is contracted until the electrodes just fit the interior, the stray electrons will not reach their objective but will stick to the sides of the glass which will accumulate a charge on the interior of the walls, and little conduction will result.

Consider the effect of a transverse magnetic field as in Fig. 276 on an electron which has a charge e and is travelling

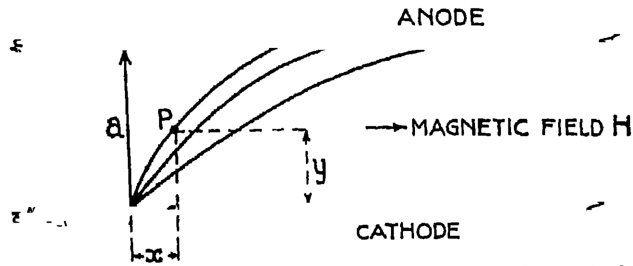

FIG. 276.—Path of an electron subject to a magnetic and electric field.

towards the anode with a potential gradient X between the electrodes. If m is the electron ratio of charge to mass

$$m\frac{d^2y}{dt^2} = eX - He\frac{dx}{dt},$$

and

$$m\frac{d^2x}{dt^2} = He\frac{dy}{dt},$$

which shows that the curve of travel is a cycloid.

Integrating the second equation

$$m\frac{dx}{dt} = Hey + C,$$

which with the terminal conditions results in $C = 0$, and

$$m\frac{dx}{dt} = Hey$$

and also by the principle of the Conservation of Energy

$$\frac{mv^2}{2} = eXy,$$

where v is the electron velocity.

Now if the electron is to pass the electrode towards which it is moving without touching it

$$\frac{dx}{dt} = v \text{ and } y = a \text{ and } E = Xa$$

or

$$H = \frac{\sqrt{2E}}{a\sqrt{\frac{e}{m}}}.$$

Filling in the value for e/m

$$H = \frac{3{\cdot}35\sqrt{E}}{a} \quad . \quad . \quad . \quad . \quad (8)$$

where H is the critical field strength in gausses and a is the distance apart of the electrodes in centimetres and E is in volts.

Therefore a condition that ionisation by collision may take place is that the electron shall just miss its objective and shall have its path extended so that a molecule may be encountered and so ionised. When this happens the positively charged molecule will return to the cathode and by bombardment, liberate other electrons which will find their way ultimately to the anode, and the condition of fulfilment of current passing is that of equation (8).

Hence if such a tube is constructed and is surrounded by a magnetic field which is gradually increased in strength, a point will be reached, viz. that of the critical field strength, where free conduction will take place, and theoretically there is no current limit. In practice the limiting factor is the heating of the electrodes and the leading-in wires, but comparatively heavy currents can be rectified for short periods by this device.

Rectification in practice is effected by an alternating field superposed on a unidirectional field and by adjusting the fields so that the resultant flux is correct for passing current during one half cycle, but not during the subsequent half cycle.

Such a tube may be constructed from the above data and the strength of the field calculated. But as in so many similar instances it is impossible to analyse the current-voltage relation as in all cases where ionisation by collision is the *modus operandi* uncertain factors must enter into the calculations.

Size of Bulbs.—Tubes have been constructed on this principle which act as perfect rectifiers, and one such 17 8 cm. long permitted a current flow of 230 milliamperes continuously with a voltage drop of 150 to 200 volts. Several amperes may be passed through the tube for short periods.

The characteristics will depend largely on the variable parameters of the tube design; in the above example the voltage regulation is 10 per cent. from no-load to full load.

By using suitably curved electrodes the combined alternating and direct fields may be substituted by a permanent magnet. Inert gases are the best to use for filling the bulb and with helium and aluminium electrodes a voltage drop of 150 volts may be expected.

Space Charge Control.—So far attention has been directed to control of conduction by an external magnetic field, but advances in design have been made by employing the space charge to effect a measure of control, tubes have been constructed which will pass currents up to 120 milliamperes with a voltage drop of about 175 volts. Such tubes have no external control and appear to have long lives on high voltage circuits.

This type of tube, which is illustrated in Fig. 277, would appear to be especially suited to small wireless transmitting sets, where unidirectional currents of the order of 100 milliamperes at 800 or 900 volts are required from an A.C. supply. As has been stated above no external control or filament supply are necessary.

The characteristics of these tubes are as follows:—

Starting voltage . . .	300 volts A.C.
Maximum „ . . .	1000 „ D.C.
Normal drop . . .	125 „
Internal resistance . .	1250 ohms.
Normal reverse current .	3-6 milliamperes.

The principle employed in the operation of these non-magnetically controlled rectifiers is by no means simple, but is bound up with the respective mass of electrons and positive ions. It must be remembered that in this class of tube, the electrodes are

included in a bulb filled with some inert gas at a predetermined pressure, and that when a potential difference is applied to the electrodes, the gas is ionised, and the electrons will travel much more rapidly in the one direction, than do the positive ions in the other. Thus there will be a preponderance of positive ions in the space between the electrodes. Due to the relative shape of the electrodes this cloud of positive ions is not removed at each half cycle, and rectification ensues.

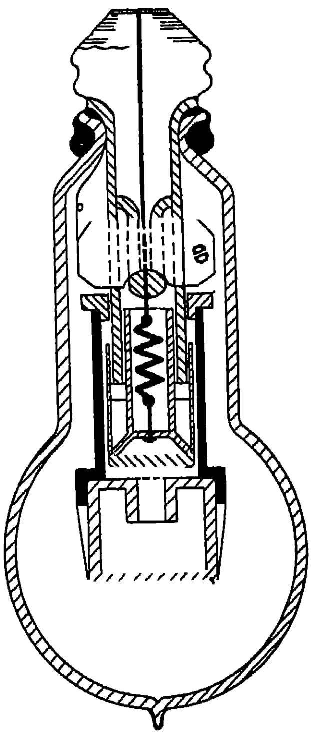

FIG. 277.—"S" tube.

Thus when the hollow electrode is made negative the tube will pass current, and under these conditions the positive ions are falling on to the interior surface of the hollow cathode, where they liberate electrons, which in turn are projected to the anode, ionising neutral gas molecules thus again forming positive ions. The electrons originally present, together with those liberated, pass through the hole in the cathode, and reaching the anode complete the circuit for conduction in one direction. When the potential is reversed, however, positive ions will pass comparatively slowly through the hole, and will impinge on the solid cathode. When an ion reaches the hole it may fall on the cathode and release an electron which will fly back to the positive ion cloud, but the current flowing will be small and limited to the number actually arriving at the anode.

By varying the method of connection the D.C. voltage may be increased by using transformers of a higher step-up ratio, and the characteristics under four various conditions are given in Fig. 278.

It should be noted that when "S" tubes are operated in parallel, balancing resistances of at least 500 ohms are required to equalise the load in each tube.

In Fig. 278 *a*, *b*, and *c*, a condenser is required across the load, of 0·5 microfarad capacity, but in Fig. 278 *d*, the capacity of the condensers in the bridge circuit, affects the load characteristics considerably, and the figures on the curves are indicated by the following circuit conditions :—

Curve	I	Condensers each 4 microfarads.
,,	II	,, ,, 1 microfarad.
,,	III	0·5 microfarad added across the load in II.
,,	IV	Voltage on no-load.
,,	V	Voltage on 100 milliampere load, and condensers each 4 microfarads.
,,	VI	Voltage on 100 milliampere load, and condensers each 1 microfarad.
,,	VII	Voltage on 100 milliampere load, condensers each 1 microfarad, and ½ microfarad across the load.

It will be apparent from what has been said of the development of these tubes, that it is easy to damage them by overload, and it is highly important that they should be treated carefully.

Some points of note are :—

(i) A resistance should always be inserted for one or two minutes before switching on the full load, to warm up the rectifier.

(ii) The rectifier is not a "current limiter" in the same sense as a thermionic rectifier has a saturation value, and if the applied voltage is increased the load current will also increase, and will be determined solely by the internal resistance of the tube, and the load resistance. Thus if the normal voltage is exceeded the tube may be destroyed.

(iii) The load should never be removed suddenly, as damage may result from too high a voltage being applied across the tube as well as too high a load current. If the transformer has a good voltage regulation this trouble will not be experienced.

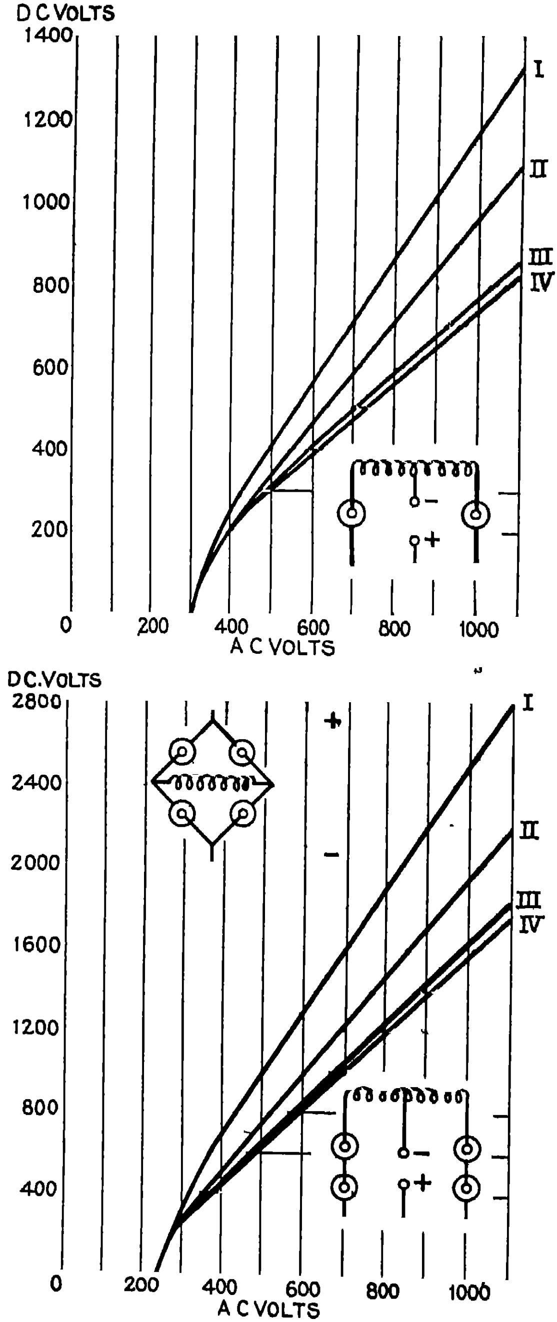

FIG. 278 *a* and *b*.—Characteristics of "S" tubes.

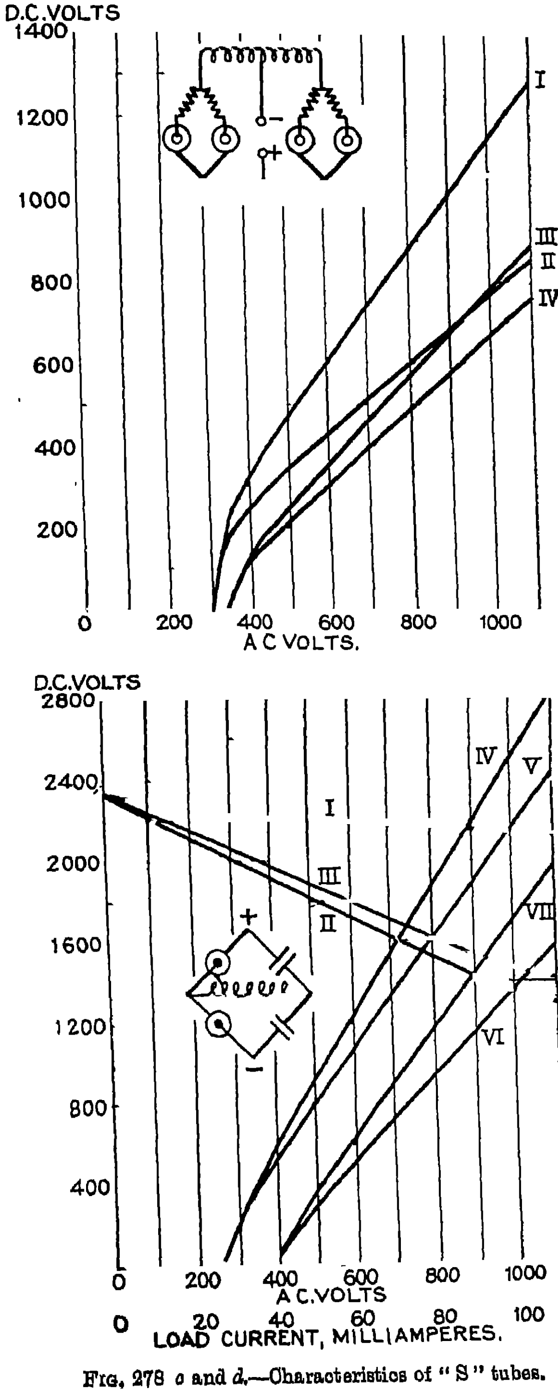

FIG. 278 *c* and *d*.—Characteristics of "S" tubes.

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CHAPTER XV.

POINT TO PLATE DISCHARGE, VIBRATING FLAME, CORONA AND PHOTO-ELECTRIC CELL RECTIFIERS, AND HALL EFFECT.

General Description.—In this chapter three devices for rectifying high voltages are described; they are unsuitable as at present constructed for large currents, but are simple to manufacture and might be of use for small powers and where apparatus of little cost is required. The disadvantages of these rectifiers is that they are liable to cause or to emphasise the effect of surges, and should therefore be employed with discretion where transformers, which may be liable to break down from waves with steep wave fronts, supply the E.H.T. voltage.

It is unfortunate that they are not more commonly employed, as especially in the case of the Corona rectifier no external supply is needed beyond that of the transformer. In the other two types very little extra apparatus is required, and all that is necessary can be manufactured in a laboratory workshop. But in the case of the Point to Plate rectifier a compressed air supply is also an essential constituent part and is not always available.

The fourth device—the Photo-electric Cell—is entirely different in its characteristics. This cell is only of value for the rectification of minute currents at relatively low voltages, and only has a practical application in wireless circuits and small current measurements, but so far has not been extensively employed for these purposes. It is therefore described in this chapter in the event of some special requirement rendering a description useful, rather than for any commercial utility it may possess.

Point to Plate Discharge Rectifier.

Construction of Apparatus.—It has been known for some time that an ordinary point to plate discharge has rectifying properties under certain conditions. Walcott and Erickson describe such an arrangement where a needle point and plate with a strong current of air blowing on to them are connected to an alternating current supply as shown in Fig. 279. Compressed air is delivered to the tube B through the nozzle E and passes through the tube parallel to the needle and finally impinges on to the plate. The actual position of the needle appears to be immaterial, but if it is situated inside the tube, the tube itself must consist of some insulating material, or be internally lined with such.

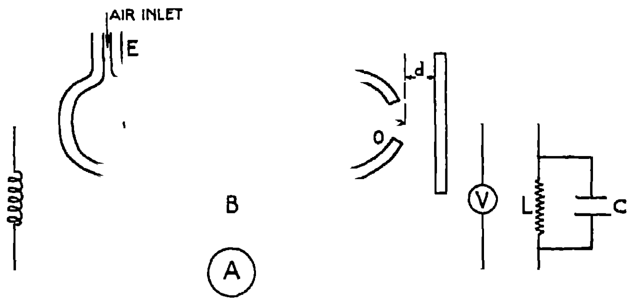

Fig. 279.—Point to plate discharge rectifier.

In one particular instance the diameter of the plate was 4 inches, and the aperture at O, 0·157 inch. The diameter of the electrode from which the needle was made was 0·0625 inch, and the dimension *d*, viz. the distance of the point from the plate, 0·68 inch. A condenser C consisting of eleven plates 8 feet by 3 feet, and spaced 8 inches apart (= 0·00095 microfarad) was connected across the load L. The transformer was wound for a secondary pressure of 40,000 volts at 60 cycles per second.

Wave Form.—Fig. 280 indicates that with no air blast no rectification ensues, and the ordinary type of A.C. spark discharge takes place. When air issues from the tube with a sufficient velocity, the current wave is rectified as shown in

Fig. 281, the condenser being disconnected at the time the oscillograms were taken.

In this case the rectified current obtained was 12·3 milliamperes at 14·5 K.V.; a heavier current is obtained by using the condenser, but the wave form is more peaky in character (page 418). 104 milliamperes at 15·2 K.V. are delivered to the load under these circumstances.

FIG. 280.—Wave form with no air blast—point to plate rectifier.

Transformer Surges.—In rectifiers of this type where a definite spark discharge is the medium by which the current passes it is important that the transformer should be designed so as to reduce the possibility of a breakdown to a minimum, as a surge is almost sure to occur to a greater or less extent depending on the circuit constants.

Thus a surge, which consists usually of a wave with a steep front, will result in a big potential gradient between successive turns of the secondary winding, but on account of its high frequency it will probably not penetrate more than a few turns, and it is advisable to strengthen the insulation of these turns.

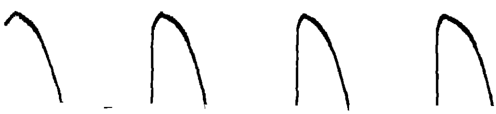

FIG. 281.—Wave form with air blast.

On the other hand, by so doing the capacity between the layers of the secondary will be affected, and as surges occasionally travel via this capacity, consisting as they do of a train of high frequency waves, it is essential to grade the capacity accordingly. The only safe method when surges are deliberately introduced into the circuit is to provide a suitable discharge in the shape of a definite load directly across the secondary terminals. The

glycerine-copper sulphate resistance described on page 136 is well adapted for this purpose. It may necessitate a larger transformer but will most probably result in the saving of expense later.

Effect of Pressure.—Cohen has conducted some experiments on point to plate discharges, but in his apparatus the chamber in which the electrodes of aluminium and copper were situated was filled with various gases and kept up to a constant pressure, which in certain cases rose to a value of 200 pounds per square inch.

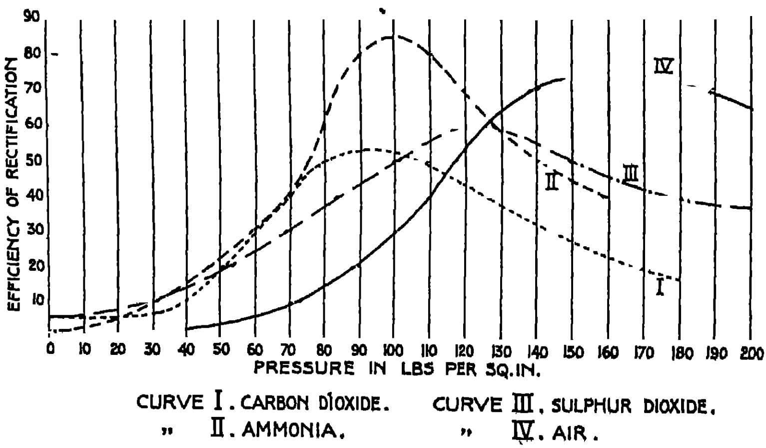

FIG. 282.—Point to plate rectifier—efficiency.

The supply was from a transformer giving a secondary voltage of 20,000 R.M.S. and the curve below in Fig. 282 indicates the efficiency of rectification, although no data is available as to the actual amount of power rectified.

Uses of Method.—As so few types of rectifier plant exist which can be made at a low cost and which will satisfactorily cope with high voltage supplies, it is unfortunate that the data are so meagre in a method which promises so well; and as there is a wide field for experimentation it is to be anticipated that some design will ere long be evolved which will enable rectifiers to be constructed for high voltages and large powers. There

appears to be no fundamental reason why currents of the order of 0·1 ampere at voltages of 100 K.V. should not be capable of treatment by these means.

The mathematical analysis has not been attempted as the number of variables on which the characteristics depend, such as atmospheric pressure, temperature, humidity, etc., are outside the control of the operator.

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	Cohen	El. World.	1919, p. 573.
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Vibrating Flame Rectifier.

It has been mentioned in Chapter VII. (page 167) that flames are capable of emitting free electrons and hence if two electrodes are placed a distance apart, so adjusted as to preclude the passage of a spark, and a flame is then allowed to approach the electrodes, ionisation by collision will ensue, and an electron stream will flow from the cathode to the anode. This, however, will not take place until the potential gradient reaches a predetermined value, and thus a point of cut-off will be apparent on the alternating current wave below which conduction will not take place.

Incomplete rectification can take place in properly designed apparatus, and J. J. Dowling and J. T. Harris have constructed a simple mechanism which is illustrated in Fig. 283. A flame from an ordinary gas supply is caused to oscillate with one-half synchronous frequency between the electrodes, and a supply of electrons is thereby available during one half cycle, when the voltage is of the right polarity.

The rectifier contains a Koenig's manometric capsule M, attached to the diaphragm of which is a soft iron armature P.

An electromagnet N supplied from the main A.C. supply attracts this armature.

It is important to note that if rectification is to ensue the flame must only vibrate once every cycle, and hence it is necessary to polarise the electromagnet, as otherwise the vibration would occur every half cycle. A steady direct current is therefore passed round the coils in addition to the A.C. supply current, and this is controlled by the rheostat R.

It has been found necessary to select a jet so that the height of the flame will be from 8 to 10 centimetres; whilst the gas

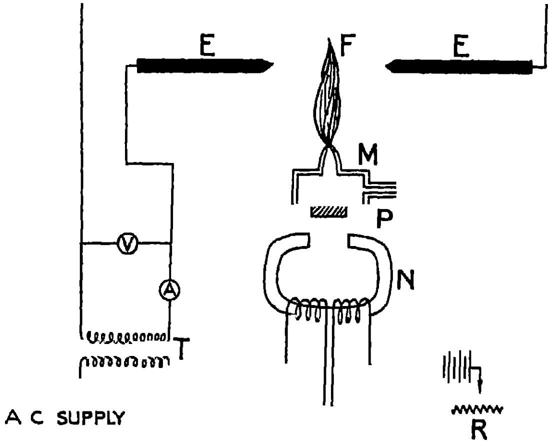

FIG. 283.—Vibrating flame rectifier.

supply tube should be lightly plugged with cotton wool to ensure a steady flow of gas. With a capsule 2 cubic centimetres in volume and a jet tube of 1 cubic centimetre, making 3 cubic centimetres capacity in all, the flame oscillates between heights of 1 and 10 centimetres, and the flame height when the magnet is disconnected is 9 centimetres.

When 6000 volts R.M.S. is applied to the spark gap EE which consists of two carbon rods, the reading on the moving iron milliammeter is 20 milliamperes, showing that the rectified power amounts to about 40 watts.

No oscillogram is available as to the wave form obtained.

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CORONA RECTIFIER.

Corona is a phenomenon manifested when an electrode in the form of a fine wire is subjected to a high potential above its surroundings. If the voltage is increased it is the forerunner of a breakdown of the gap; but if the voltage is not increased it may persist indefinitely as a glow discharge in the form of cylindrical luminous tufts surrounding the wire.

In Chapter VII. (page 166) the cause of the phenomenon is explained and is shown to be due to a state of ionisation by collision which is taking place at a critical voltage, where the potential gradient at the surface of the electrode is sufficient to cause ionisation of the molecules in the immediate vicinity of the electrodes, but not sufficient to cause complete ionisation for the whole length of the gap.

The use of corona for effecting high voltage rectification may appear to be an abnormal method and one unlikely to lead to any useful results; nevertheless the efficiency of such a method is extraordinarily high, although, as might be expected, the rectified currents are not usually large enough to be of much commercial value.

Very little work has been done on the subject, but Davis has carried out some interesting experiments on this particular phase of high voltage dielectric phenomena.

It will be expected that as the voltage at which corona appears depends on the potential gradient, it will largely be affected by the radius of the electrode. Further, that as ionisation by collision entails a passage of current, a loss of energy will ensue, unless the electrode is of sufficient dimensions.

This loss of energy is the medium by which the rectified current flows, but it should be restricted to the rectifier chamber. F. W. Peek has demonstrated the effect of the diameter of the wire electrode and has evolved the following empirical formulæ connecting the variables concerned :—

$$E = E_0 p + C\sqrt{p}$$

and

$$E = E_0 p\left(1 + \frac{b}{\sqrt{pr}}\right)$$

where E is the critical intensity or intensity of the electric field at the conductor surface, p is the pressure in percentage of atmospheric pressure, r is the radius of the wire, and E_0, b, and C are constants, which amongst other things depend on the surrounding dielectric.

Corona between a positive conductor and a negative shell is termed positive corona, and conversely; it is apparent from investigation of the oscillographic records that large currents may be obtained from a negative wire at voltages which will give little or no current with a positive wire. Thus if the wire is made the cathode the cylindrical shell will become the anode and rectification will ensue. This type of rectifier is somewhat analogous to the neon tube and perfect rectification cannot therefore be expected.

Particulars are given of the currents in hydrogen which are the results of experiments by Davis and Breese, and from which it will be seen that a considerable current discharge is obtained.

TABLE XXXI.

Diameter of Wire, mm.	Diameter of Shell, cm.	Gas Pressure, mm.	R.M.S. Volts.	R.M.S. Amperes.
200	4·45	258	—	0·0152
200	4·45	434·5	8640	0·0182

With an improved design of apparatus rectified currents can be obtained of the order of 0·12 ampere with alternating voltages of about 8000.

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PHOTO-ELECTRIC CELLS.

Photoelectric cells are made with a glass envelope evacuated to a high degree, the inside surface of which is coated with some electro-positive metal. A convenient, but not entirely correct picture of the action may be envisaged by considering the electrons, which are not closely united to their positive nucleii

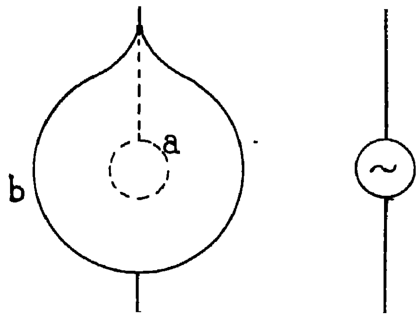

FIG. 284.—Photo-electric cell.

in electro-positive metals, are set into a condition of such violent oscillation by the light waves that they are completely withdrawn from the metallic film, and are attracted to the internal electrode.

The usual method of constructing photo-electric cells consists of coating the inside of a glass bulb about 4 centimetres in diameter and roughly spherical in shape with a layer of pure potassium, which forms one electrode, as shown in Fig. 284 The other electrode takes the form of a metal ring rigidly suspended about the centre of the bulb. When light falls on a small window in the cell, a current will flow due to the photo-electric effect, and if the electrodes are connected to a galvanometer a deflection of the needle will be observed.

As regards the rectifying properties of the cell: if the electrodes are connected to a source of alternating current, when the potassium electrode is negative and light falls on the aperture, electrons are released by photo-electric effect, and are projected by the influence of the electric field to the ring electrode. If, however, the field changes its direction no electrons can be released when the voltage between the electrodes exceeds that of the photo-electric effect.

The characteristics of the current flowing are largely dependent on whether or not gas is present in the bulb. In Fig. 285 two typical curves are illustrated:—

I—the bulb is highly evacuated and the volt drop becomes constant after a certain current is exceeded;

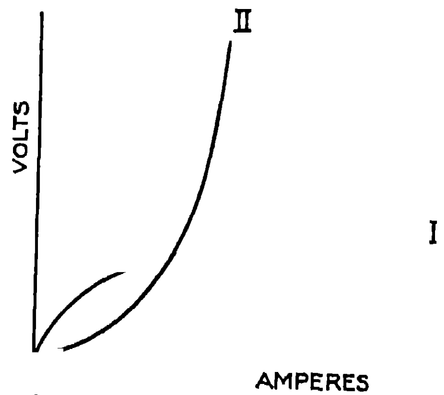

FIG. 285.—Current-voltage characteristics of photo-electric cell.

II—the current increases owing to ionisation by collision as gas is present in the bulb.

The volt-ampere characteristic of a gas bulb (i.e. curve II) is a complicated function containing two exponential terms, and the equation connecting the circuit constants where an inductance is present is

$$iR + x\frac{di}{d\theta} = E \sin \theta + f(i) \quad . \quad . \quad . \quad (1)$$

where $f(i)$ is of the form

$$\frac{(\alpha - \beta)\epsilon^{(\alpha-\beta)l}}{\alpha - \beta\epsilon^{(\alpha-\beta)l}}$$

and where a and β are the numbers of ions produced by collision by a negative and positive ion respectively when moving through 1 centimetre of gas, and l is the distance between the electrodes. Equation (1) is incapable of solution unless some simplification is possible, by taking a particular case and evolving an expression for $f(i)$.

In the case of an evacuated bulb, the volt drop is constant above a certain value and the equations used in the mercury vapour rectifier analysis will apply with some slight modification.

The value of the current depends on the amount of light falling on the window, but in a particular case where the voltage applied was about 500 maximum, the rectified current (maximum value) was found to be 0·005 ampere. Even this current is high and as a general rule it can only be measured in microamperes. The application of such a device is strictly limited and so far little use has been made of it as a rectifier.

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Hall Effect.

It has been noticed that when a metal plate preferably of bismuth is rotated in a magnetic field, the flux passing through the plate, and when current is passed along its length, a potential difference is obtained across the plate. Further, if the flux and the current are alternating in character, and moreover are in phase, the voltage is pulsating, and if an alternating voltage is applied across the plate, a rectified voltage results.

Only very small currents have so far been obtained, but the method is an interesting one, and worthy of development.

The actual experiments were made with a sheet of bismuth 26 millimetres square and 3·5 millimetres thick; placed in a solenoid, a current of $0·3 \times 10$ amps. was obtained.

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PART IV.

LIQUID CONDUCTION.

PART IV.

LIQUID CONDUCTION.

CHAPTER XVI.

CONDUCTION OF ELECTRICITY THROUGH LIQUIDS.

Behaviour of Electrolytes on the Passage of Current.—Two fundamental facts in connection with electric conduction in liquids are (i) that electrolytes behave in a similar fashion to metals in so far as their obedience to Ohm's Law is concerned, this general statement being subject to certain conditions which will be elaborated later, and (ii) that electrolytic is different from metallic conduction in that, in the former case a displacement of matter accompanies the passage of current, whereas in the latter case the conductor is undisturbed in this sense.

Partial dissociation of the molecule of the electrolyte already exists in the solution, and the passage of current results in a gradual movement of one component of the molecule, or ion to the anode, and the other component to the cathode. Thus if two copper electrodes are immersed in a solution of copper sulphate, the net effect of a current will be the transference of metallic copper from the anode to the cathode. This is the result of a migration of the copper ions of the copper sulphate molecule from the anode to the cathode, and of the SO_4 ions in the reverse direction. As an equal number of copper ions are withdrawn from the anode to those deposited on the cathode, the number of SO_4 ions in the electrolyte remains the same, and the concentration and nature of the solution does not change.

If the copper electrodes are replaced by electrodes of platinum a similar state of affairs exists with the exception that

a copper ion cannot now be extracted from the anode, and the SO_4 ion therefore attaches itself to the nearest ion in the vicinity for which it has the greatest affinity (in this case the hydrogen ion, making up the molecule of the water of solution) is taken and forms with the SO_4 ion a molecule of sulphuric acid. Atoms of oxygen will be set at liberty by this change, and will appear as bubbles of gas at the surface of the electrode, whereas copper will be deposited on the other platinum electrode. The net effect will therefore be a gradual migration of the copper ions from the solution to the platinum electrode, the liberation of oxygen from the electrolyte, and its conversion from copper sulphate to sulphuric acid.

Each case of electrolytic conduction therefore depends on the chemical affinity of the various constituents, and the result can only be determined when these facts are known.

Faraday's Law.—These phenomena were observed in the early nineteenth century by Faraday who after years of classical research on the subject enumerated two laws which assert:—

(i) That the amount of electro-chemical decomposition is a precise measure of the quantity of electricity flowing, and (ii) that the mass of a substance liberated is proportional to its chemical equivalent.

These two laws (the second of which in effect follows from the first) may be summed up in the following equation which is of fundamental importance:—

$$w = Izt$$

where w equals the weight of substance liberated in grammes, I is the current flowing in amperes, t is the time in seconds, and z is a constant called the electro-chemical equivalent.

It is usual to calculate the value of w for the case of silver, and ascertain the value of z by a ratio of atomic weights and valencies according to the equations—

$$\text{Chemical Equivalent} = k \times \text{Electro-chemical Equivalent}$$

$$\text{and Chemical Equivalent} = \frac{\text{Atomic Weight}}{\text{Valency of the Element}}.$$

In the case of silver

$z = 0\cdot0011183$ gramme per ampere per second

and $k = 0\cdot965 \times 10^5$ for all elements,

and from these formulæ the weight of any material deposited or gas liberated can be calculated, if the atomic weights and valencies are known.

Polarisation E.M.F.—There are, however, certain limitations to this general theory, which can best be explained by means of an example. Consider the case of two platinum electrodes immersed in acidulated water and connected to a source of direct current, the voltage of which can be varied at will. Arrange the controlling resistance or potentiometer in such a way that the voltage between the electrodes is kept at a constant value, less than 1·7 volts; if observations are made of the current which flows through the electrolyte it will be noted, in the first place, that there is no visible and violent chemical decomposition such as would be expected, although small gas bubbles are noticed clustering around the electrodes; and, in the second place, that the current quickly decreases to a value which is a small proportion of the initial current. If the supply is disconnected at this point and the electrodes are connected to a galvanometer a minute reverse current is apparent and the gas bubbles gradually disappear, thus indicating that these bubbles function as a gas battery which has a very limited capacity. Increase the voltage to a value greater than 1·7 and it is observed that chemical decomposition at once commences, and further that the current no longer diminishes after a few seconds, and that a much larger current is permitted to pass. If the supply voltage is now disconnected a reverse current will still be observed to flow for a short period. Thus the voltage of 1·7 determines a critical point in the electrolytic conduction, and it is important to note the reason for this discontinuity in the current-voltage characteristic.

The fact of a reversal of current taking place on the cessation of the supply must necessitate the existence of a counter E.M.F. which can only have its seat at the surface of the electrodes. This is called the counter E.M.F. of Polarisation

and has been found to be due to the deposition of small quantities of the elements of the electrolyte and the electrodes on the surface of the electrodes; and thus a local battery action is set up which acts in such a way as to attempt to prevent the passage of current, until the supply voltage exceeds its own voltage of 1·7. As these small batteries are caused by the passage of current they will grow in number as the current continues, and the resistance of the cell will therefore increase to such a point that the main current almost ceases. An increase in the supply voltage will result in an increased current but only the excess of voltage above the figure of 1·7 is available to force the current through the cell.

This effect of polarisation is the seat of many troubles in the history of the primary battery: it is well known that in many forms of Lechlanche or Dry battery a continuously high discharge current cannot be maintained, that the current falls away more or less rapidly, and that a period of recuperation is needed before the cell can be expected to return to its former state. Apart from the back E.M.F. this reduction of output may be sometimes explained by a fall in the hydrogen ion concentration near the block of the cell, which is slowly corrected by diffusion. After a period of rest the bubbles are absorbed and the cell will function again in a normal fashion. Thus many improvements in the construction of primary batteries centre round the provision of a more perfect depolariser.

Resistance of Electrolyte.—It is now possible to enlarge on the statement made on page 397 that electrolytes behave in such a manner that they obey Ohm's Law, or in other words that the current flowing is independent of the resistance of the electrolyte. This has been the basis of much experimental work, but eventually Kohlrausch found that this law is substantially true if the excess voltage of the supply over the E.M.F. of polarisation is considered. It is possible therefore to write.

$$I = \frac{E - e}{R}$$

where e is the counter E.M.F., E is the supply voltage, I is the

current and R is the resistance of the cell. It must not be assumed that R is absolutely constant as the electrolytic resistance may vary owing to the production of gas bubbles, etc., but if this fact is taken into consideration Ohm's Law is valid for liquid conduction.

Nearly all modern methods of determining the resistance of electrolytes depend upon the use of alternating current to avoid this polarisation. Ohm's Law is then correct if the number of cycles is sufficiently high.

General.—The whole subject of electrolytic conduction is one of some complexity, and what has been said only represents a sufficiently brief description to enable the theory of electrolytic rectification to be followed, so far as it is known at the present time.

Before passing on to a consideration of the rectifying properties of an electrolytic cell, a word of caution is necessary to avoid a confusion of terms. In the example given of two copper rods immersed in copper sulphate solution, copper and SO radicles have been termed ions, a copper and an SO_4 ion combining to form a copper sulphate molecule. The ion of SO which leaves the cathode is called a cation and the copper radicle the anion. It is unfortunate that this terminology has been adopted, as confusion is probable with the ion of gaseous conduction.

CHAPTER XVII.

ELECTROLYTIC RECTIFIERS.

Theory of Electrolytic Rectification.—It was observed early in the history of electrolytic conduction that a combination of certain anodes and electrolytes resulted in a unidirectional current, when an alternating voltage was impressed across the electrodes, but no satisfactory theory accounted for the phenomenon until 1902 when Guthe propounded the oxide-gas film theory, which suggests that on the anode, a solid oxide or hydroxide film is formed which increases in thickness with the passage of current; at the same time a thin film of gas is formed on the solid film which further increases the resistance of the cell. The action of rectification was attributed, therefore, to the ease with which free electrons, which are present on the surface of the anode, can penetrate the oxide and gas layer owing to the high potential gradient, and traverse the electrolyte to the cathode; whereas the heavier cations are more or less completely held up by the film on account of their greater mass. This would result in the production of a high counter E.M.F., or E.M.F. of polarisation, which opposes the passage of a reverse current.

This theory has been amplified to a great extent by Professor Gunthe Schulze, and his theories based on experimental evidence are admirably set forth in his treatise and its translation by de Bruyne on the subject. It is not possible within the scope of this book, to consider in detail the complete theory which has been described elsewhere—such a description would of necessity include the effect of the many variables, which are to be found in the apparently simple electrolytic rectifier. Professor Schulze describes at least twelve such, each of which depends

in some unknown way on such factors as, the metals of which the electrodes are made, the nature of the electrolyte, its concentration and temperature, and the current and voltage, etc.

Before proceeding to discuss the theory, consider the facts which are at present available, as to what actually happens when two electrodes are subjected to a potential when immersed in an electrolyte.

Formation.—In the first case if these two electrodes are subjected to a constant potential, the current will rapidly fall from its initial to a small value, in a time which depends on the area of the electrode and the applied voltage. For example, Holler and Schrodt have found that with an aluminium anode of area 1 square centimetre, and 25 impressed volts, the film formed almost at once; whereas with an area of 300 square centimetres several hours were required for a complete formation with 120 impressed volts. Fitch has contended that this film grows at a rate which is proportional to the time, and that this film is a combination of an oxide and a gas as stated by Guthe, the gas film reducing rapidly in thickness on a reversal of the current. In the light of later experiments this final suggestion does not appear to explain the true facts, although it may be that the oxide film increases with time; but the growth cannot continue indefinitely but probably initially follows a logarithmic law.

If the conditions of this experiment are so arranged that a constant current is made to pass between the electrodes then it will be found that the voltage increases up to a point when a definite sparking is noticeable over the surface of the aluminium, and subsequently the voltage increases proportionately to the time of application. At a certain point, however, an increased sparking is apparent and the voltage no longer rises and the formation of the plates is complete. This voltage is termed the "maximum voltage of formation," and at this point the resistance of the cell is constant. A further rise in the voltage will result in a complete breakdown, and normal conduction accompanied by electrolysis takes place. This effect and the current curves are shown on Figs. 286 and 287.

It can be shown that for small concentrations the maximum voltage can be represented by

$$(514 \log D - 330) \text{ volts}$$

where D is the dilution in litres per gramme equivalent.

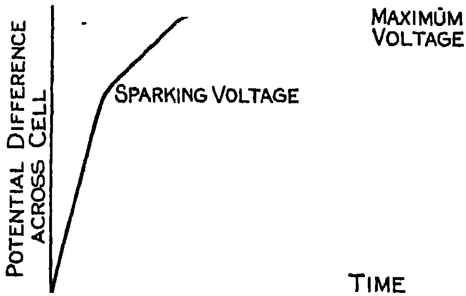

FIG. 286.—E.M.F.—Time characteristics.

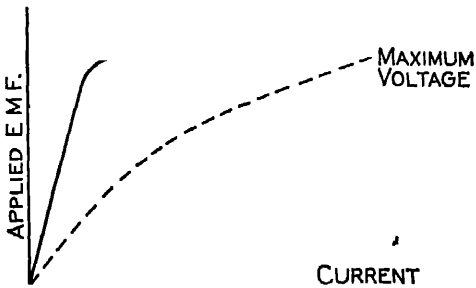

FIG. 287.—E.M.F.—Current characteristics.

Valve Action.—The valve action has been largely studied by Schulze, and although the theories are incomplete yet it is supposed that the existence of a permeable and impermeable film depends on the gas-electrolyte boundary. It is known that, although it requires a high potential to free electrons from a cold metallic surface, yet in the case of an electrolytic surface, a still greater potential is required. Further, if the gas pressure is high it increases the difficulty of passage as the mean free path is smaller. During the process of formation the oxide film is being deposited on the anode, and as this film is partially

responsible for the process of rectification the plates must be correctly formed if the apparatus is to function satisfactorily, and at the same time in this porous layer oxygen is evolved. This process represents what Schulze terms the characteristic of the rectifier in so far as its resistance properties are concerned ; but it does not wholly explain the mechanism of rectification, which requires some factor to limit the current to one direction only. It is found after experiment that with a certain direction of current flow, and below a certain minimum voltage "mindest spannung," the cell becomes conducting, and Schulze states that under these conditions the cell behaves like a *mercury vapour rectifier ;* thus as Schulze points out the term "electrolytic rectifier" is not strictly a correct terminology, as the cell is only electrolytic in so far as its constituent parts resemble a cell operating on the electrolytic principle.

The values of this minimum voltage in the case of tantalum, with a forming voltage of 85, and in a 0·05 N concentration of solution of electrolyte, is given in Table XXXII.

TABLE XXXII.

Potassium hydrate	9·6 volts
Sodium bicarbonate	13·2 ,,
Potassium nitrate	11·7 ,,
Sodium sulphate	9·0 ,,
Potassium phosphate	9·3 ,,
Sodium chloride	18·2 ,,

It appears that the behaviour of the cell as a more or less perfect rectifier depends on the solubility of the anode in the electrolyte; and thus tantalum which is unaffected by acids is likely to be preferable to aluminium. This is found to be the case, and in fact it was this characteristic of the partial solubility of aluminium in all of the electrolytes tried, which obscured the issue in the earlier work on the subject.

The actual thicknesses of the respective films, oxide and gas (Δ and δ) have been investigated by Schulze who has shown that for tantalum these layers vary approximately as the formation voltage; and where ϵ is the dielectric constant, the ratio

δ/ϵ is affected only by the element of which the anode is composed, and by the formation voltage, and in this case Fig. 288 can be verified experimentally for both aluminium and tantalum.

Professor A. Smits discusses the assumption made in the above theories and arrives at the conclusion that the explanations are not sufficient to meet all the facts, such as for instance that amalgamated aluminium shows no rectifying effect, and secondly that under certain circumstances a chlor-ion concentration renders valve action impossible. He comes to the conclusion

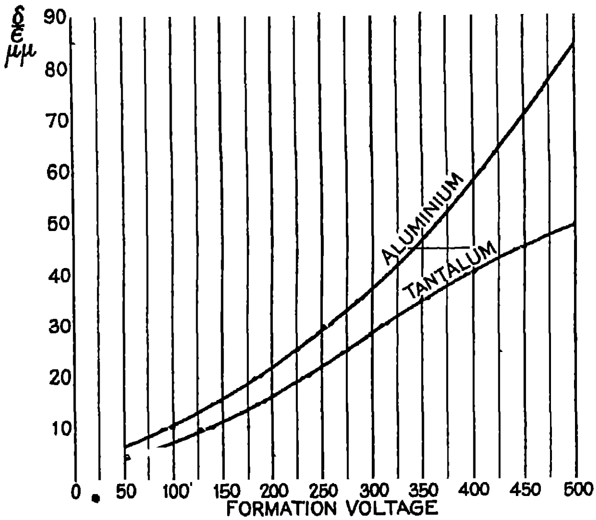

FIG. 288.—Formation voltage and film thickness.

that the electromotive equilibrium theory will prove the solution to the problem.

The whole question of the actual cause of the valve effect is one of great complexity, and a complete solution of the problem has not yet been accepted. The correspondence in the foreign technical press leaves the matter over for further data in the light of experiment. There is one point, however, which must not be omitted in a consideration of the theory, viz. the fall of potential with time is not such as would be expected if a condenser discharge were the sole cause of rectification. If the

graph be plotted to a log scale it is curved in form, whereas the graph of a discharge from a condenser would show a linear relation.

Effect of Temperature.—Owing to the high resistance of the films, and the relatively high current passing during the periods of current flow, the I^2R losses are high, resulting in a considerable rise of temperature. This has a dual effect in certain classes of rectifier, as, for instance, where aluminium electrodes are employed, the solubility of the anode increases, and as has been stated above, rectification becomes imperfect. At the same time an undue increase in the temperature will

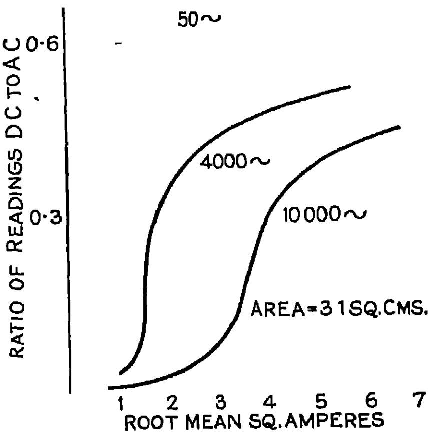

FIG. 289.—Effect of frequency on wave form.

have an adverse effect on the oxide film, and will render the rectification imperfect—in some cases if the current density is increased to too great a value the rectifier ceases to function, and current will pass in both directions. On the other hand, rectification is dependent on a high current density so that there is an optimum point at which the rectifier will function in the best fashion, and it is at present a matter of experiment to determine where this optimum point exists with any one type of cell.

In certain cases it may be advisable to include some form of water cooling in order that the requisite current may be obtained without too high a cell temperature being attained.

Effect of Frequency.—As electrolytic rectifiers are inherently leaky condensers the capacity current, which will inevitably flow, will vary with the frequency, and this will largely affect the

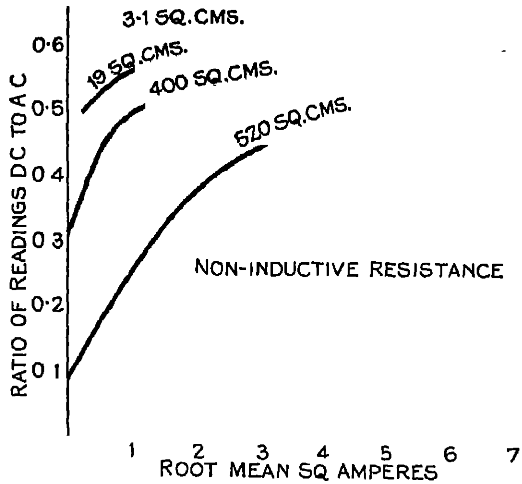

FIG. 290.—Effect of electrode area on wave form (no inductance).

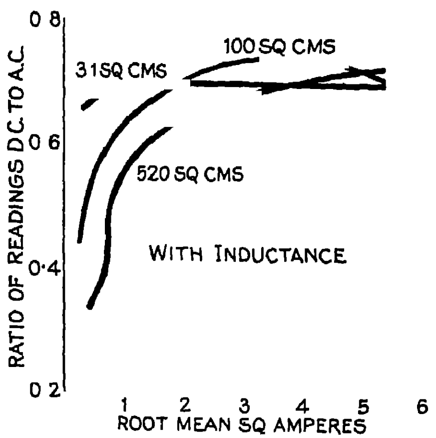

FIG. 291.—Effect of electrode area on wave form (no inductance).

performance on load. If ρ is the ratio of the readings of the rectified current on a moving coil and a square law instrument, ρ will normally equal 0·63 for a single phase circuit, and for

a sinusoidal wave form (page 34). As the frequency increases this ratio will decrease as shown in the following curve due to Zenneck.

Effect of Electrode Area.—For the same reason the capacity current will cause the ratio ρ to fall as the electrode area increases, and this effect is indicated in Figs. 290 and 291, which are also due to Zenneck.

In Fig. 291 where inductance is included, the effect of the inductance is largely to neutralise the effect of capacity, producing an inherent power factor closer to unity than is found with a pure resistance load.

Emission of Light from the Anode Surface.—As has been stated above, when the critical value of the voltage is about to be reached, careful observation will reveal a faint luminescence over the surface of the anode, which as the voltage is further increased, changes into small scintillating sparks, due no doubt to the breaking down of the hydroxide film owing to the high potential gradient. In some cases the emission is intense enough to be easily discerned, but at ordinary pressures the anode usually has to be carefully examined to reveal it.

Special Forms of Rectifier with Colloidal Anodes.—In order to reduce the inefficiency of a rectifier, and its consequent heating many devices have been proposed. Perhaps the best of them is that devised and patented by André, in which the cathode is of nickel and the anode of colloidal silver, but any metal will function as an anode which can be produced in a colloidal state, and whose oxide is conducting. Difficulties have been experienced in the past owing to the fact that after a certain period of time the rectifier ceases to function due to strings of colloidal silver which short-circuit the leading-in wires.

Briefly the results of this interesting experimentation may be summarised as follows:—

If two strips of silver are immersed in a solution of concentrated sulphuric or phosphoric acid, and alternating current is passed between them (at a current density of about 0·1 ampere per square centimetre of electrode area) electrolysis occurs; and the current steadily rises to a point when the electrochemical

reaction ceases, and the electrolyte has a conductivity comparable with that of a metal, and the solution is found to contain micelles of colloidal silver.

It is then found that if one of the silver electrodes is replaced by one of nickel, and the electrolyte replaced by a paste consisting of powdered pumice and sulphuric acid, as shown in Fig. 292, and if the electrodes are allowed to touch lightly then electrolysis first takes place, which as colloid silver is formed, gives place to a rise of current, and at the same time rectification takes place, and current will flow from the silver to the nickel, but only a minute amount in the opposite direction. The amount of rectification

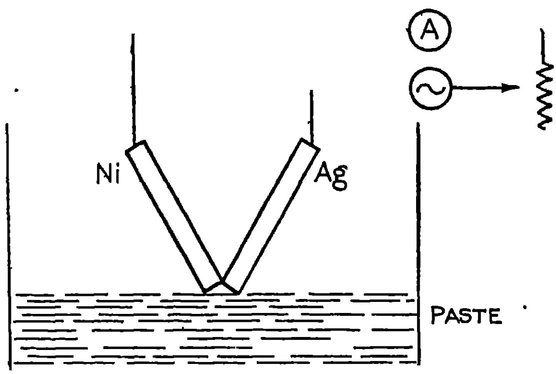

FIG. 292.—Colloidal anode.

can be measured on the D.C. ammeter A. In this case it is found that the cell functions only so long as the metal strips are moist, but that during that period the internal resistance is practically zero. It would appear that it is not strictly correct to term the silver strip an electrode, as there is reason to believe that the colloidal silver is the true electrode, and that the metal strip merely functions as a leading-in wire, and the actual operation of rectification probably takes place at the surface of the nickel.

Thus in Fig. 293 the nickel strip is coated with a layer of oxide A which is exposed to the anodic silver colloid B, which again is in metallic contact with the silver strip.

It is perhaps previous to speculate on the actual cause of the valve effect, but as sparking occurs under certain conditions, it may be that the same theory may be suggested here, as has been put forward to account for the phenomenon in the ordinary electrolytic rectifier, but so far there does not seem to be sufficient evidence to warrant a definite statement.

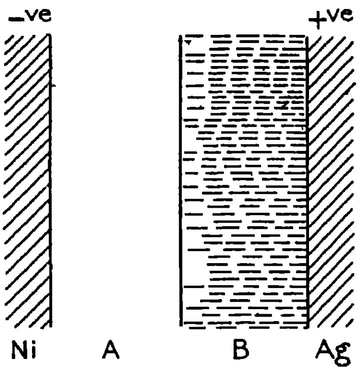

FIG. 293.—Colloidal anode.

There are but few elements which will function satisfactorily as cathodes, as the chief requirements are (1) the electrode must not be attacked by hot concentrated acid, (2) it must oxidise rapidly when current passes, and (3) the oxide must be unattacked. Those elements so far tried are given in Table XXXIII.

TABLE XXXIII.

Element	Maximum Direct Current	Voltage for Best Rectification
Lead	60	12
Nickel	60	18
Soft iron	100	25-28
Copper	50	8
Aluminium	70	20
Silicon	100	50
Molybdenum	100	10

With regard to the anode material, as the metal must be unattacked by strong acid, and its oxide must be similarly

immune, and further must be electrically conducting, silver is practically the only metal that can be employed.

The frequency of supply to which this rectifier can be applied is dependent on the speed of formation of the oxide film.

With regard to the life of the rectifiers, it has been found that they only function satisfactorily for long periods if there is a reverse E.M.F. in the circuit, and thus the ideal purpose for which they are most suitable is that of battery charging. For example, whereas a rectifier will operate for 1000 hours as a battery charger, it will only rectify on a resistance load for 50 hours. As battery chargers they will pass a current of from 2 to 3 amperes (biphase).

These rectifiers, which are small in bulk, behave peculiarly when connected in series or in parallel. In series, unless each one is shunted by a suitable adjustable resistance, so that the

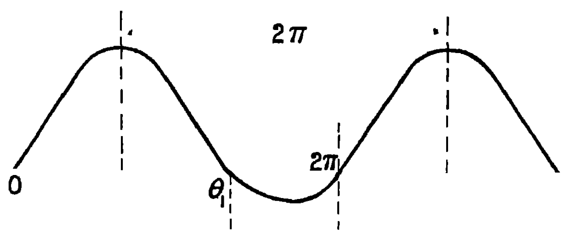

FIG. 294.—Current wave form in electrolytic rectifier.

voltage is equalised across each cell, one cell alone will drop all the voltage, and will eventually fail, this process continuing until only one cell remains. In parallel the same thing happens, as the internal resistance rapidly falls on load, and one or two cells will first take the load and will eventually heat up and fail. The remedy in this case is to connect a resistance in series with each cell.

Theoretical Analysis of the Current-Voltage Relation.—It is clear that the current-voltage relation will be periodic and, moreover, that it will repeat itself after a time which will be equal to the time of oscillation of the supply. The current will then generally take the form of the curve in Fig. 294.

It is known that the cell behaves very much in accordance with Ohm's Law (see Chapter XVI., page 400), excepting that

the resistance to the current is greater in one direction than in the other, and that there is a slight negative current flowing during the reversed cycle. Call the resistance of the cell in the first half cycle r_1 and in the rectifying cycle r_2. Then if an inductance x and a resistance r are included in the circuit, during the period represented by 0 to θ_1, the equation for the E.M.F. can be written

$$i_1 r_1 + i_1 r + x\frac{di_1}{d\theta} = E \sin\theta - E_1 \quad . \quad . \quad (2)$$

where E_1 is the counter E.M.F. of the rectifier or minimum voltage, E_1 cannot always be assumed to be small in value; for example, with tantalum and an electrolyte of nitric acid, it may reach a value of 46 volts. Equation (2) may be simplified by putting

$$r' = r_1 + r,$$

whence

$$\frac{di_1}{d\theta} + \frac{i_1 r'}{x} = \frac{E \sin\theta - E_1}{x} \quad . \quad . \quad (3)$$

The solution of this equation is

$$i_1 = A\epsilon^{-\frac{r'}{x}\theta} + \frac{E}{Z}\sin(\theta - \phi) - \frac{E_1}{r},$$

where $\tan\phi = \frac{x}{r'}$ and $Z = \sqrt{r^{2'} + x^2}$.

When $i_1 = 0$, $\theta = 0$, whence

$$A = \frac{E}{Z}\sin\phi + \frac{E_1}{r},$$

and when $i_1 = 0$, $\theta = \theta_1$, whence

$$i_1 = \frac{E}{Z}\left\{\epsilon^{-\frac{r'}{x}\theta_1}\sin\phi + \sin(\theta - \phi)\right\} - \frac{E_1}{r'}\left(1 - \epsilon^{-\frac{r'}{x}\theta}\right) \quad (4)$$

and

$$\frac{E}{Z}\left\{\epsilon^{-\frac{r'\theta_1}{x}}\sin\phi + \sin(\theta_1 - \phi)\right\} = \frac{E_1}{r}\left(1 - \epsilon^{-\frac{r'}{x}\theta_1}\right) \quad (5)$$

Similar equations can be evolved for the negative portion of the wave, but as this loop is of small magnitude owing to the high resistance of the rectifying cycle, the current can be neglected and equations (4) and (5) represent the characteristic equations of the wave form.

If it can be assumed that E_1 is zero, a condition not always possible, the mean current flowing is

$$I_M = \frac{1}{2\pi}\int_0^{\theta_1} i_1 d\theta = \frac{E(1 - \cos\theta_1)}{2\pi r'} = \frac{E\sin^2\frac{\theta_1}{2}}{\pi r'}, \quad . \quad (6)$$

and if θ_1 is put equal to π a condition of no inductance in the circuit, I_M reduces to the equation

$$I_M = E/r'\pi = I/\pi$$

for a single-phase wave form.

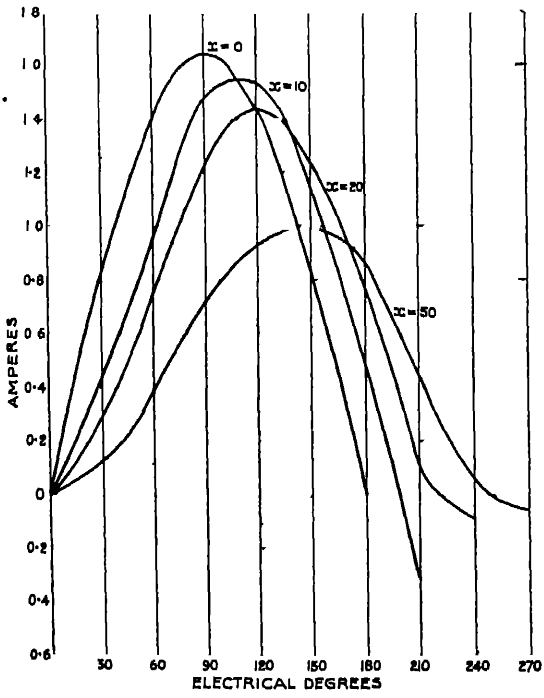

FIG. 295.—Current waves with different values of the inductance.

If i_1 is differentiated with regard to θ

$$\frac{di_1}{d\theta} = -\left\{\frac{r'}{x}\epsilon^{-\frac{r'}{x}\theta}\sin\phi - \cos(\theta - \phi)\right\}\frac{E}{Z},$$

and if $\frac{di_1}{d\theta}$ is equated to zero a relation is obtained between ϕ

and θ_m which is the value of θ for a maximum value of i_1. This substituted in equation (4) results in

$$i_1\ (\text{max.}) = I = E \sin \theta_m / r' \qquad . \qquad . \quad (7)$$

where θ_m is given by

$$\epsilon^{-\frac{r'}{x}\theta_m} = \frac{\cos(\theta_m - \phi)}{\cos \phi}.$$

The maximum value of the current wave is thus dependent on the inductance in circuit and is related to the overlap θ_1 by equations (5) and (7). It is impossible to separate the exponential terms from the trigonometrical functions, and if the one is wanted in terms of the other the only way is to make each calculation separately with numerical examples.

A series of curves for differing values of x is given in Fig. 295, which indicate the reduction in the crest current and the consequent smoothing.

These curves are calculated for the following values of the constants :—

$$r = 20,\ r_1 = 10, \text{ i.e. } r' = 30 \text{ and } E = 50 \text{ volts.}$$

Analysis in Battery Charging Circuit.—In the case of a battery charging circuit equation (2) becomes

$$i_1 r_1 + i_1 r + x\frac{di_1}{d\theta} + e = E \sin \theta$$

where e is the sum of the minimum voltage and the back E.M.F. of the load circuit. A solution of this equation is

$$i_1 = A\epsilon^{-\frac{r'}{x}\theta} + \frac{E}{Z_1}\sin(\theta - \beta) - \frac{e}{r'}$$

where the symbols have the usual significance. The condition that when $i_1 = 0$, $\theta = 0$ gives

$$A - E/Z_1 \sin \beta - e/r' = 0,$$

or

$$A = \frac{E}{Z_1}\sin \beta + \frac{e}{r'},$$

and therefore

$$i_1 = \frac{E}{Z_1}\sin \beta \,.\, \epsilon^{-\frac{r'\theta}{x}} - \frac{e}{r'}\left(1 - \epsilon^{-\frac{r}{x}\theta}\right) + \frac{E}{Z_1}\sin(\theta - \beta).$$

The further condition is that when $i_1 = 0$, $\theta = \theta_1$, and hence

$$\frac{E}{Z_1}\sin\beta\,.\,\epsilon^{-\frac{r'}{x}\theta_1} - \frac{e}{r'}\left(1 - \epsilon^{-\frac{r'}{x}\theta_1}\right) + \frac{E}{Z_1}\sin(\theta_1 - \beta) = 0 \quad (9)$$

which gives the value for θ_1, the angle of cut-off. The final condition is that for a battery of any given capacity the charging current is known, and thus

$$I_M = \frac{1}{2\pi}\int_0^{\theta_1} i_1 d\theta$$

which reduces to

$$I_M = \left\{\frac{Ex^2}{r'(x^2 + r'^2)}\left(1 - \epsilon^{-\frac{r'}{x}\theta_1}\right) - \frac{e\theta_1}{r'} + \frac{ex}{r'^2}\left(\epsilon^{-\frac{r'}{x}\theta_1} - 1\right) - \frac{r'E}{x^2 + r'^2}\cos\theta_1 - \frac{Ex}{x^2 + r'^2}\sin\theta_1 + \frac{Er'}{x^2 + r'^2}\right\}\frac{1}{2\pi}.$$

In this equation the circuit constants x and r' are given, and θ_1 has been obtained from equation (9), and it is, therefore, possible to calculate the crest value E of the supply voltage required or conversely E may be given and x or r' calculated.

This case is, therefore, simpler than that of the mechanical rectifier in that the cell is not short-circuited during the period of current reversal. It is not usual, however, to insert a reactance in the case of a battery charging circuit as the undulatory character of the wave form is not objectionable; considerable simplification results if the inductance is eliminated, and the equation for the current wave becomes

$$i_1 = \frac{E\sin\theta}{r'} - \frac{e}{r'}$$

and
$$\theta_1 = \sin^{-1}\frac{e}{E}$$

and
$$I_M = \left\{\frac{E}{r'}(1 - \cos\theta_1) - \frac{E\theta_1}{r'}\right\}\frac{1}{2\pi}.$$

This case is analogous to that of the mechanical rectifier, where in Chapter V. (page 111) a numerical example is given.

Effect of Capacity of the Oxide Film and Gas Layer.—The capacity of the oxide film and gas layer can be substituted by an equivalent capacity C in the circuit diagram of Fig. 296.

The equations connecting the various quantities are

$$e + ir' + x\frac{di}{d\theta} + x_c\int i d\theta = E\sin\theta$$

where x_c is the condensive reactance and equals $1/pC$.

Differentiating

$$x\frac{d^2i}{d\theta^2} + r'\frac{di}{d\theta} + ix_c = E\cos\theta.$$

This equation is analogous to that of a damped vibrating system under the influence of an external impressed force of a sinusoidal character, and results in a general solution

$$i = \frac{E}{Z}\cos(\theta - a) + A\epsilon^{-\frac{r-l}{2x}\theta} + B\,.\,\epsilon^{-\frac{r+l}{2x}\theta}$$

where $Z = \sqrt{r'^2 + (x - x_c)^2}$ and $l = \sqrt{r'^2 - 4xx_c}$

and $$\tan a = \frac{x - x_c}{r'}.$$

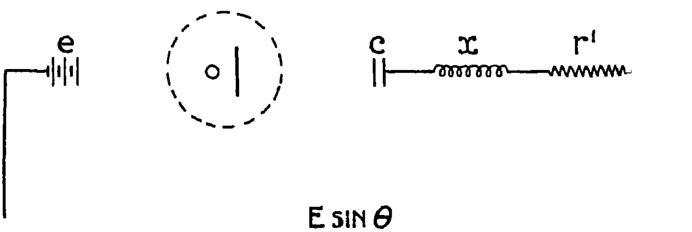

FIG. 296.—Circuit diagram with capacity and inductance.

The constants A and B are determined by the terminal conditions

$$i = 0, \text{ when } \theta = 0$$

$$\text{and} \quad i = 0, \text{ when } \theta = \theta_1,$$

whence

$$A = \frac{\frac{E}{Z}\cos(\theta_1 - a) - \frac{E}{Z}\epsilon^{-q\theta_1}\cos a}{\epsilon^{-q\theta_1} - \epsilon^{-p\theta_1}}$$

and $$B = \frac{\frac{E}{Z}\epsilon^{-p\theta_1}\cos a - \frac{E}{Z}\cos(\theta_1 - a)}{\epsilon^{-q\theta_1} - \epsilon^{-p\theta_1}}$$

where $$p = -\frac{r - l}{2x} \text{ and } q = -\frac{r + l}{2x}.$$

To evaluate θ_1 it is necessary to take into consideration the final condition that when $i = 0$, $E = e$, the counter E.M.F., and $\theta = \theta_1$.

Thus θ_1 can be ascertained, but as is the case with the forced mechanical vibration, the above is not the complete solution, which varies in form depending on whether $r^{2\prime}$ is greater than, equal to, or less than $4xx_c$.

The resonant condition is that

$$r^{2\prime} = 4xx_c$$

which will result in a current wave with a high peak value.

The example is not of great practical importance as it is unusual to insert an inductance in such a circuit. Assume, therefore, that the inductance is zero, and the characteristic equation which represents the voltage relationships is

$$e + x_c \int i d\theta + ir' = E \sin \theta \quad . \quad . \quad . \quad (10)$$

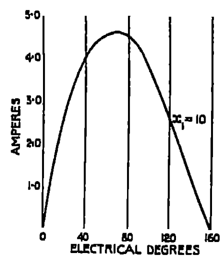

FIG. 297.—Current wave form with condensive reactance.

The solution to equation (10) is

$$i = A\epsilon^{-\frac{x_c}{r'}} + \frac{E}{Z} \sin (\theta + \phi)$$

where $\qquad Z = \sqrt{r^{2\prime} + x_c^2}$ and $\tan \phi = \frac{x_c}{r'}$.

When $\quad i = 0, \quad \theta = 0$ and θ_1,

and hence

$$A = -\frac{E}{Z} \sin \phi$$

and $$i = \frac{E}{Z} \sin (\theta + \phi) - \frac{E}{Z} \sin \phi \,.\, \epsilon^{-\frac{x_c}{r'}\theta}$$

and also

$$\sin (\theta_1 + \phi) = \sin \phi \,.\, \epsilon^{-\frac{x_c}{r'}\theta_1}$$

which gives a value for the point of cut-off.

Taking $r' = 21{\cdot}5$ ohms, and $x_c = 10$, and $E = 110$,

$$i = 4{\cdot}65 \sin (\theta + 25) - 1{\cdot}96\epsilon^{-0{\cdot}465\theta},$$

and this curve is shown in Fig. 297. It will be noted that the inclusion of a capacity tends to retract the curve without increasing its peak value, and therefore, shortens the point of cut-off.

Effect of Different Electrolytes.—It has been postulated that any electrolyte which will cause the formation of a gas

TABLE XXXIV.

Applied Voltage.	Solution.	Anode Material.	Volts Between Electrodes.	Current.	Reverse Current.
85	Potassium fluoride	Aluminium Bismuth	85 85	3·7 4·1	2·9 4·0
85	Ammonium fluoride	Magnesium Aluminium	16 20	3·9 3·8	0·1 0 1
85	Ammonium fluorsilicate	Magnesium Aluminium	54 54	3·4 3·5	1·2 1·2
85	Ammonium carbonate	Aluminium	15	3·8	0
85	Ammonium oxalate	Aluminium Bismuth	4 45	1·7 2·65	0 2 0
85	Ammonium phosphate	Aluminium Bismuth	4 20	2·1 2·85	— 0·1
85	Double phosphate of ammonium and potassium	Aluminium	8	3·25	0

Cathode of Graphite, and temperature kept constant.

layer and any electrode, which on the passage of current will cause a stable oxide film to be formed, will function as a rectifier. Numerous substances will give the required result, but there are one or two which are more efficient than the others. Nodon has provided particulars of various metals and solutions which are suitable, and some of those tried are mentioned in Table XXXIV.

From these results it will be seen that amongst the elements tried aluminium alone gave rise to a true valve effect. (The expression "true valve effect" is used with the reservation that

it is highly probable that there is no such thing as a perfect electrolytic rectifier, but as the resistance to the reverse current is high, ordinary indicating instruments may be insufficiently sensitive to record it.)

A further point to note is that the resistance of the cell as measured by the voltage drop between the electrodes is least in the case of aluminium, and it is apparent that the best electrolyte to use is ammonium phosphate, because it gives rise to the least I^2R losses and consequent internal heating.

TABLE XXXV.

Anode.	Cathode.	Electrolyte.	Current Used.	Ohms per Cubic Centimetre on Closed Circuit.
Lead	Lead	Double phosphate of potassium and ammonium	D.C.	6·29
Aluminium	Lead	Double phosphate of potassium and ammonium	A.C.	60·0
Lead	Lead	Ammonium carbonate	D.C.	8·84
Lead	Aluminium	Ammonium carbonate	D.C.	18·90
Lead	Lead	Ammonium carbonate	A.C.	10·62
Aluminium	Lead	Ammonium carbonate	A.C.	30 0
Aluminium plus 5 per cent. nickel	Lead	Ammonium carbonate	A.C.	61·5
Lead	Aluminium plus 5 per cent. nickel	Ammonium carbonate	D.C.	8·84

Measurements of the resistance of electrolytes have also been made, and Table XXXV. will show the divergence obtained under varying conditions.

M. A. Codd conducted some experiments on the apparent resistance (or impedance) of electrolytes, using electrodes of aluminium and iron, where alternating current was used for the measurements with a non-inductive load in circuit. The cell was disconnected from the supply after a period and connected to a 12 volt accumulator, and its apparent resistance measured;

the areas of the electrodes were 11 square inches each, spaced 2 inches apart, and a saturated solution was employed in every case. Table XXXVI. indicates the results obtained.

These last tests show very strikingly that the last salt used as electrolyte was incapable of forming the oxygen film, and that little rectification ensued in consequence.

It should be noted, as stated in Chapter XVI., that the only way of obtaining true and consistent results is to employ alternating current of high frequency in making internal resistance tests, otherwise the large inherent capacity due to the effect of the double dielectric on the surface of the anode will lead to erroneous deductions.

TABLE XXXVI.

Electrolyte.	Mean Amperes.	R.M.S. Amperes.	Form Factor.	R.M.S. Volts.	Current at 12 Volts.		Apparent Impedance during Rectification.
					Flowing.	Rectified.	
Potassium phosphate	1·8	3·5	1·95	14·7	8·8	0·0024	5000
Sodium bicarbonate	1·4	2·65	1·90	15·5	4·6	0·0011	11,000
Potash alum	1·1	2·25	2·01	16·25	3·3	0·0014	8570
Ammonium phosphate	1·1	2·25	2·01	16·0	8·0	0·0009	13,330
Ammonium bicarbonate	0·95	2·02	2·13	16·75	8·0	0·0015	8000
Sodium phosphate	0·90	2·05	2·26	16·0	2·5	0·0017	7050
Potassium bitartrate	9·40	0·55	1·38	18·25	0·25	about 0·100	—

Effect of Different Electrodes. — As regards the metals which can be used as electrodes, beyond those already described, tantalum has been tried with considerable success on account of its inert behaviour towards acids and salts in solution; and its life appears to be indefinitely long, but it is an expensive metal to purchase and difficult to obtain in sheet form. For the passage of 3 amperes to charge a 3 or 4 cell battery a thin sheet 2 inches square or a wire 2 to 3 inches long and 0·010 inch in

diameter used as an anode with a lead plate as a cathode will give satisfactory results.

In the case of a wire electrode, on account of the strength of the electric field, sparking often occurs even at low voltages, and it is preferable to employ sheet metal where at all possible.

As tantalum is sensitive to contact with other metals and is able to absorb many times its volume of hydrogen it must be carefully handled, and should be heated to a high temperature before use.

Tungsten has also been tried successfully as an anode in various solutions. L. H. Walter experimented with this element in electrolytes of nitric, sulphuric and hydrochloric acids, all of which resulted in good rectification. So far, however, the experiments were only possible with small filament wires, but now that tungsten is available in sheet form, a useful type of rectifier may be obtained, especially with an acid electrolyte where the resistivity is low. To give some idea of the possibility of such a rectifier—a filament from a 60 watt, high volt lamp, i.e. with an electrode surface of about 0·1 centimetre, a current of 0·5 ampere was successfully rectified in an electrolyte of sulphuric acid (S.G. 1200) and a lead cathode. A 5 per cent. solution of potassium bichromate also gave satisfactory results.

Oscillograms of Current Waves.—Oscillograph records of

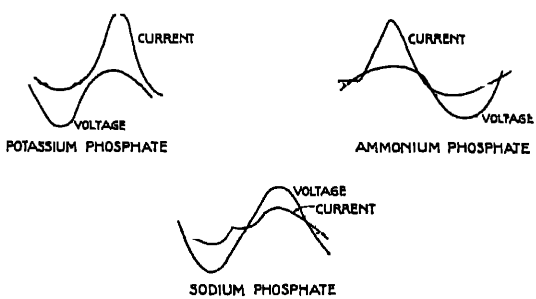

FIG. 298.—Oscillograms of current with differing electrolytes.

the rectified current show a number of interesting points as the current curve varies in shape with the electrolyte used.

Fig. 298 illustrates some of these curves taken for the

three electrolytes: potassium phosphate, sodium phosphate, and ammonium phosphate. Potassium phosphate will give excellent results, but the salt is expensive and also difficult to keep on account of its deliquescent properties.

In these curves the negative portion of the wave is very pronounced and the net rectified current, therefore, considerably reduced in value. This is possibly due to the temperature at which the cells are run, as well as the chemical constitution of the electrolyte, as some further curves taken in a cell with ammonium phosphate as the electrolyte give much better rectification (see Fig. 299).

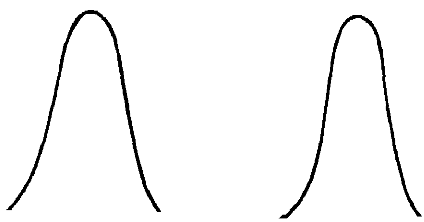

FIG. 299.—Current wave with ammonium phosphate electrolyte.

Too much reliance must not be placed on these isolated cases as there are many variables entering into the question, but the above oscillograms are given as examples of the type of current wave to be expected, and also to show that while the distortion is not serious the rectification factor may be poor.

Efficiency.—As regards the efficiency of electrolytic valves, the only losses which are likely to occur are

(i) Transformer loss

and (ii) I^2R loss in the cell.

The latter is almost constant and part of the former varies with the load. Fig. 300 is a curve showing a typical overall efficiency with varying load, and indicates how constant it may be over a wide range of loads. This property is useful, especially if the rectifier is only required for short duty.

Efficiency curves are, however, an unreliable guide to the actual performance, and it is dangerous to draw too optimistic a

conclusion. It is of small advantage if a rectifier has a high efficiency, and yet very soon ceases to function, and this unfortunately is the chief trouble with electrolytic rectifiers.

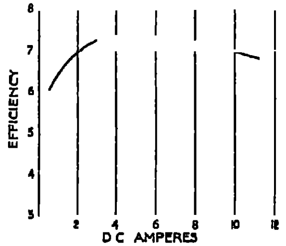

FIG. 300 —Efficiency curve for electrolytic rectifier.

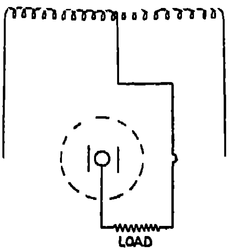

FIG. 301.—Biphase electrolytic rectifier.

Polyphase Rectification.—Biphase and polyphase connection can be employed as in the case of thermionic rectifiers, with the added simplification that both electrodes can be immersed in the same electrolyte—a parallel case to that of the mercury vapour rectifier.

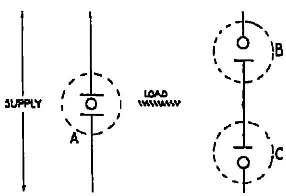

FIG. 302.—Greatz connection for electrolytic rectifiers.

Fig. 301 indicates such an arrangement, but as in other similar instances, a loss of one half the transformer voltage is incurred (see page 177).

The Wheatstone Bridge or Greatz method of connection can also be adopted, as shown in Fig. 302, where cell A has two electrodes, and B and C are valves of the ordinary type.

An ingenious device of Messrs. Siemens enables a biphase supply to be maintained, but on a different principle (see Fig. 303).

The load is in series with a cell A, and in parallel with a cell B, and during the positive half cycle current flows through A and the load, but cannot pass through B; on account of the condensive reactance of B it charges B up, till its potential equals that across the load. During the next half cycle B discharges through the load and smooths out the current wave.

This could equally be accomplished by a condenser in place of B; but the natural evolution leads to the circuit shown in Fig. 304, where the electrode B is what may be called the

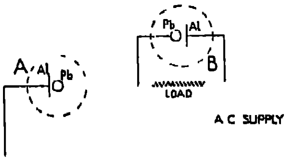

FIG. 303.—Siemens connection for electrolytic rectifier.

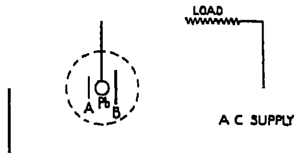

FIG. 304.—Modified Siemens connection for electrolytic rectifiers.

condenser electrode, and A the one used for the passage of the main current. Bairsto has investigated this particular type of cell and has shown that with any given ratio of plate area A to

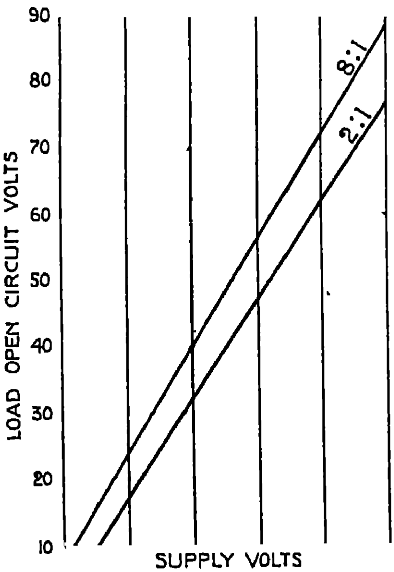

FIG. 305.—Voltage relationship with Siemens connection.

B the supply voltage bears a linear relation to the open circuit load voltage.

Fig. 305 illustrates this point, where curves for plate area ratios of 8 : 1 and 2 : 1 are given. In the case of the 8 : 1 ratio,

B consists of eight plates each equal in superficial area to the plate A. The B plates were formed in a solution of ammonium borate at 150 volts, and the single plate A at 300 volts. The lead cathode was shaped to interleaf with those of A and B.

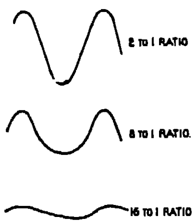

FIG. 306. — Current waves with Siemens connection.

Fig. 306 shows the type of current wave obtained with various plate ratios, and as would be expected the increasing capacity with the increasing ratios tends to smooth out the wave.

The voltage regulation with such an arrangement is given in Fig. 307.

This method may be applied to two or even three phases of supply, and Fig. 308 indicates the connection of anodes required for two- and three-phase systems.

In the case of these multiphase supplies similar wave forms

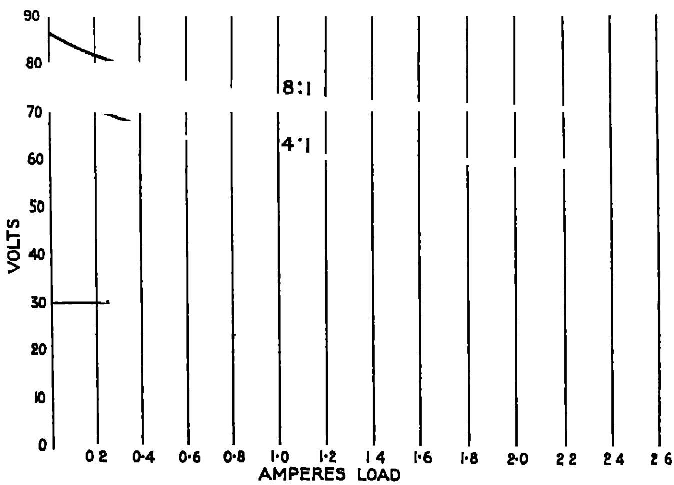

FIG. 307.—Voltage regulation with Siemens connection.

are obtained, viz. a decreased undulatoriness with increased ratios of B to A_1, A_2 and A_3.

Balkite Rectifier.—A practical rectifier employing tantalum and lead electrodes is now available for the charging of small 6 volt batteries, and can be used for the supply of high tension

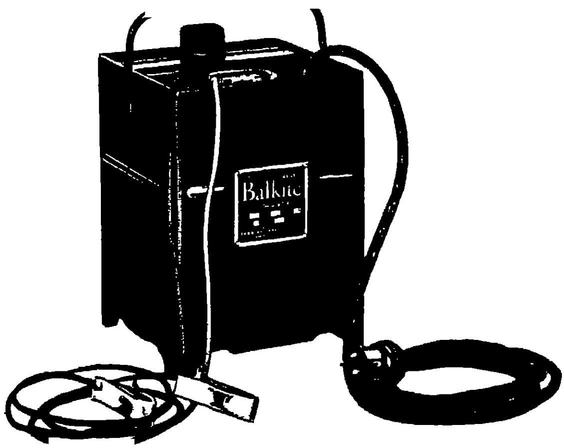

FIG. 309.—Balkite rectifier.

[*To face page* 427.

to wireless sets. The general form of the apparatus, which embodies a transformer in its containing case is illustrated in Fig. 309.

It will be noted that the apparatus is simple to operate, and as it employs the same electrolyte, at the same specific gravity as an ordinary accumulator (1200 S.G.), little is required beyond the ordinary equipment. The only exception to this is the addition of a small quantity of ferrous sulphate (two to three ounces of concentrated solution to a rectifier, or one ounce of solution to every pint of sulphuric acid of S.G. 1200). If a rectifier is used without this addition it will function, but with a greatly decreased current capacity. This is indicated by experiments carried out by Robinson, the results of which are given in Table XXXVII.

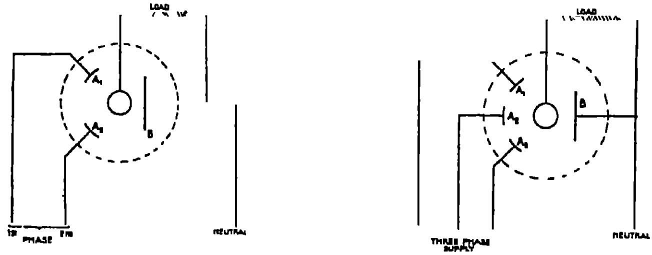

FIG. 308 *a* and *b*.—Polyphase connection.

TABLE XXXVII.

Without Ferrous Sulphate.

Volts to Neutral.	D.C. Current.	Remarks
15	2·6	No battery in circuit
25	6·0	No battery in circuit
15	0·75	6 volt battery ,,
25	2·0	6 volt battery ,,

With Ferrous Sulphate.

Volts to Neutral.	D.C. Current.	Remarks
15	8·2	No battery in circuit
25	12·3	No battery in circuit
15	3·5	6 volt battery ,,
25	7·5	6 volt battery ,,

At the same time this addition of ferrous sulphate prevents the disintegration of the tantalum which appears otherwise to be very rapid at the surfaces of the electrodes.

The above tests were taken with a biphase system of rectification.

Nitric acid with carbon and tantalum electrodes would produce a more perfect rectifier, but the use of this electrolyte is not recommended.

Design of Rectifiers.—In the design of electrolytic rectifiers, it is important to bear in mind the physical characteristics briefly enunciated above. Schulze lays down the following essential conditions:—

(i) Perfect rectification must be obtained.

(ii) The maximum voltage of formation (page 403) must be higher than the peak value of the alternating current supply.

(iii) Electrostatic capacity must be small.

(iv) The minimum voltage (page 405) must be small in the permeable direction.

(v) The specific resistance of the electrolyte must be low.

(vi) The cooling surface must be such as to prevent the temperature from rising above 40° C. at full load.

These points are interdependent to a large extent, as if a high rectified voltage is required the minimum voltage will be high, and the electrolyte must be diluted so that the maximum voltage of formation is well above the alternating peak voltage of supply. This means a high specific resistance, and consequently higher losses, and a greater temperature rise.

As regards the practical manufacture of an aluminium rectifier, if it is required for use on a large scale, or in a position where little attention is possible, it is advisable to obtain the advice of the makers as to the suitability of any particular type, and to purchase from a reliable firm. But in the case of small charging sets it is possible to manufacture a rectifier with little trouble, which, if it is not entirely satisfactory, at least does enable a small 6 volt accumulator to be charged from an A.C. supply where otherwise expensive converters would be required. The following details are therefore put

forward, not in any way as a design for a completely satisfactory rectifier, but as one where, if the enthusiast does not object to a certain amount of trouble and attention, he may accomplish this operation at a moderate cost.

A lead cylinder should be obtained which may consist of a plate 10 centimetres wide by 15 centimetres long, bent into the form of a cylinder 5 or 6 centimetres in diameter, which forms the cathode of the cell. The anode is made from an aluminium plate 9·5 centimetres high by 1·7 centimetres and 0·3 centimetre thick. Both anode and cathode are immersed in the electrolyte in an earthenware pot. The electrolyte should consist of a saturated solution of ammonium phosphate. With an alternating supply voltage of about 90, 27 volts (mean value) direct current will be obtained. The current should not be allowed to fall below 0·1 ampere per square centimetre of plate area, and in this example will give the best results if kept at about 1·6 amperes. The voltage can be regulated by a series resistance or a small transformer across the supply mains, with tappings at intervals. As the polarisation sets in, the current will decrease, but after about half an hour it will attain a steady value. It is always advisable to keep the supply voltage low, as if the critical voltage is exceeded the cell will not rectify, and current will pass in either direction. It is preferable, if it can be arranged, that a transformer should be inserted to reduce the voltage to about 45 and this will result in a rectified voltage of 13, which is sufficient for a topping up charge of a 12 volt accumulator. The performance of the cell will be improved if cooling arrangements can be introduced in the form of a lead or composition pipe circulating cold water round the cathode.

If ammonium carbonate is employed, the temperature should under no circumstances be allowed to exceed 50° C. as decomposition sets in, at that point. On the other hand, the conductivity increases at higher temperatures, and thus the best temperature is of the order of 40° C.

Use of Electrolytic Rectifiers at High Temperatures.—The use of rectifiers at high temperatures has proved an interesting speculation, and has actually been accomplished in

spite of the fact that with ordinary electrolytes a critical temperature is reached beyond which rectification ceases. Schulze has experimented with a cell where the electrolyte consists of molten sodium nitrate at a temperature of 330 degrees centigrade, and the type of wave form obtained is indicated in Fig. 310.

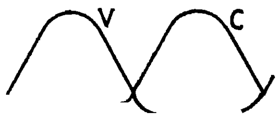

FIG. 310.—Current wave form in high temperature rectifier.

The curves show that under these circumstances the voltage is constant during the period of current flow and therefore the cell does not obey Ohm's Law, and functions in a different fashion from that previously considered. The nature of the reaction is obscure and no satisfactory explanation has yet been put forward.

Use of Electrolytic Rectifiers at High Pressures.—Rectifying cells have also been constructed which operate under pressure. Carman and Balzer have successfully obtained a cell which will function under a pressure of 20 atmospheres. They found that the rectification decreased with increasing pressure, and are inclined to deduce from this fact that Schulze's theory of the mechanical action of the gas film is supported by experimental data, as it would be expected that this gas film would decrease in thickness under increasing pressure.

Future Work.—Experimental work is still continuing with a view to establishing a theory which will meet all the facts known in electrolytic rectification. So far this has only met with partial success, and the problem must be relegated to the band of those still unsolved. The present state of the work is not due to lack of tried observers but to the difficulty of making the observations, and obtaining consistent results.

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PART V.

SOLID CONTACT RECTIFIERS

PART V.

SOLID CONTACT RECTIFIERS.

CHAPTER XVIII.

WIRELESS CRYSTAL RECTIFIERS.

THE improvement in capacity of several types of crystal and disc rectifiers renders them important for the charging of small batteries and similar purposes.

The physical action of none of them is so far well understood, and little can be written beyond a description of their characteristics and a brief discussion on the more important theories.

The greatest amount of work has been done on the wireless crystal, viz. that which is only available for minute currents; but there are two recent additions to the solid contact rectifier which are of considerable interest, and are considered in the next chapter.

Later in this chapter various theories of crystal rectification are developed, but in order that they may be more clearly understood it is necessary to bear in mind three phenomena which may attend the passage of current through a conductor.

Peltier Effect.—When a current passes across the junction of two dissimilar metals or conductors, heat is absorbed or liberated in accordance with a reversible process. Thus heat is absorbed or produced according to the direction of the current, as for example in the case of a copper-iron circuit, if the current is forced from the copper to the iron, the junction is cooled and conversely. This heating is accompanied by the generation of

an E.M.F., called the Thermo-electric E.M.F., and if the external current is made to operate in the direction of the thermo-electric current the junction is cooled, and if in opposition to it the junction is heated; further, the heat generated or absorbed is proportional to the first power of the current, or

$$H_P = a\int idt.$$

It will be appreciated that there must always be two junctions present, and thus the heating of one is accompanied by the cooling of the other, the current doing reversible work.

Joulean Effect.—Joule discovered that if a current passes along a metal conductor, heat is evolved, the amount of heat being proportional to the square of the current, and to the resistance of the conductor, whence

$$H_J = b\int i^2 Rdt.$$

Thomson Effect.—If a current passes through a conductor XY, where X is at a temperature T_1 and Y at T_2 then the heat developed in XY when unit electricity flows from X to Y is

$$H_T = \sigma(T_1 - T_2)$$

where σ is the specific heat of electricity.

The Thomson Effect will not be apparent when the temperature gradients are small and may as a rule be neglected.

Theories of Crystal Rectification.—There is no definite proof that the correct theory of crystal rectification has yet been discovered, although numerous theories have been advanced, and of the physical phenomena which have been suggested as being the cause of rectification, electrostatics, electrolysis, thermo-electricity and combinations of these principles have severally and collectively received the attention of those seeking to solve the problem. Till recently the only theory which had survived the criticism of experimental evidence was that which was advanced by Dr. Eccles where it was supposed that when current passes across the junction of two dissimilar materials, and the resistance of the contact is high, the current flowing causes

Joulean heating, and therefore a Peltier E.M.F. both in its positive and negative periods. In one half cycle the Peltier E.M.F. assists, and in the other it opposes the current. Thus perfect rectification will only result when the respective E.M.Fs. accurately balance.

Dr. Eccles has further shown that the heat is likely to be localised in a small volume of the crystal round the point of contact ; but at the same time it is continually conducted away from that point: nevertheless high temperatures may be reached which have been estimated by some at 300 degrees centigrade.

Later developments have suggested that the high resistance contact which is necessary for the generation of the Joulean heat is due to a molecular layer on the crystal face, and that it is only the presence of this layer which permits rectification to take place. This has been borne out to a certain extent by experiments conducted at the Research Laboratories of the General Electric Co. on the use of galena as a rectifier, where it was discovered that the heating of the crystal in an atmosphere of sulphur considerably enhanced the rectifying properties.

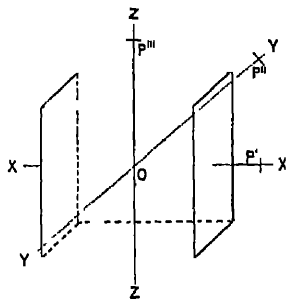

FIG. 311.—Crystalline structure of galena.

Owen has enlarged upon the thermo-electric theory and suggests that experimental data is indicative of two regions of heat development. For instance, the immediate contact surface would consist of a molecular layer where Peltier heat would be developed; surrounding this molecular layer, but of much smaller dimensions, is a region of the crystal in which Joulean heat and Thomson heat are set free.

Several objections have been advanced against the thermo-electric theory, but perhaps the most serious is that of the inequality between various crystals in their rectifying properties, even when especial precautions are taken to ensure that the results shall be consistent.

A theory based on the crystalline structure of galena has

received some support, although again it is not complete in its explanation of all the observed data.

If a cube is considered, as shown in Fig. 311, and its true centre ascertained, which will be the intersection of straight lines drawn through the centres of opposite faces, it will be situated at O; if now equal intercepts are taken on the three axes OX, OY, OZ, as shown at OP′, OP″, and OP‴, a plane drawn through the three points P′, P″, and P‴ will intersect the cube in what is called the (1, 1, 1) plane. It is apparent that eight such planes can be drawn and the solid resulting from cutting off the intersections of the planes will be an octohedron. (A model can readily be made up in plasticine to illustrate the effect.)

In such a crystal it would be expected that the metal ions would move more freely in the (1, 1, 1) plane than in any other, where they would collide with sulphur ions, by having to pass through a network of sulphur ions. If an alternating current passes through the crystal, lead ions will pass to the crystal causing metallic conduction across the crystal. When the current reverses the sulphur film is restored and the resistance to the passage of current rises.

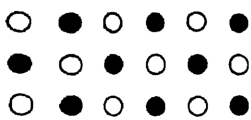

Fig. 312.

Thus it would be expected that the greatest rectification would occur in the (1, 1, 1) plane, or when the current passes in a certain direction across the crystal. James has shown that the rectification ratio in the (1, 1, 1) plane is 15·9 to 1, and in other planes it is of the order of 1 or 1·5 to 1, thus confirming this aspect of the theory.

Galena is lead sulphide in which there are equal numbers of atoms of lead and sulphur. If, therefore, the atoms are so arranged that a sulphur atom is always adjacent to a lead atom, as shown in Fig. 312, in which the black dots represent the lead and the circles the sulphur atoms, then it can be shown that the octohedron face will always expose either a surface of lead atoms or sulphur atoms, with resulting unbalanced electrostatic forces.

James, in the "Philosophical Magazine," has advanced many arguments, the preceding one of them, for the acceptance of an electrolytic theory of crystal rectification, and instances the formation of metal threads by electrolytic action, which would explain the hysteretic nature of the steady current curve. The effect of the addition of impurities in the crystal also appears to bear out the possible truth of the theory.

On the other hand, Strachan has lately suggested in an important series of articles that the loose contact forms a metal contact in one direction and a dielectric in the other. This phenomenon is probably associated only with such substances as galena, which are neither insulators nor good conductors of electricity, and which normally possess a high specific resistance when good contact is made with the conductor. This will also be bound up with the fact that at all surfaces the molecules have a greater degree of freedom than in the interior of the substance; and in the particular material or crystal which is functioning as a rectifier the orientation of the molecules on the surface may result in a given orientation of "axes of conductivity," which are assumed to be reversible when a good contact is made, as against a non-reversible direction when only loose contact is made.

A molecular layer of sulphur would be obtained if octohedral galena could be prepared, but so far this has not been possible, so as to test the theory thoroughly, as galena has a cubic structure, and to obtain it in any other form is difficult. The theory is ingenious and may have some foundation in fact; at any rate it is worthy of consideration. Against this theory is the fact that X-ray examination fails to indicate any preponderance of the (1, 1, 1) face in specimens of good galena as compared with poor specimens.

Effect of Pressure, Temperature, Frequency and Current.—Flowers has investigated the effect of numerous variable conditions to which crystals are usually subjected:—

Effect of Pressure.—Variations of pressure from 4 to 550 grammes and from 60 to 100,000 cycles produced very little variation in rectification. This does not altogether accord with

actual practice in the case of galena-metal contacts where there is certainly a marked optimum pressure range. With carborundum and molybdenite there is apparently no such optimum pressure.

Effect of Temperature.—Rectification decreases with increasing temperature. Between 200° and 300° C. rectification disappears altogether.

Effect of Frequency.—It is considered likely in view of later experiments that frequency has little effect.

Effect of Current.—Experiment shows a marked increase in rectification with increasing current, and an investigation would indicate a square law connecting current and rectification ratio.

Effect of Current Density or Size of Contact Point.—Increasing the size of the contact point would appear to decrease the rectification ratio, and as would be expected higher voltages have to be employed to obtain the same current values.

Instability appears, however, when very small currents are used, when also, under certain circumstances, reversal of current is experienced.

The above results may be combined and would indicate:—

(i) That for good rectification a certain minimum current density must be exceeded;

(ii) Resistance in series with the crystal decreases the rectification;

(iii) For large currents the rectification at high frequencies is likely to be greater than with lower currents at low frequencies.

Effect of Time.—The effect of time on rectification is marked, and it will be observed that this is in accord with the thermo-electric theory, whereby the heat which is generated necessarily takes a certain time to leak away. Thus when the time of application of the voltage across a zincite crystal was 3×10^{-3} seconds the resistance was found to be about 1500 ohms, whereas when the duration of current flow was increased to 15 seconds the resistance increased to 2000 ohms.

Materials Used.—Numerous types of crystal have been used as rectifiers with more or less success. During the war, the

Army Wireless sets were equipped with two types, viz. Perikon which is a combination of one crystal of zincite and one of chalcopyrite; and carborundum, which was used in conjunction with a hardened steel disc. At the present time most of the crystals employed in small wireless sets consist of lead sulphide or galena. This is used either in the natural state and specially treated, or is manufactured from its constituent parts. Herzite, Permanite and other trade names are given to galena crystals, but this list does not by any means exhaust the materials from which rectification can be effected. A bismuth-aluminium combination will rectify although the results are not so good on account of the better conductivity of the metals. On the

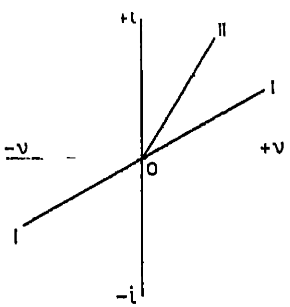

FIG. 313.—Characteristics of crystal rectification.

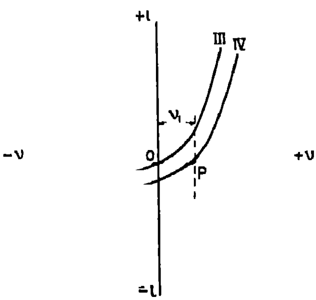

FIG. 314.—Volt-ampere characteristics of crystal rectifier.

other hand, molybdenite-graphite or copper is an excellent combination.

Current-Voltage Characteristics.—In Fig. 313 curve I represents a state of affairs where rectification does not take place, because for a given positive voltage a certain positive current will flow and for the same negative voltage an equal negative current will be allowed to pass. Also curve II, which continues in linear form to the origin, and then is coincident with the axis $(0, -v)$, will represent a good rectifier as no negative current will pass with a negative voltage. It is then legitimate to conclude that as the slope of curve II is a measure of the resistance of the rectifier contact, if this resistance is zero the

best form of rectification will ensue; this is represented by the coincidence of curve II with the axis (0, + i), and it is to be inferred that the line (+ i, 0, − v) is the volt-ampere characteristic of a perfect crystal rectifier. This is never attained in practice, because it has been postulated that for a high thermo-electric E.M.F. a high resistance contact is of importance. Hence a compromise must be effected to ensure the best results. Such curves usually take the form of curve III in Fig. 314

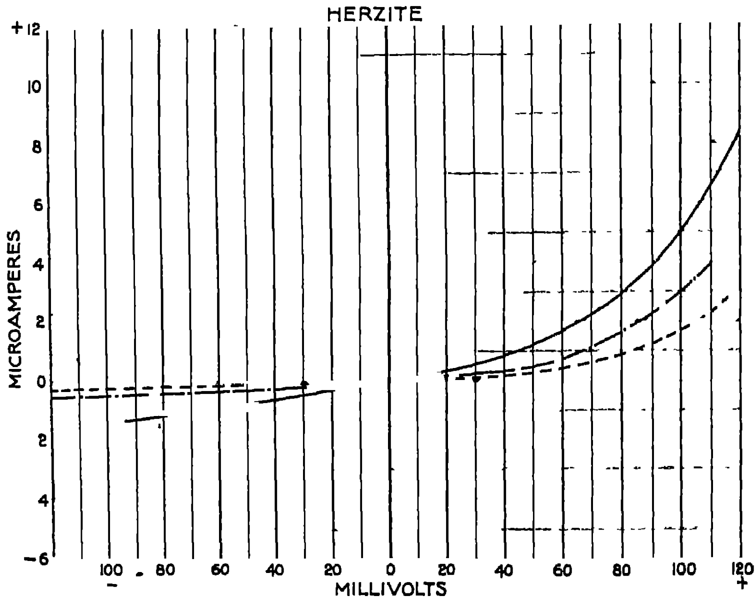

FIG 315.—Herzite crystal with low voltages.

where a slight negative current is permitted to pass when the voltage is negative. The measure of a good crystal therefore is primarily concerned with the portion of the curve in the (− −) quadrant, and it is only after it is clear that this portion of the curve is relatively unimportant that a consideration of the curve in the (+ +) quadrant is worth undertaking.

It is sometimes found that a characteristic curve takes the form of IV in Fig. 314, where it is noticed that the voltage axis is intersected at a point which is not the origin. This is no

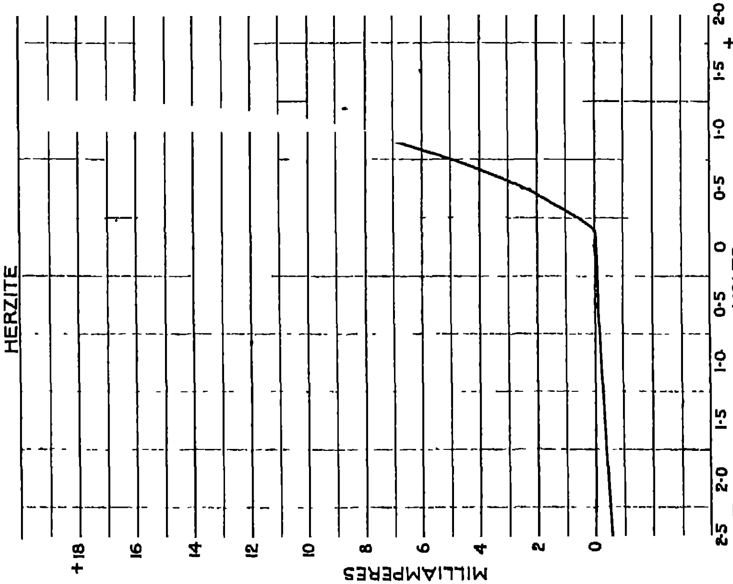

FIG. 316.—Herzite crystal with high voltage.

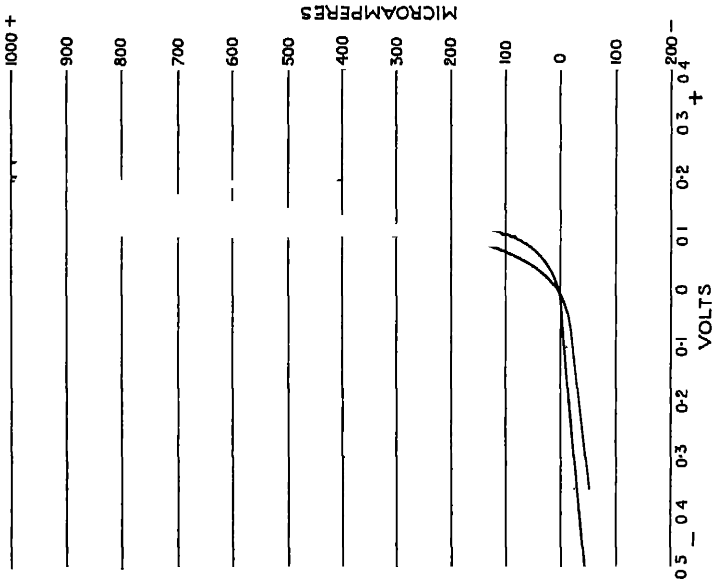

FIG. 317.—Wembley crystal with high voltage.

detriment to the rectification properties of the crystal, if by any means it is possible by suitable arrangements to shift the axis $(+ i, - i)$ to such a position that P will become the new origin. This is attained by impressing a constant voltage $(- v_1)$ on the crystal before the characteristic curve is taken. A curve of this nature is found in the carborundum-steel combination and in consequence in the wireless sets during the war a potentiometer and a dry battery were included which enabled this voltage to be permanently applied; the exact setting of P was obtained by a sliding contact on the rheostat until the best signals were received, which corresponds to the new origin being situated at P.

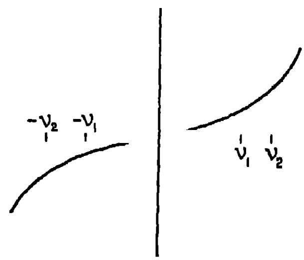

FIG. 318.—Characteristic curve under extra high voltage.

Three sets of curves are given of the characteristics of various crystals. Figs. 315 and 317 indicate the type of rectification obtained with low and high voltages and with different crystals, and Fig. 316 shows that the crystal should have a voltage of 0·12 impressed on it to obtain the best results.

If the characteristic curves are taken for higher voltages, and therefore larger currents, a curve such as that depicted in Fig. 318 will be obtained, and it will be observed that whereas at the lower voltage v_1 the rectification is good, when the voltage is increased to v_2 the negative current corresponding to $- v_2$ is considerably greater and the rectification is thereby reduced. Although this curve is not taken from any one particular case it is typical of what may be expected.

Effective Polarity of Crystals.—Strachan has shown that for practical purposes, crystal rectifiers may be considered as

having a definite polarity in that the direction of the rectified current across the crystal determines to a certain extent its magnitude. This terminology must be used with discretion as it must not be assumed that the crystal acts as a generator of electrical energy.

To illustrate the effect of this so-called polarity the current across a galena crystal in one direction may be 0·25 milliampere, whereas in the other it may be 0·75 milliampere. And, on the other hand, with a copper silver sulphide crystal (Straneyerite) the respective currents were 0·3 and 0·1 milliampere respectively. This would suggest that for wireless reception, where telephones are suited for a current in one direction, it may be advisable to connect the circuit in the right direction.

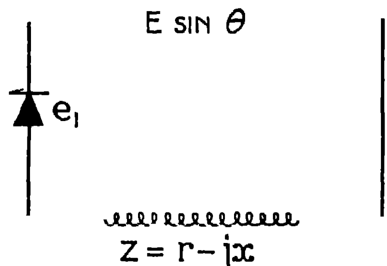

FIG. 319.—Simple crystal circuit.

In such cases as are illustrated in Fig. 314 where a given potential is applied to the crystal by means of a potential divider, a higher potential will be required in one direction than that necessary for the same signal strength in the other.

The importance of this phenomenon which has not yet been satisfactorily explained is of such an order that it may be of considerable advantage to take the trouble to ascertain the correct polarity of the circuit before it is finally completed.

Theoretical Analysis.—As regards the mathematical analysis of the circuit, in Fig. 319 an alternating E.M.F. $E \sin \theta$ is impressed on a crystal in which the drop in voltage is

$$e_1 = f(i)$$

and the rectified voltage is absorbed in a load of impedance

$$Z = r - jx.$$

The differential equation connecting the various quantities will then be

$$E \sin \theta - e_1 = ri + x\frac{di}{d\theta}.$$

This equation cannot be solved unless e_1 is known, and is some simple function of the current. In any particular case if it may be assumed that the voltage variation is small, the relation between the two may be considered to be linear, or

$$x\frac{di}{d\theta} + ri + ki = E \sin \theta.$$

The solution of this equation is similar to that employed in considering the relationships for a mercury vapour rectifier, and is of the form

$$i = A \sin (\theta - \phi) + B\epsilon^{-c\theta}$$

in which A, B, c and ϕ are determined from the initial conditions. Usually the exponential term in the calculations can be omitted.

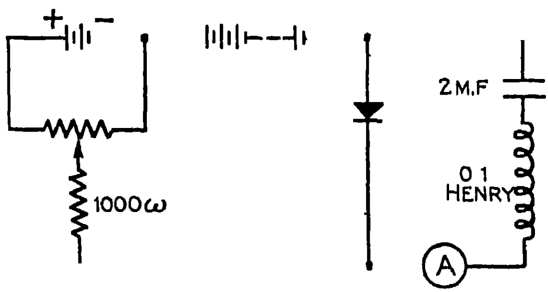

FIG. 320.—Oscillating crystal circuit.

Oscillating Crystals.—Oscillating circuits formed by crystals and suitable impedances have been suggested as a replacement of thermionic triodes, and already a considerable amount of research has resulted in the production of a combination which will give satisfactory results once it has been finally adjusted.

Lossev has shown that such oscillations can be generated from a circuit as is illustrated in Fig. 320, and states that when the circuit oscillates, scintillations of light accompany the passage of current.

Lossev shows that the characteristic curve is as shown in Fig. 321, where the property of negative resistance is indicated, and he contends that the shape of the curve depends on the production of a minute arc across the contact.

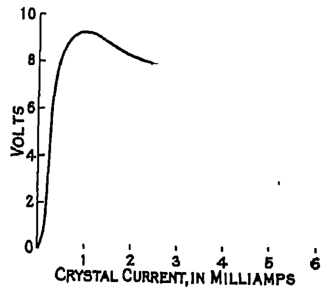

FIG. 321.—Characteristic of Oscillating Crystal.

General Conclusions.—It will have been gathered from the foregoing that whilst more data is now available on the way in which crystals function as rectifiers, it still cannot be said that a complete solution of the problem has been found. In this case there is the factor that the crystal has been largely superseded by the thermionic valve, and development has been restricted to the latter, the former problem being considered to be of academic interest only. It may easily transpire, however, that the triode will again have a rival in the crystal, in which case there is little doubt that the relative claims would be settled on a basis of cost.

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CHAPTER XIX.

LARGER CAPACITY SOLID CONTACT RECTIFIERS.

THE advent of the relatively high capacity solid contact rectifiers, which will rectify currents of the order of from two to three amperes, has created considerable interest in wireless circles. The obvious disadvantages of the electrolytic rectifier on account of the employment of acid or alkali, the sparking of the vibrating reed, and the renewals of the gas tube, all tend to depreciate their value. A dry rectifier with a reasonable life, therefore, will be received with satisfaction especially as there is reasonable expectation of high efficiency and long life.

Copper Disc Rectifier.—A new and highly interesting rectifier has recently been described in the "Journal of the American Institute of Electrical Engineers" (March, 1927), by Grondahl and Geizer. This together with the pin-crystal represent the simplest rectifier yet developed, and both are worthy of the greatest consideration.

The only information obtainable up to the present is that in the publication above mentioned; and this description is therefore restricted to a reproduction of several of the characteristic curves there given.

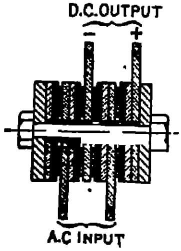

FIG. 322 *a*.—Copper disc rectifier.

The original discovery was of the fact that a copper disc covered with a layer of cuprous oxide had a greater resistance in the one direction than in the reverse: in the first case in the ratio of three to one.

Figs. 322 *a* and *b* illustrate the development of this result into a practical rectifier of the biphase

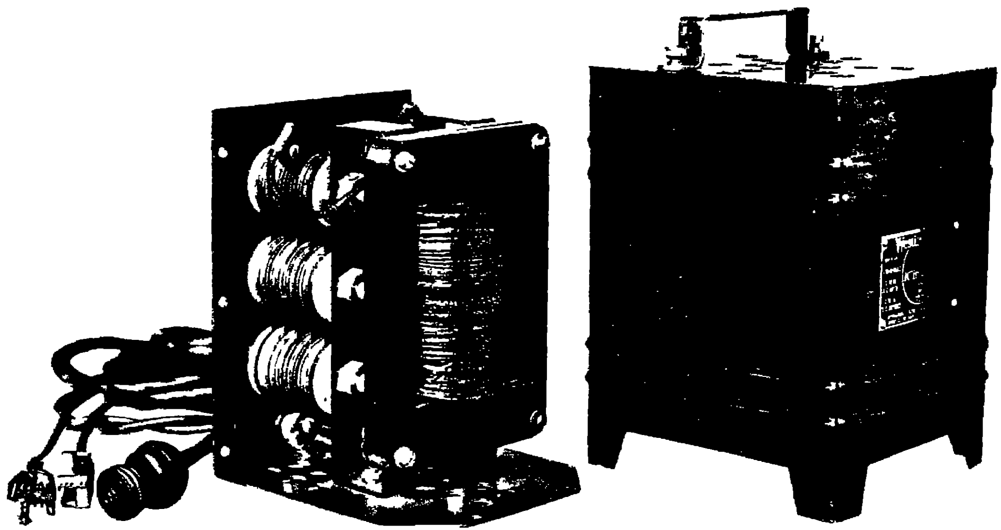

FIG. 322 *b*.—Copper disc rectifier.

[*To face page* 450.

type. Fig. 322 *b* shows the apparatus as marketed by the Westinghouse Co. of America.

The elements are normally 1·5 inches in diameter and the four elements making up the unit shown in Fig. 322 *a* will supply current at 6 volts. With regard to the current density, this depends largely on the cooling possible, and in some cases ventilating fins may be advisable for extra radiation. When the current density exceeds two amperes per square inch, the elements should be immersed in oil, and if fins are added the current density can be increased to 3·5 amperes per square inch.

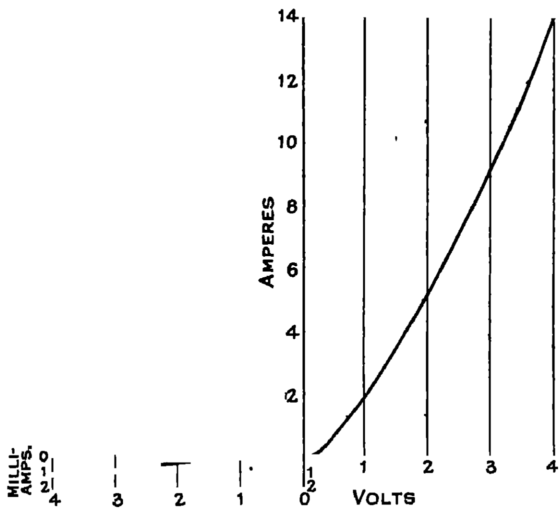

FIG. 323.—Current-voltage characteristics of copper disc rectifier.

It is emphasised that the rectifier is remarkably consistent, and that large currents can be carried on account of the low internal resistance. This point is indicated in Fig. 323 which shows a resemblance to the characteristics of the crystal rectifier in Figs. 315 to 317.

These two curves lead to the resistance-voltage curves of Fig. 324, and the rectification ratio curve of Fig. 325.

As regards efficiency, the curves in Fig. 326 show the variations

due to changes in watt input, which is given as the ratio of direct current output to alternating current watts input (see page 221). These curves refer to a rectifier of 1·5 inch elements as shown in Fig. 322.

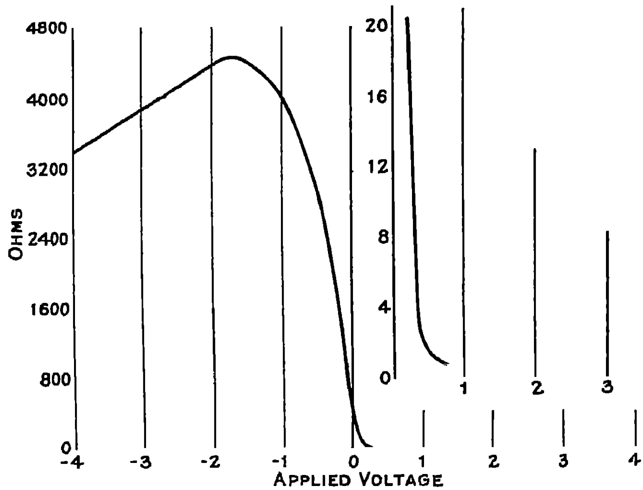

FIG. 324.—Resistance-voltage curves of copper disc rectifier.

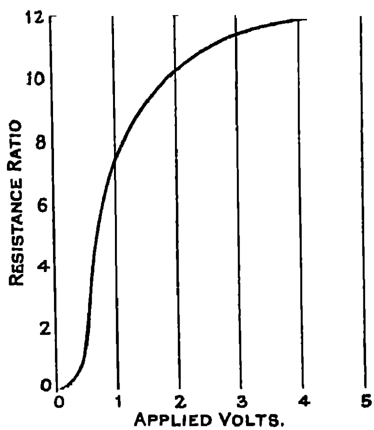

FIG. 325.—Rectification ratio of copper disc rectifier.

The oscillograms of Fig. 327 indicate a form factor of 1·13 which varies to 1·25 with different units. The resistance of the unit is not constant but varies with the voltage, and thus on a

battery charging circuit decreases as the charging rate decreases. The voltage regulation of the rectifier can be changed by varying the weight of copper; and, for example, it is increased from

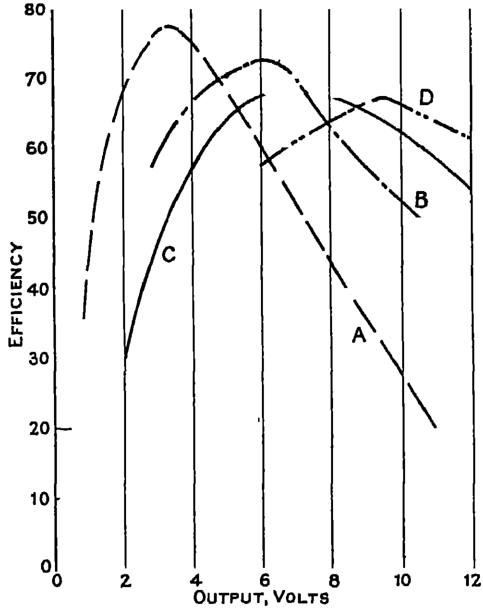

FIG. 326.—Efficiency curves of copper disc rectifier.

16·5 per cent. to 8·5 per cent. by doubling the weight of the metal.

As regards the bulk of a rectifier, if it is assumed that 1·5 inch elements are used, it is found that 200 are required to rectify 1 K.W. Therefore, the bulk of a 4 K.W. unit is 1 cubic foot and weighs 80 pounds.

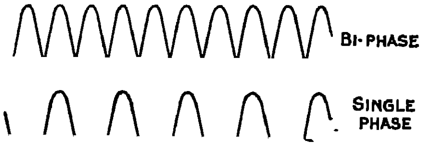

FIG. 327.—Oscillograms of copper disc rectifier.

The life of the copper disc appears to be very long: thus after one year's continuous operation at a constant current

density of 0·5 ampere per square inch, there appeared to be no products of electrolysis.

The theory of operation appears to be obscure, but it is suggested that it does not depend on thermo-electricity or electrolysis. One of the authors has proposed a theory that due to the intimate relationships of the copper and the oxide, electrons are able to escape in quantities from the copper to its oxide, and the action is analogous to that of the hot cathodic filament in a thermionic rectifier. The potential gradient carries the electrons from the copper to the oxide, and the diffusion of electrons from the copper, or in other words, the space charge, opposes their passage in the reverse direction on the reversal of the E.M.F.

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Pin-Crystal Rectifier.—This type of rectifier is essentially of the crystal variety, and consists in one form of a silver pin and a crystal of iron pyrites with which the pin is pressed in tight contact, as shown in Fig. 328.

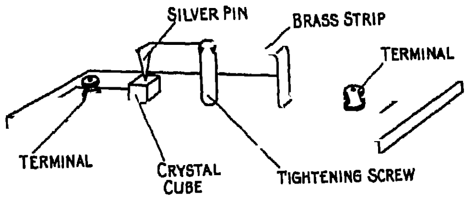

Fig. 328.—Pin-crystal rectifier.

With regard to the capacity of the unit, which has been described in the Patent Specification, each crystal will safely rectify 10 milliamperes, the current being limited by the heat dissipation at the point of contact; and it is unsafe to apply a greater potential than 7 volts to any one unit.

Thus theoretically for "trickle" charging a 15-volt battery at 0·120 ampere, twenty-four units are required, twelve in parallel and in two banks in series; whilst to rectify the high tension supply for a typical wireless receiving set, from the supply mains, several such units would be wanted connected in series.

In the manufacture of the rectifier, considerable care in formation is necessary, and at the same time the pressure of the silver pin is important.

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PART VI.

WIRELESS RECTIFIERS.

PART VI.

WIRELESS RECTIFIERS AND RADIO SUPPLIES.

CHAPTER XX.

WIRELESS RECTIFICATION AND THREE-ELECTRODE VALVES.

Wireless Detectors.—Detectors in wireless engineering and radio reception come into the category of rectifiers because devices which will rectify an alternating current will in general, and with suitable connections and apparatus, enable high frequency oscillations to be detected and made audible in a telephone or loud speaker.

In modern wireless practice, detection is restricted to the use of two types of rectifier :—

(i) The diode or triode, which is chiefly used on medium and large sized installations, and

(ii) The crystal in combination with the triode for certain medium power receivers, and alone in the smaller types.

The present chapter and Chapter XVIII. are devoted to a consideration of items (i) and (ii), although it is believed that electrolytic rectifiers and photo-electric cells amongst others have been used for this purpose, but there is no evidence at the present time that they will replace existing methods.

The mechanism of detection may be understood by a consideration of the following statements, which are general in their application :—

In Fig. 329, R is any form of rectifier with the above restrictions, inserted in the aerial circuit, and C is a small condenser. High frequency oscillations received on the aerial pass

through R, and so long as the current in the circuit is assymmetrical the rectification factor does not matter. There is thus a net bias on the current either in the positive or negative directions, and again the direction of this bias does not affect the argument. A net charge is received on the condenser across which the potential increases so long as the signals are being received. When they cease the condenser commences to discharge (unilaterally) through the telephone and audible signals are produced.

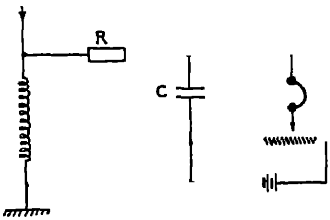

FIG. 329.—Simple radio receiving circuit.

This cycle of events is illustrated graphically in Fig. 330, where curve (*a*) indicates the receipt of a train of waves of high frequency such as might be obtained from a spark set; (*b*) shows how the rectifier is able to eliminate the reverse

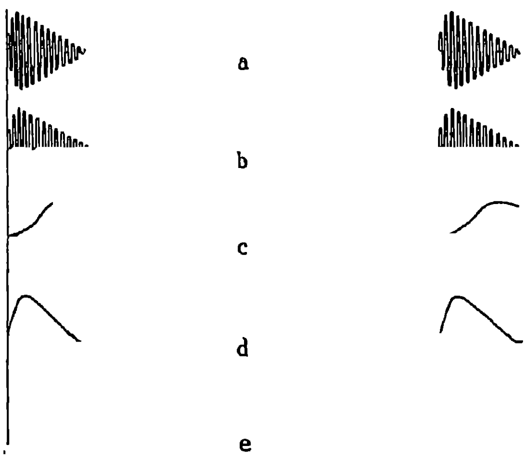

FIG. 330.—Curves of condenser charge and telephone current.

current, in (*c*) the condenser, which commences with a zero charge, becomes charged as the signals proceed, and thence gradually discharges; this is shown in (*d*); and in (*e*) the current through the telephone is illustrated, the area of the

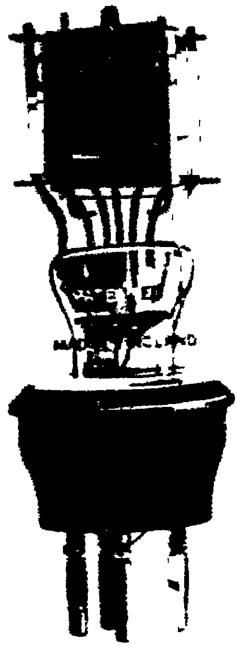

FIG. 331 a.—D.E. 5-valve.

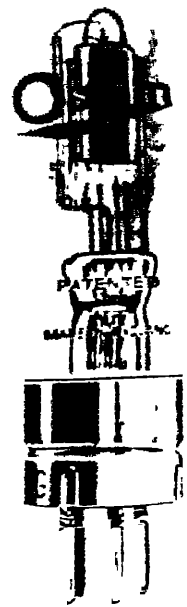

FIG. 331 b—L.S. 5-valve.

[To face page 461.

curve being equal to the difference of the areas of the loops of (*d*).

Three-Electrode Valves.

Characteristics.—As this chapter is only concerned with the detection of high frequency oscillations, the chief uses of the triode as an amplifier or an oscillator are not considered, but as regards its characteristics it is more reliable in action than its parent, the diode or two-electrode valve, as the controlling effect of the grid is much more certain than that of the filament emission and variation of plate voltage in the latter case.

In design and form the triode is similar to the diode, with the exception that between the anode and the filament a loosely-coiled spiral or mesh of wire is placed which is termed the grid. In the larger valves the mesh is commonly employed, but in the case of the smaller special valves, viz. the DE5 and LS5 valves, illustrated in Fig. 331 *a* and *b* respectively, the grid takes the form of a more or less closely wound spiral, the pitch of which largely determines the impedance of the valve.

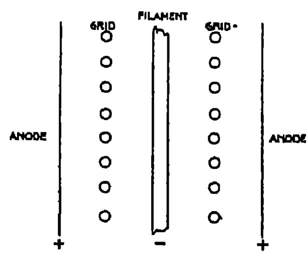

Fig. 332.—Section of triode.

A diagrammatic representation of a triode is given in Fig. 332, where the small circles represent the grid spiral. Thus it will be seen that the influence of a potential on the grid will be either to retard the flow of electrons or to accelerate it, as they are emitted from the filament, according to whether the grid is positive or negative with respect to the filament.

The effect of this grid potential is greater than would be expected, and in Fig. 333 *a* and *b* the characteristics of two types of valve are given, which illustrate this point.

From these curves it may be noted that a small variation in grid potential for a given anode voltage produces a greater

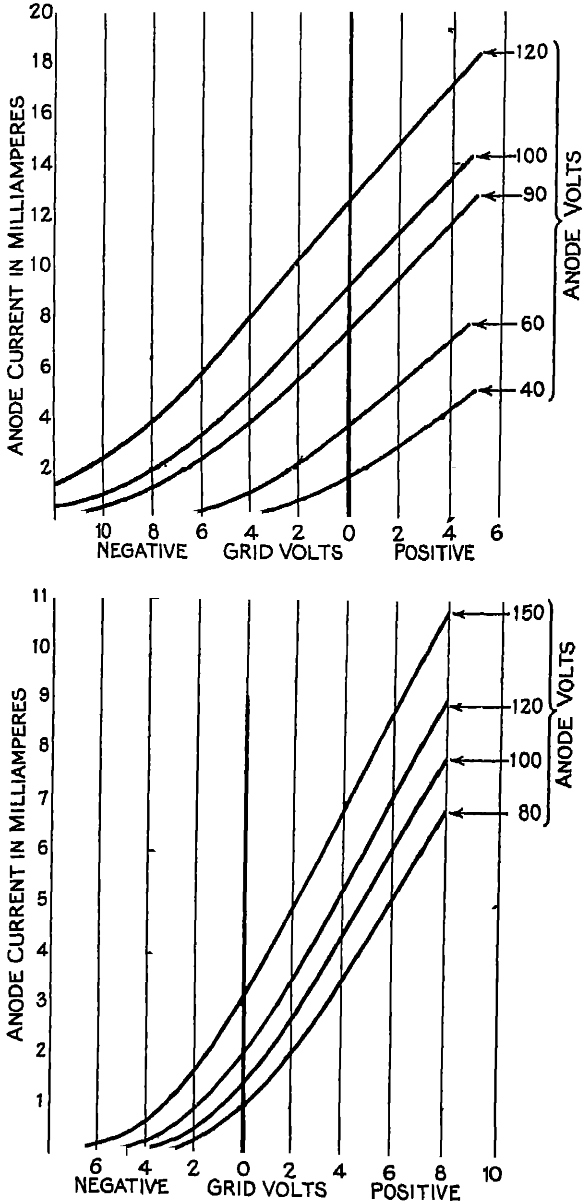

FIG. 333 *a* and *b*.—Characteristics of DE5 and DE5*b* valves.

variation in anode current than a similar variation in voltage would do if the grid were not present.

There are two principal methods of detection of signals

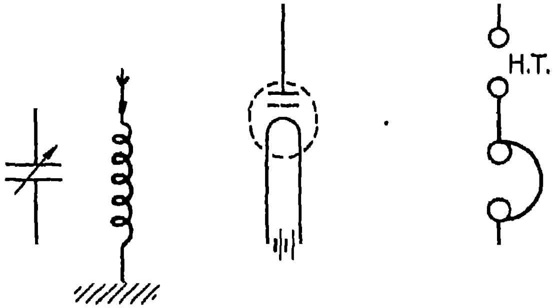

FIG. 334.—Anode rectification.

which are in use, (1) anode rectification, and (2) grid leak rectification. These methods are briefly considered below, but for any detailed analysis, text-books on the subject should be consulted.

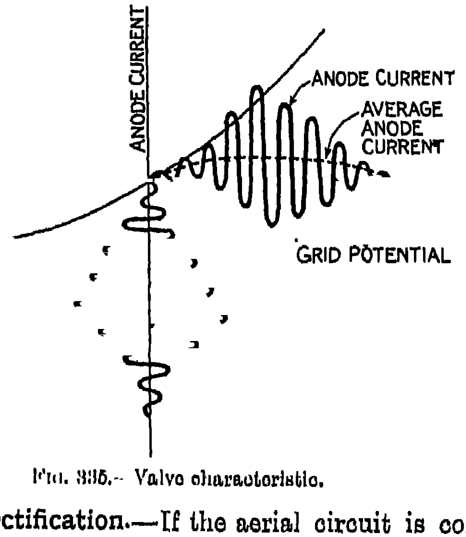

FIG. 335.—Valve characteristic.

(1) **Anode Rectification.**—If the aerial circuit is connected directly to the grid of a valve, as shown in Fig. 334, which has characteristics represented by the continuous curve in Fig. 335,

then the grid voltage is seen to be oscillating in nature, and from the valve characteristics, the actual anode current variations can be deduced graphically; but owing to the fact that the characteristic is not linear, this anode current will have an average value which is not zero, and which will take the form of the dotted line in Fig. 335. The effective current in the telephones thus depends on the amplitude of the audio frequency current or average anode current, and as this amplitude is dependent on the curvature of the characteristic, the particular portion of the curve at which the incoming wave train operates, will determine the signal efficiency, and it is possible, if the characteristic is approximately linear for no signal to be audible. The particular portion on which the signals operate

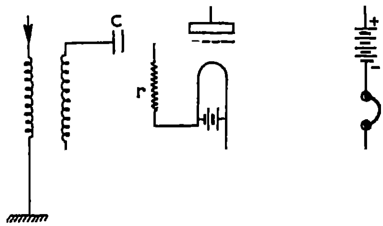

FIG. 336.—Grid leak detector circuit.

can be varied by including an external E.M.F. impressed on the grid, or to a less extent by varying the anode potential.

(2) **Grid Leak Detection.**—In this case a small condenser C and a resistance (grid leak) are included in the circuit as shown in Fig. 336.

It is now necessary to consider the grid current characteristic of the valve.

During the time when no signals are received, it is assumed that the valve is functioning in a stable fashion. It is usual to connect the resistance r to the positive leg of the filament, in which case the drop of voltage in the filament is represented by the length of the line OB in Fig. 337, and if cot a is made equal to r, the point of intersection of BA with the curve A will indicate the normal potential of the grid, viz. OC. If oscillating signals are impressed on the grid, the grid voltage will swing

about the point A. Thus a similar series of distorted oscillations will occur about the point A and electrons will accumulate on the condenser plate connected to the grid; this will cause the

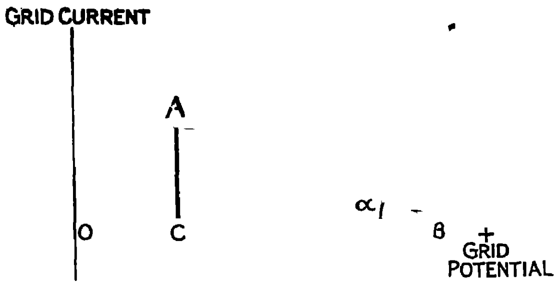

FIG. 337.—Grid current characteristic.

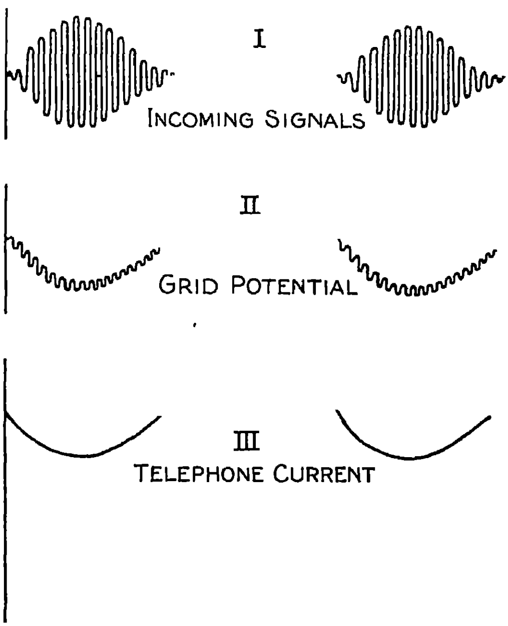

FIG. 338.—Current waves with grid leak.

potential of the grid to be lowered, which will react on the plate current in accordance with the curvature of the valve characteristic. This accumulated charge must be allowed to leak away

before the next train of waves arrives, and this is the function of the leak, the time of leakage depending on the circuit constants r and C.

The form of the oscillations is shown in Fig. 338.

This description of the detecting properties of the three-electrode valve is necessarily brief. Any further consideration would lead to a lengthy description and mathematical analysis. Moullin and Turner have discussed the action of the triode as a detector in an excellent article which considers the problem in all its aspects, and to which the reader is referred for further details.

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Exp. W.	1924, p. 93.
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Elect.	1926, p. 288.

CHAPTER XXI.

RADIO SUPPLIES, HIGH AND LOW TENSION.

The increase in the number of users of wireless apparatus renders the supply of electrical energy in small quantities, but under stringent conditions, imperative, and up to the present time the only commercial methods available are the use of accumulators for filament supplies, or, in certain particular cases, dry cells; and dry cells for the anode current, or so-called high tension supply.

These two supplies require further specification as to the amount of energy and the constancy of voltage required; and it is therefore necessary to consider each case separately.

Filament Supply.

When a lamp filament is subjected to the passage of a variable current, its candle-power or light output will also vary to a greater or lesser degree depending on the diameter of the filament, its length and the cooling due to the leading-in wires, its constitution and the frequency of supply. The fluctuations may vary from the seventh to the eleventh power of the current due to these causes. This is exemplified in the case of a lamp on a 25 cycle supply where the flicker is clearly visible, whereas on a 50 cycle supply no objectional candle-power variation is noticeable. In the case of a thermionic valve the electron emission follows a different law from that of the light output (see page 297), and the functioning of a lamp provides little data as to the operation of a valve on a variable supply. Even assuming it is possible to increase the temperature lag of the filament by some means, it is probable that the alternating electrostatic field will cause an objectional hum.

That this is of little import in certain valves has already been proved, but apparently both the detector and power amplifiers should be operated from a constant source. If, therefore, direct current is required for any filament supply it is probably cheaper to make provision for it for all the valves.

On a direct current main the commutator ripple is so small that the filaments can be lit from the domestic supply in series or in parallel as may be desired.

Thus the conclusion is reached at once that on A.C. circuits, filaments can only be supplied through a rectifier, or by a separate battery or accumulator, the latter alternative entailing charging arrangements or sending the battery to a local contractor to be charged.

The small rectifiers which can be supplied for charging batteries are now so reliable that it can only be on the grounds of expense that they are not universally employed in the domestic house; and if the problem is considered from the financial side, there is no doubt that the interest on the capital outlay of any of the rectifiers enumerated below will be less per annum than the cost of charging by the local contractor.

For houses with direct current supplies there are five alternatives available, viz. :—

(*a*) Supplying the filaments in parallel from the mains.
(*b*) Supplying the filaments in series from the mains.
(*c*) An accumulator and charging at home
(*d*) An accumulator and sending out to charge.
(*e*) A thermopile.

Thus the question of the filament supply can be stated in tabular form as follows :—

TABLE XXXVIII.

FILAMENT SUPPLY.

A.C Supply.	D.C. Supply.
(*a*) Transformer with rectifier, and supplying direct. This method is not at present satisfactory.	(*a*) Supplying from the mains with filaments in parallel.
(*b*) Accumulator, and charging at home with a rectifier.	(*b*) Supplying from the mains with filaments in series.
(*c*) Using a valve with a big temperature lag. These valves in the form of the indirectly heated cathode are now available.	(*c*) Accumulator and charging at home (*see below*).
(*d*) Accumulator and sending out for charging.	(*d*) Accumulator and sending out for charging.
(*e*) Thermo-electric methods.	(*e*) Thermo-electric methods.

Consider the practicabilities at the present time :—

Alternating Current Supply.—(*b*) The following types of rectifier can be employed :—

(i) Electrolytic, vide the Balkite (page 426).

(ii) Vibrating reed (page 137).

(iii) Thermionic rectifier (page 300).

(iv) Gas-filled tube (page 350).

(v) Synchronous commutator (page 100).

(vi) Plain motor generator set, which has not received attention in this book, as it presents no unusual features.

(vii) Solid contact rectifiers (see page 450).

It is not proposed to deal at all with the proper care and maintenance of accumulators; suffice it to say that whichever of these devices is chosen, it is preferable that it be selected with a view to providing sufficient current to charge at the proper rate which is stated on the case of the accumulator, and to remember that a higher charging rate is better than a lower one. It is a choice of two evils, but the former is to be preferred.

A further qualification is that it is better to employ a rectifier which does not spark, as this may cause annoyance to other wireless apparatus in the neighbourhood

The other alternatives (*a*) and (*d*) are not considered as they are outside the realm of practical politics in the case of (*a*), and are unhappily too well known in the case of (*d*).

(*e*) *Thermo-electric Methods.*—This type of supply depends on the fact that a junction of dissimilar metals, when heated, is a generator of electrical energy (page 435), and it can be stated definitely that if the practical difficulties of design could be overcome, a thermopile would provide the most useful form of supply, as it could be independently heated by an oil flame, gas jet, or electric heating element, and no rapid fluctuation of the supply would cause an equivalent variation of the filament voltage. Unfortunately no such device has yet been designed, which is completely satisfactory, but there is some hope that in the near future the ideal thermopile will be available for very small power outputs.

Direct Current Supply.—Where filaments are supplied from the mains, care must be taken to avoid earthing the electricity

authority's mains, as this is contrary to the regulations. The difficulty can usually be overcome by inserting a two-microfarad condenser between the wireless set and the earth.

There is little to choose between alternatives (*a*) and (*b*). In the former case, the filament rheostat must be in parallel with the filament, and, in the latter case, in series with it; a lamp or resistance is also required in series with the filament circuits and their rheostats, to reduce the voltage to approximately its proper value, and to absorb the excess energy. The size of valves employed will decide this loss of energy, which will be equal to the total current taken from the mains, multiplied by the voltage.

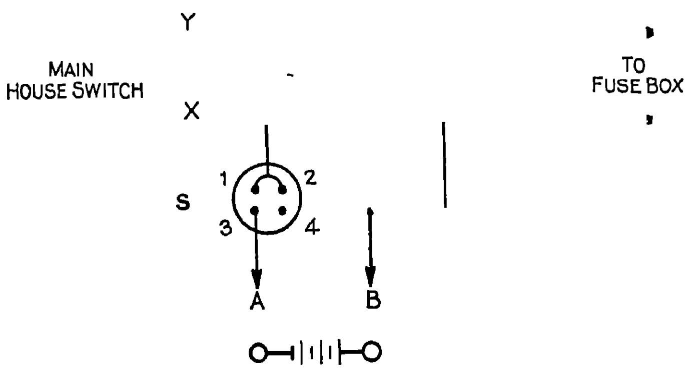

FIG. 339.—Battery charging from direct current supply.

Further, it is to be remembered that if the filaments are in series, the voltage from valve to valve increases.

(*c*) (1) There is a device open to all users of direct current, and it is surprising that it is not more frequently employed.

It is illustrated in Fig. 339, and merely consists in installing a switch S with four terminals 1, 2, 3, and 4; 1 and 2 of which are permanently strapped together; in one position of the switch arm terminals 2 and 3 are connected, and in the other connection is made between 1 and 4. An H and H switch can be purchased cheaply which will perform this function. X and Y are the main supply cables interrupted at a convenient point; and the two leads A and B are led from the switch and are labelled + and − *ve*. The battery to be charged is connected

to A and B, and when the switch is operated the lights in the house will be slightly dimmed when the battery is correctly joined to the circuit. If the lights are brightened, the leads should be reversed. By these means all the current taken in the house lighting circuit will be employed to charge the battery, but it cannot be too strongly emphasised that usually only a trickling charge will be obtained, and the life of the battery will thereby be impaired.

The convenience of the method, however, and the fact that battery charging costs literally nothing, will probably make it worth while on financial grounds alone, to renew the battery more often, and have the added advantage of charging on the premises at a cheap rate. It will also be apparent that charging only takes place when lamps are switched on. In introducing the switch S into the circuit, it is advisable to insert it in the lead of the supply which is at earth potential. The correct terminal can be ascertained by temporarily connecting one side of a lamp to a water pipe, and the other side to either point X or Y. The lamp not lighting will indicate the earth lead. Finally, it is advisable to stand the battery on charge on paraffin wax blocks, so that no leak to earth is possible.

(ii) The other method of charging cells at home is to use a simple circuit with lamps in series to reduce the voltage and absorb the excess energy. By these means the correct charging current can be obtained, but the method is too costly to be frequently employed.

(*e*) *Thermopile.*—The same arguments apply here as have been used above.

Anode Supply.

The normal method is to employ a dry battery, which can now be obtained in a reliable form, but which is more costly to maintain than is often appreciated. The next alternative is a battery of small accumulators, which must be charged at frequent intervals, if their rated output is to be maintained. The charging of these small batteries at home is possible, but on account of their small size they require expert attention, and their use by unqualified attendants is not recommended.

Supplying current from the mains is more easily performed for the anode circuit than is the case with the filaments, and the different methods available are now enumerated. At the same time it must not be overlooked that the anode current is much more susceptible to fluctuation than are the filaments, and greater precautions must accordingly be taken to avoid all current variation.

The various methods available are indicated in Table XXXIX.

TABLE XXXIX.

Alternating Current Supply	Direct Current Supply
(a) Thermionic rectifier and filter	(a) Plain filter circuit
(b) Neon rectifier, and filter or smoother	(b) Bucket control
(c) Thermopile	(c) Thermopile
(d) Synchronous Commutator	
(e) Solid contact rectifier	

Alternating Current Supply.—

(a) *Thermionic Rectifier.*—This is a convenient method to employ, and is easily manufactured by those who have access to the necessary tools. A transformer is required with a secondary voltage of about 340 volts R.M.S., and provided with a middle tapping; at the same time tappings are wanted near the midpoint of the winding, for the supply of the rectifier filaments, which in this case can be supplied by alternating current.

A biphase wave form will result, and this is smoothed by a suitable filter or smoothing device, both of which are described below (page 309). The advantage of this system is that the anode circuit is isolated from the mains, and the wireless set can be earthed as required.

The general arrangement of connections in Fig. 340, and the design particulars are as follows :—

Primary Winding of Transformer.—Two coils of 600 turns of No. 36 D.S.C. wire.

Secondary Winding of Transformer for Filaments.—Two coils of 42 turns of No. 20 D.C.C. wire.

Secondary Winding of Transformer for Anode Circuit.—Two coils of 750 turns of No. 36 D.S.C. wire, each tapped at 250 and 500 turns.

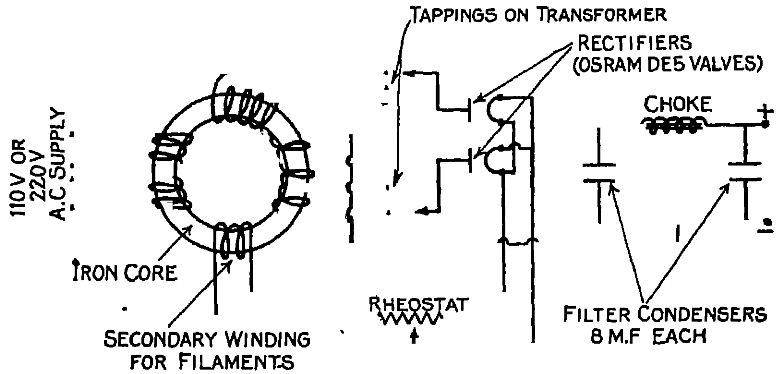

FIG. 340.—Rectifier circuit for wireless supply from A.C.

The iron circuit is made up of stampings of the size shown in Fig. 341.

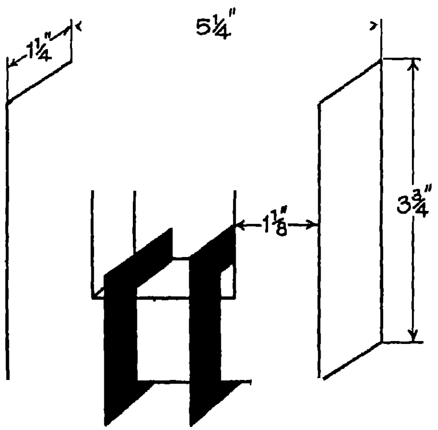

FIG. 341.—Transformer stampings.

The filter circuit is considered later, but the output curves of such a rectifier are given in Fig. 342.

The primary windings can be connected in series for a

220 volt circuit, or in parallel for a 110 volt supply, and the transformer has been designed, primarily for a frequency of 50 cycles.

With regard to the valves to be used, most of the three electrode amplifiers will be suitable, if the grid and anode are connected together. If greater anode currents are wanted, a set of four valves can be constructed, consisting of two banks of two in parallel. This is possible with the above

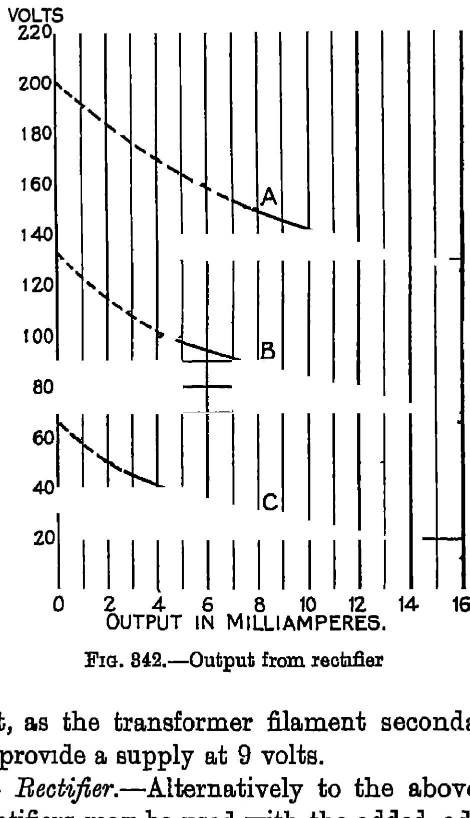

FIG. 342.—Output from rectifier

arrangement, as the transformer filament secondary has been designed to provide a supply at 9 volts.

(*b*) *Neon Rectifier.*—Alternatively to the above, large neon gas-filled rectifiers may be used with the added advantage that no filament circuit is required. In this case, however, a 400 volt transformer is required, with an earthed neutral point, as neon rectifiers will not operate satisfactorily much below 200 volts. The transformer illustrated and described above would do equally

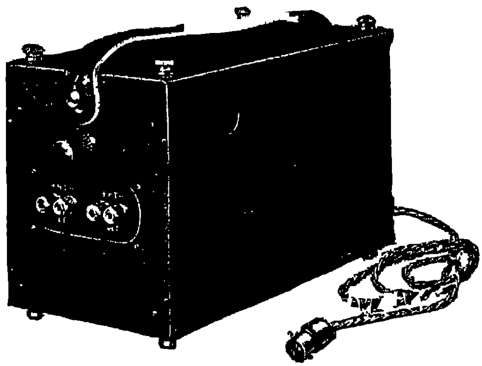

FIG. 343.—G.E.C. H.T. eliminator.

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well for these rectifiers, only the filament circuit would not be wanted.

A filter circuit or smoother must be included as a necessary feature of the circuit.

Commercial types of neon rectifier are now available, and one manufactured by the General Electric Company is illustrated in Fig. 343 ; it employs a tube of the biphase type illustrated in Figs. 247 and 248.

The diagram of connections is given in Fig. 344, and it will be noted that there are three output terminals, viz. a common negative, a low voltage positive with a voltage range of from

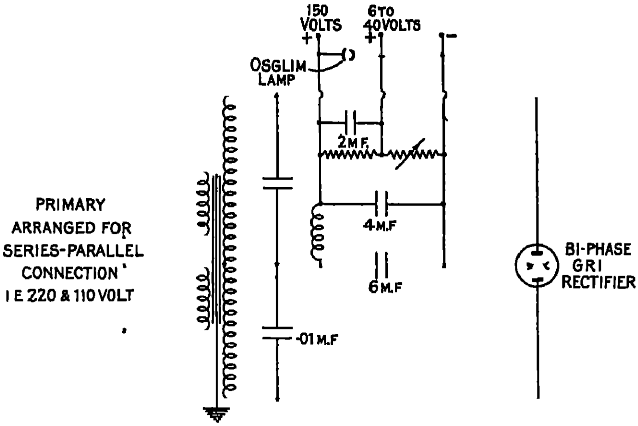

FIG. 344.—Diagram of connections of H.T. eliminator.

6 to 40 volts controlled by a variable rheostat, and a main amplifier positive of 150 volts.

Characteristic curves are given in Fig. 345 for this particular unit, for three primary voltages, viz. 234, 220, and 200. The upper curves refer to the rectifier when the glow discharge lamp is removed, and the lower ones to the state of affairs when it is inserted. It will be apparent that although a lower voltage is obtained in the latter case, yet the voltage regulation is greatly improved.

The excellence of this type of rectifier is well illustrated by the oscillographic records in Fig. 346.

The eliminator must of necessity embody a filter as shown in the diagram of connections, as well as a transformer for raising the voltage of supply to the requisite value.

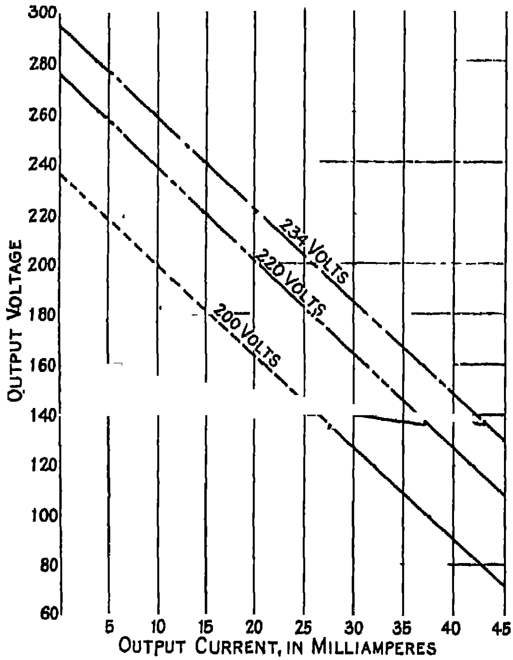

FIG. 345.—Characteristic curves of H.T. eliminator.

Items (*c*) and (*d*) require little amplification here—the thermopile has been discussed above, and the commutator rectifier is expensive, and entails rotating plant.

FIG. 346.—Oscillographs of H.T. eliminator.

(*e*) *Contact Rectifier*.—This has been described on page 450, and little more is needed at this point. It is certainly a possible competitor of the other methods and deserves attention; but

up to the present it is not sufficiently far advanced as a commercial practicability for supplying H.T. The difficulty is in the accurate matching of the several crystals or discs. Thus if each one had identically the same characteristics the voltage drop would be the same; this, however, is rarely if ever the case, and what happens is that the drop differs widely from crystal to crystal: the weakest breaks down and the others follow suit one by one.

Direct Current Supply.—

(a) *Plain Filter Circuit.*—All that is necessary is a plain filter circuit, and two methods are described

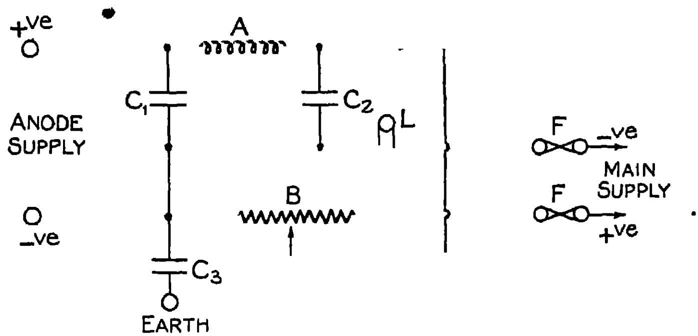

Fig. 347.—Filter circuit.

(i) In Fig. 347 is shown a filter circuit of the ordinary type which can be cheaply constructed. The supply is taken from the mains to the potential divider B and thence to the lamp L and back to the mains.

The slider, or a tapping B is led to the choke coil A and forms the positive terminal, the negative being connected directly to the lamp.

The particulars of the apparatus for a 200 volt supply are as follows:—

Potential Divider	1700 ohms to carry 0·3 ampere.
Lamp	200 volt 40 watt vacuum.
Condensers C_1 and C_2	8 microfarads each.
Condenser C_3	2 microfarads.
Fuses F	1 ampere capacity.

The anode choke consists of 8000 turns of No. 36 D.S.C. wire with a resistance of 600 ohms, wound on the central limb of an iron stamping illustrated in Fig. 348.

The reasons for the complications of Fig. 347 are as follows: Lamp L is inserted in case of a short-circuit of the condensers C_1 and C_2, and generally functions as a current limiter.

As an alternative to this lamp, which will have an internal resistance of about 400 ohms at the temperature at which it operates, a further rheostat may be employed, of the same resistance. Its contact point or slider will then afford a convenient supply of voltage for the grid bias, and will provide any voltage

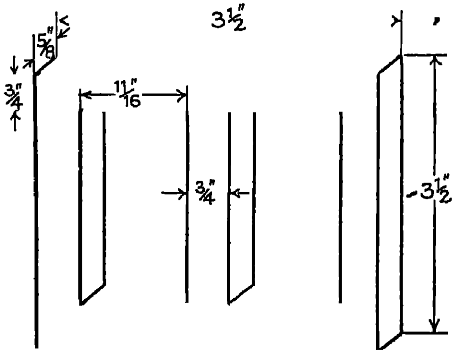

FIG. 348.—Iron stampings for choke.

from zero to 50 volts, but a small filter must under these circumstances be supplied in each grid lead.

Condenser C_3 is provided for the isolation of the supply authority's earth from the earth on the wireless set.

Such a filter circuit can be used indiscriminately where either the positive or negative sides of the supply are at earth potential, but, in the former case, it should be noted that the filament battery and the aerial circuit are 100 to 150 volts below earth potential. If due precautions are taken there is little danger from such an arrangement.

(ii) A new form of filter circuit which so far has not been employed is described in Patent Specification 211268/24 (J. W. Ryde), and depends for its operation on the peculiar

characteristic of the neon discharge tube, and the fact that the cathode fall is independent of the current flowing within certain well-defined limits, and depends only on the design of the tube and its electrodes, and the nature and pressure of the gas.

Thus in Fig. 349, L is the tube, and R_1 is a rheostat especially adjusted to provide a normal cathode fall, and R_2 is a potential divider from which any voltage from zero to the supply voltage, less the drop in the apparatus, can be obtained.

The most important condition of operation is that the supply voltage must not be lowered below a minimum value, determined (*a*) by the fact that the glow disappears, and (*b*) the cathode fall is no longer constant.

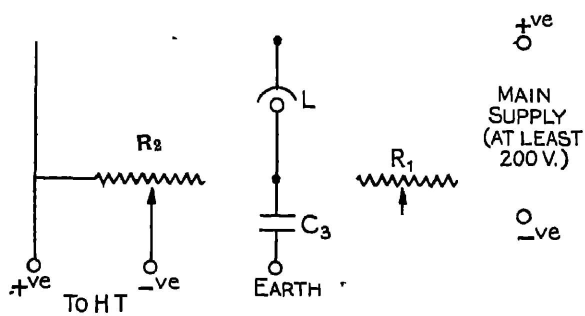

FIG. 349.—Neon tube smoothing device.

There seems to be little reason why such a smoothing circuit should not be combined with any of the rectifiers described above, and it would at the same time embody a cheap and simple system to instal.

A condenser C_3 and earth terminal are also provided to guard against earthing the supply authority's mains.

(*b*) *Bucket Control.*—This method of supply is useful where it is essential that the anode supply should be completely isolated from the mains. This condition is fulfilled by the device illustrated in Fig. 350, where S consists of a four pole change-over switch, or its equivalent (the precise form of which is mentioned below), and F is a form of filter circuit. C_1 and C_2 are condensers, the capacity of which depends on the load

current required. The operation of the arrangement is as follows:—

When the mains are connected to C_1, C_2 is discharging to the load and conversely, and it will be seen that if the switch is operated synchronously on an A.C. supply, the apparatus will provide direct current to the filter, but in the case of a D.C. supply, the speed of operation can be chosen at will, and will affect the smoothness of the current wave to the filter only.

The switch S can take several forms; it can consist of a vibrating reed controlled by an electromagnet working on the electric bell principle with the necessary contacts on the armature; or, again, a form of synchronous commutator could

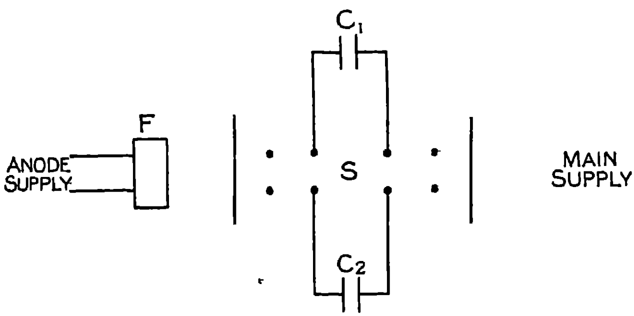

FIG. 350.—Connections of bucket filter.

be designed to function as a four-way change-over switch, but would have the disadvantage of rotating mechanism.

General Precautions.—The above outline of the methods available for the supply of electrical energy to wireless sets is not complete without a word of caution which is necessary in view of the later developments in sound reproduction. The characteristic hum from a fifty cycle supply is of such a low frequency that the loud speakers chiefly to be found at the present time will not reproduce anything objectionable on such a supply. But the improvements of the last few months have completely changed this state of affairs, and filter circuits must now be designed with a view to eliminating the lower frequency harmonics, which will be unpleasantly apparent when changes are made to the newer forms of reproducer.

Forms of Supply.—It will have been noted from what has been said that wireless apparatus can be supplied from the mains when the voltage takes the form of the following:—

200 to 250	volts	alternating	current	of any	periodicity	
100 to 120	,,	,,	,,	,,	,,	
200 to 250	,,	direct current				

and that the real difficulty occurs in the case of the 100 to 120 volt direct current supply, where usually higher voltages than 100 are required for the anode circuit. To cope with this voltage, which will be suitable for all forms of battery charging and filament supply, it is necessary to raise the voltage by some means, and here two methods at least have been developed. Both of them enable the voltage to be raised to any amount (within practical limits) and are worthy of brief description.

High Voltage D.C. Transformation.

(i) *Rotating Commutator and Transformer.*—A mechanism for the transformation of a direct current at low voltage into a

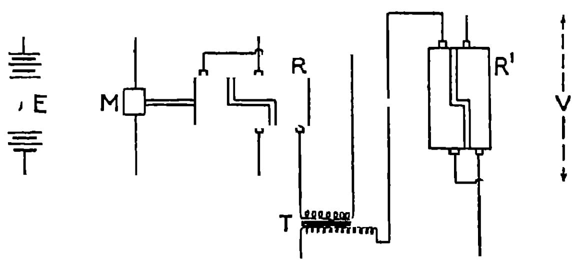

FIG. 351.—High voltage D.C. transformer.

high voltage has been tried in France with successful results, and the method employed is described by Barthelmy (see Fig. 351).

A battery or other source of direct current is connected to a commutator R which is coupled mechanically with another commutator R′ insulated for high voltages. The two commutators are driven by a motor M which need not be supplied from an alternating source as it is not necessary to rotate the commutators synchronously. R converts the D.C. voltage E

into an alternating voltage of irregular wave form which feeds the primary of the transformer T. A high alternating voltage is delivered to the commutator R′ where it is again rectified into a unidirectional voltage. The commutators RR′ are simple to manufacture and little power is required to drive the rotating parts. The speed at which the commutators revolve can be varied and thus the frequency of the supply to the transformer can be made to suit the iron circuit.

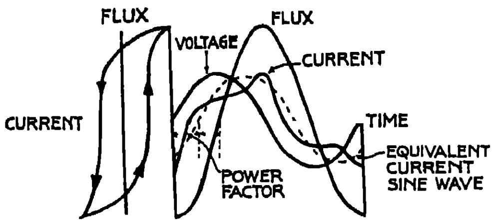

FIG. 352.—Current wave distortion by hysteresis.

The wave form of the input current will be distorted by the iron of the transformer on account of the hysteresis effect, but the amount of distortion can be predicted if the conditions are known.

In Fig. 352, E is the voltage wave of the primary circuit, and since

$$e = k\frac{d\phi}{dt}$$

the flux curve is in quadrature with the E.M.F. if the supply is sinusoidal, as indicated by the curve ϕ. The induction is the flux per unit area of cross-section of iron, and, therefore, from the flux curve which is also sinusoidal the magnetic force curve can be drawn by projecting ordinates from the flux curve on to the cyclic curve of magnetisation. The current is in phase with the magnetic force and therefore the current wave can be obtained.

The voltage of a transformer is proportional to the product

flux × periodicity × number of turns

and as the latter is constant for any one transformer a variation of the periodicity will affect the flux if the voltage is also constant. If the flux is maintained at low values the distortion due to hysteresis is lessened, and the wave form improved on the secondary side; the net result is that the higher the speed of the motor the less the wave distortion, and the better the rectification.

It is not known whether such a device has given trouble in regard to commutation although the idea has been successfully tried as regards the fundamental principles.

(ii) *Condenser Method.*—This method has been developed by Scroggie, and is described in detail in "Experimental Wireless."

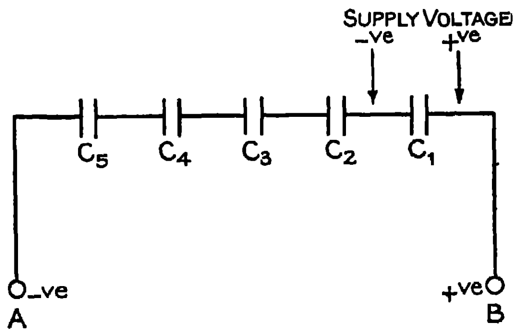

FIG. 353.—Direct current transformer.

It is only intended to give a brief outline of the general arrangements of the circuit.

In Fig. 353 a number of condensers C are connected in series, the outermost terminals forming the anode supply terminals A and B. The number of condensers employed determines the voltage transformation required, due allowance being made for the voltage drop in the system.

The supply voltage is applied to each of the condensers in turn by means of a rotating commutator, the speed of which must be ascertained from the time taken for the condenser to charge and discharge, or from the time constant of the circuit.

The resulting wave form will not be smooth enough for most purposes, and it will usually be necessary to include a filter, such as has been described, between the terminals A and B, and the anode circuit.

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PART VII.

INVERTERS.

PART VII.

INVERTERS.

CHAPTER XXII.

CONVERSION FROM DIRECT CURRENT TO ALTERNATING CURRENT.

IN the Introduction it has been stated as a well-known fact that there is a need for transmission schemes both overhead and underground, which will show a marked improvement over the present standard designs.

The three-phase system, which now predominates, requires a larger factor of safety for a given effective voltage than should be necessary, due to the fact that designs have to be prepared on a basis of a 50 per cent. over-voltage, on account of the peak attained. In the case of insulation troubles this is a serious consideration, but where corona losses are concerned, it may be much more troublesome to cope with, as corona depends on the size of the conductor, and it may happen that the size of the copper installed is a factor of corona rather than its legitimate purpose of carrying current. This is uneconomical, and as corona commences at a certain definite critical voltage anything which will lower this voltage will be welcomed.

Thus on every score a high voltage direct current transmission would be of advantage at the present juncture.

On the other hand, there is no doubt that for generation and transformation, alternating current has great advantages; and a combination of the two schemes would therefore appear to present the solution of problems which cause many troubles. The problem of inversion, viz. the conversion from direct to alternating current is, therefore, of equal import to that of

rectification, and will have to be studied from new standpoints, if the ideal schemes are to be evolved.

There is little known about inversion except in America, and lately in France, where one or two experimental plants have been installed, and so far these experiments do not do more than indicate that theproblem can be solved. In this country the Transverter is now beyond the experimental stage, and would appear to be the only practical solution available. This aspect of the case has already been considered (see page 119), and only the data from the United States and France will now be described.

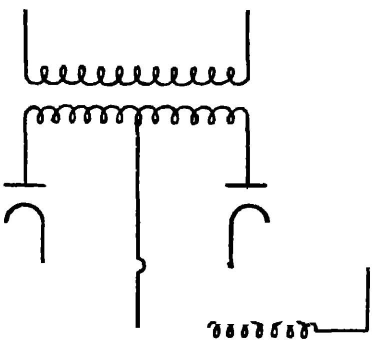

FIG. 354.—Biphase rectification.

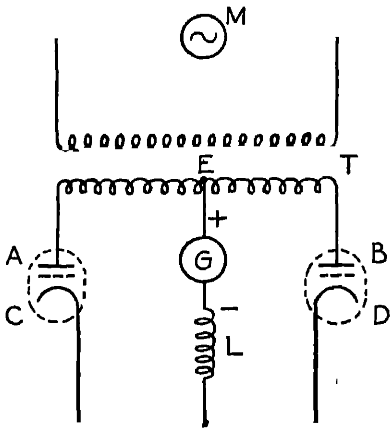

FIG. 355.—The inverter.

The four methods possible are :—

(i) By means of the three-electrode valve and grid control.

(ii) The magnetron effect.

(iii) Mercury vapour rectifier with grid control.

(iv) The transverter.

(i) *Three-Electrode Valve and Grid Control.*—Consider the two circuits illustrated above, viz. Fig. 354, the usual biphase rectifier, and Fig. 355 the inverter.

It has been shown that in the ordinary rectifier circuit (page 186) the inductance between the anode and the transformer, or in the transformer windings, causes a certain overlap of the load currents supplied from the separate anodes, and that during the period of overlap both rectifiers supply current to

the load. In an inverter such as is shown in Fig. 355 a direct current generator G is supplying a steady current to two-three electrode valves and the primary of a transformer, to the secondary of which is connected a synchronous motor M. The function of the valves is to switch over the direct current from one of the primary circuits to the other, and thus reverse the current in each half of the primary winding, in which case a form of alternating current will be available for driving the synchronous motor.

It will be apparent that there are no means available in a two-electrode valve (Fig. 354) whereby this switching operation can be effected, and it is solely due to the insertion of a grid in the diode, that this form of inversion is possible.

Neglect for the moment the precise action of the grid, and assume that some form of tap is present in the two valves which will periodically and consecutively cut off the current; also remember that the valves can only conduct current in the directions AC and BD, or when A and B are positive to C and D respectively. If valve BD is rendered conducting, current will flow via the circuit EBDG, and when AC is conducting via EACG; the current in the primary of the transformer thus changes its direction at each reversal, and induces an alternating E.M.F. in the secondary circuit including the motor. The question of the starting of the motor will be considered later.

At the point where the current is switched from one valve to the other, both valves are conducting, and it will be apparent that current can flow from G via both valves and back through the inductance L, and that if this current is allowed to continue for too long a period it will constitute a short-circuit of the supply. This current is limited by the inductance, and the rate of rise and fall of the grid control.

It is advisable to bear in mind that to all intents and purposes the drop of voltage in the valve is constant, and that so long as A and B are positive to C and D, current of any magnitude up to saturation value can flow, and that, therefore, if the current is to be made to fall gradually it must be accomplished by varying the grid voltage, and, further, any sudden changes in the current will tend to surges which are to be deprecated.

To afford this measure of control, and to eliminate the possibility of short-circuit, the grid of each valve is connected to an alternating supply as shown in Fig. 356, where T′ is taken from one of the windings of the synchronous motor.

With the arrangement shown, the synchronous motor is not self-starting, and has to be run up to its correct speed before the circuit will function correctly; but it will be noted that any circuit which will provide a suitable oscillation on to the grids of the valves will be sufficient to allow current of an alternating character to flow through the main valves. Such

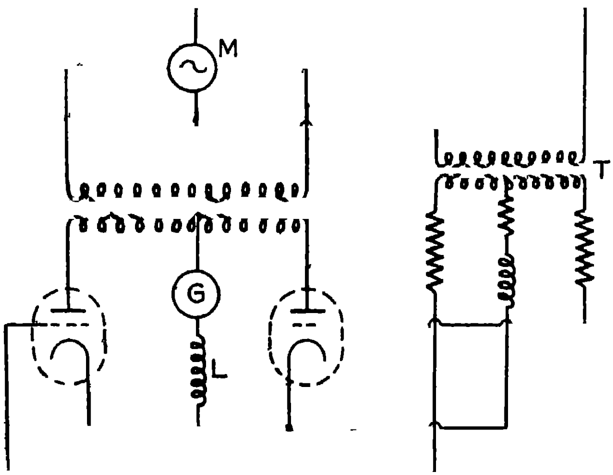

FIG. 356.—Complete inverter circuit.

oscillations can be obtained from an oscillatory circuit consisting of a three-electrode valve with inductance and condensers to provide a given frequency, and a small starting motor employed to run the motor up to speed. The final alternating current supply will be obtained from an alternator coupled to the motor shaft.

The above is only a brief survey of the possibilities of this system of inversion, but enough has been said to demonstrate beyond all doubt that only practical and minor difficulties in design have to be overcome.

(ii) *Magnetron Effect.*—The general theory of the magnetron has been given on page 318, where it was shown that if an electromagnetic field is applied axially to a two-electrode valve, for any given filament and anode diameter, electrons will fail to reach or will all reach the anode according to whether, for any given current, the potential between the anode and the cathode is less or greater than a given critical voltage.

This critical voltage is expressed by the equation

$$E_c = 0{\cdot}01882\ I^2\left\{\log_{10}\frac{d}{d_c}\right\}^2 \text{ volts}$$

where d and d_c are the diameters of the anode and cathode, and I is the filament current in amperes.

This equation can be written for a tungsten filament operating at 2500 degrees Kelvin

$$E_c = 44100\ d^3\left\{\log_{10}\frac{d}{d_c}\right\}^2$$

where d_c is in centimetres.

The magnetic field is always present when current passes through a conductor, and it may therefore happen that the filament current itself will give rise to a magnetron effect. It can be shown that such an effect is quite negligible in the case of small valves, but the following table of calculations by Hull will show that the effect increases with filament diameter.

TABLE XL.

Tungsten at 2500 degrees Kelvin.

Diameter of Anode cms.	Diameter of Filament cms.	Critical Voltage.
5·0	0·0025	0·0075
5·0	0·025	3 6
5·0	0·100	127
5·0	0·250	1140
5·0	1·000	21,600
5·0	2 50	62,300

It can be generally stated that where filaments are carrying 50 amperes or more, care has to be taken to ascertain whether magnetron effect is present or not.

In the case of the ordinary rectifier, this deflection of the electron stream is a definite disadvantage as it will reduce the current passed by the rectifier, and will distort the wave form. But for certain purposes, notably inversion, this distortion may be used to advantage.

Consider the wave form in Fig. 357 which represents the current wave in a rectifier supplied with direct current on the anode at a voltage of 488, and with 12·1 volts across the filament,

Fig. 357.—Magnetron effect.

at which potential it absorbed 1·8 kilowatts. The filament was supplied from an alternating current source, and the curves show how the anode current which should be constant in value, but for the diversion of the electrons, is interrupted. In this case the maximum current was 1·41, and the minimum current 0·08 ampere.

It will be noted that the variation of anode current is at a frequency double that of the supply, and that the effect of the

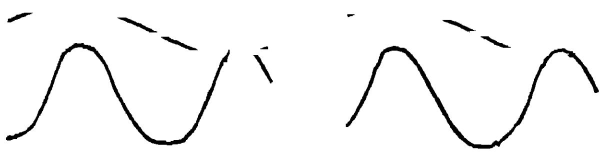

Fig. 358.—Oscillograms of magnetron effect.

variation of filament current throughout the cycle does not cause a sudden discontinuity in the anode current, but that this is a gradual effect.

Next consider another case where the magnetron effect is more marked. In Fig. 358 the filament volts were 14, and 2·4 kilowatts were absorbed in the same valve. The anode potential was 700 volts, and the variation of anode current was between 0·32 and 2·82 amperes.

The striking factor in this oscillogram is the fact that the anode current is a very close approximation to a sinusoidal curve of double frequency of the filament supply, and thus it is not difficult to envisage an arrangement whereby this anode current is passed through a transformer for conversion into alternating current. Such a method would be simpler than the grid control, but suffers from the disadvantage that a variation in anode current varies the wave form. Where this is not important it provides a helpful solution to the problem. The provision of the heating current would preferably come from a separate alternating current supply driven from a motor generator set, although it is conceivable that the starting current could be obtained by similar means to that suggested for the grid control inverter, viz. by an oscillator with inductance and condensers.

(iii) *Mercury Vapour Rectifier with Grid Control.*—This method of inversion, which is, so far as the writer is aware, entirely new, is the result of research by Maurice Leblanc, and is described by H. Giroz in the "Bulletin Société Française des Électriciens." Whether it will be capable of inverting power currents at high voltages is uncertain, but there seems to be no insuperable objection.

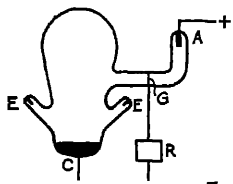

FIG. 359.—Mercury vapour rectifier with grid control.

Consider a mercury vapour rectifier as shown in Fig. 359, which embodies the special feature of the grid G, in addition to the anode A and the cathode C. EE are the auxiliary anodes which maintain the arc as well as an ionised atmosphere of vapour in the bulb, and which are an essential portion of the rectifier when acting as an inverter. The function of the reversing switch R is to apply, as desired, a positive or negative potential to the grid.

The grid fulfils two important functions, and it is essential that they should be clearly understood.

(*a*) If there is no arc struck between the anode and the cathode, it is not possible to cause current to flow if the grid is negative with respect to the cathode.

(*b*) If an arc already exists between *A* and *C* it is impossible to interrupt it merely by rendering the grid negative with respect to the cathode.

This phenomenon is explained by noting that when the arc is struck, the vapour being ionised, positive ions exist which collect as a space charge around the negatively charged grid, neutralising its potential. In the case of the thermionic rectifier, where there are no positive ions, the grid is capable of causing an interruption of the current. In addition, in thermionic emission, a varying charge on the grid will cause a varying flow of anode current. In the mercury vapour inverter, however, such is not the case, and the resistance which during one portion of the cycle is in the neighbourhood of zero, discontinuously approaches infinite values at other portions.

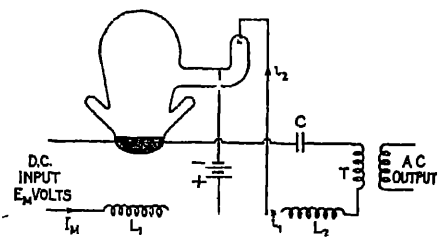

FIG. 360.—Inverter circuit.

Consider the circuit arrangements which have been proposed. In Fig. 360 a rectifier is provided with grid bias cells connected to the grid. In the main direct current circuit, the usual choke L_1 is employed, and the oscillating circuit consists of a second choke L_2, a condenser C, and a transformer T.

The direct current I_M is steady whilst those of the rectifier and oscillator circuits vary, and

$$I_M = i_1 + i_2.$$

The function of L_1 is to maintain the current I_M constant in value, and consider, therefore, the condition when i_2 is zero; this state of affairs is apparent in Fig. 361, where the current and voltage curves are shown.

The supply voltage E_M commences to charge the condenser C until the voltage across the terminals is such that the grid is

negative, and current commences to flow from the anode to the cathode, according to the curves in Fig. 361.

It is easy to see that the process is a continuous one, the periodicity being determined approximately by the expression

$$\frac{1}{2\pi L_2 C}.$$

It will be apparent that this arrangement can only be employed to supply low frequency current in view of the size and expense of the apparatus.

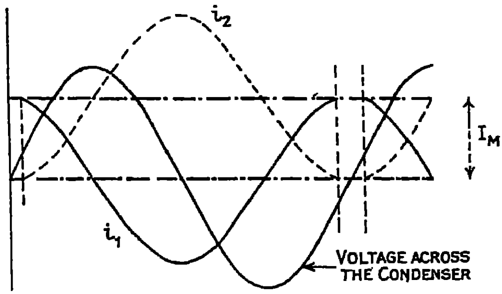

FIG. 361.—Wave forms of inverter.

The following are the values of a particular test :—

D C. supply voltage . .	500 volts.
„ „ current . .	20 amperes.
Frequency	20,000 cycles per second.
Choke L	0·01 henry.
Capacity C	0·006 microfarad.

The oscillation in the L_2C circuit, which will be approximately sinusoidal in shape, can be collected by the transformer T.

It is of interest to note that, whereas in the case of the rectifier only anodes supply current whose voltage exceeds that of all the others, the reverse is true of the inverter, viz. only those anodes whose voltage is less than all others will invert.

Enough has been said to indicate that a very interesting problem is in the course of solution ; it is one which may have far-reaching effects on power transmission in general, and developments will be awaited with interest.

(iv) *Transverter*.—So far, therefore, the Transverter is the only practical method available. It is a reversible machine electrically, and will be at once ready to convert from D.C. to A.C. at high voltages.

Which of these four methods will ultimately be employed or survive the test of experience, it is early to say, but one would preferably eliminate rotary plant, if at all possible; but, on the other hand, the life of thermionic valves is limited at the present juncture, and the dislike naturally enough, of glass bulbs in power houses and substations, is prejudicial to the other methods.

It is highly possible that there will be uses for all, for where large powers are to be converted the transverter will probably be the cheaper in the end, but for small powers of relatively only a few kilowatts there is little doubt that one of the former devices would have many advantages.

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PART VIII.

ALTERNATING CURRENT MEASUREMENTS AND GENERAL USES OF RECTIFIERS

PART VIII.

ALTERNATING CURRENT MEASUREMENTS AND GENERAL USES OF RECTIFERS.

CHAPTER XXIII.

Measurements of Small Alternating Currents and Voltages.—One adaptation of a rectifier, which so far has not been extensively employed, is in the measurement of small alternating currents and voltages. Indicating instruments can be obtained of the dynamometer type which will read to 10 milliamperes or 0·2 volt, but for accurate work their use is often inadmissible on account of their low resistance. Thermo-ammeters can sometimes be employed, but they can easily be damaged by currents only a few per cent. above their rated capacity, and they are therefore not preferred if other means are available.

The use of a rectifier has suggested itself, the actual measurement being made by any convenient direct current instrument with suitable calibration.

Dr. Clayton Sharpe of the New York Testing Laboratories has developed the use of these rectifiers in precision measurements. A synchronous commutator type rectifier is used, which essentially performs the functions of a reversing key. One of the difficulties to be overcome is the proper insulation of the motor from the galvanometer, but it has been finally surmounted by mounting the commutator on a separate bed to the motor and carefully earthing the latter.

Effect of Capacity.—It is important in considering the use of a rectifier in this connection to note the effect of capacity on the resulting indications, as it is more marked than would at

first be expected. Whitehead has investigated this side of the problem and his published results add greatly to the existing knowledge on the subject.

Two methods of connection are possible, viz. the series method, as shown in Fig. 362, in which case one half of the

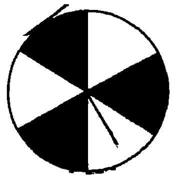


FIG. 362.—Series commutator.

alternating current wave is suppressed, and a single-phase wave form results; or the shunt connection of Fig. 363 which acts as a reversing switch, and a biphase current is delivered.

The series commutator consists of six sectors alternately conducting and insulating, and is driven as usual at synchronous speed. The shunt commutator has four segments, two conducting and two metallic. The supply leads are connected to two slip rings, one ring being joined to each segment.

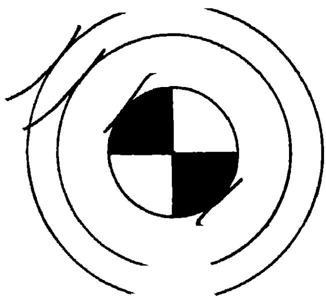
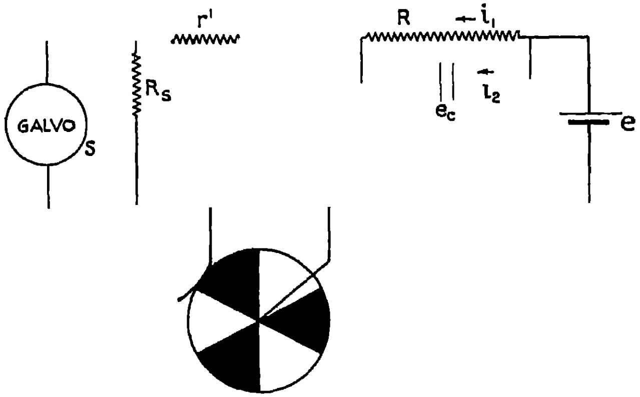


FIG. 363.—Shunt commutator.

The reading on a direct current instrument (other things being equal) will therefore be double that with a shunt commutator to that from a series commutator.

If a direct current supply is used, a condenser will have a greater effect than in the case of an A.C. supply, as the steep

wave front of the discontinuous curve will accentuate the effect of the capacity. In the experiments described a primary battery was used in conjunction with a galvanometer for measuring the current, and the ratio of running to standstill deflection was plotted against the capacity in parallel with a series resistance for both series and shunt commutators. This ratio should be 0·5 if the capacity has no effect or if it is absent. The diagram of connections employed is indicated in Fig. 364.



·FIG. 364.—Connections for testing the effect of capacity in rectifier.

That capacity has a big effect is shown clearly from the performance curves of Fig. 365, where it is demonstrated that a series commutator is much more susceptible to errors from capacity effects than is one connected in shunt. The absolute effect of capacity on a series rectifier is shown in Fig. 366, and it is obvious that great care must be exercised to reduce the capacity of the wiring if accurate measurements are to be made.

The mathematical analysis is briefly as follows :—

Assume that

e is the battery voltage,

s is the galvanometer resistance,

R_s is the galvanometer shunt,

r' is the resistance in series with the galvanometer,

and x_c is the condensive reactance of the circuit.

Then let

$$r = r' + \frac{sR_s}{R_s + s}.$$

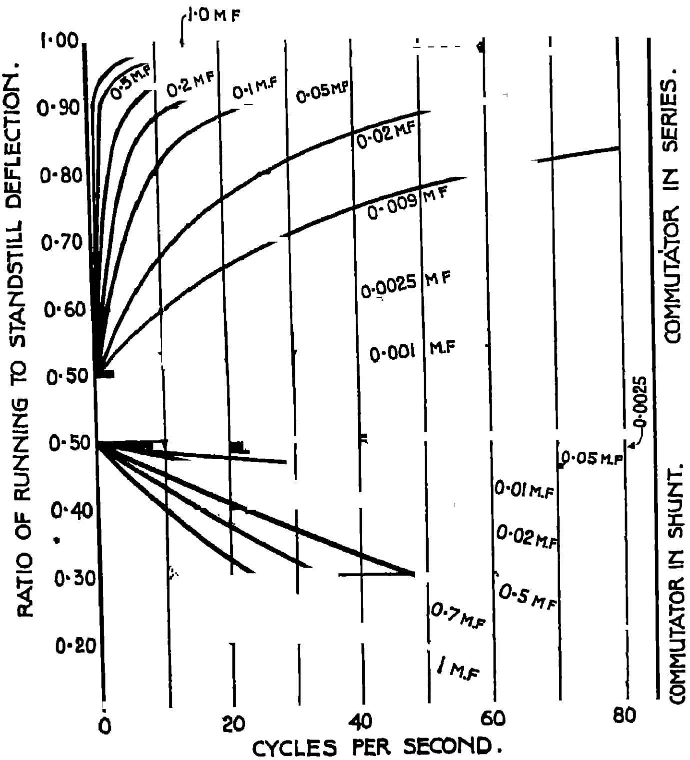


FIG. 365.—Effect of capacity in shunt on running to standstill ratio.

The equations for the various E.M.Fs. can be written down for the time when the commutator is switching the current directly to the galvanometer in the form

$$ir + i_1R = e \quad . \quad . \quad . \quad . \quad (1)$$

$$i_1R = e_o = x_c\int i_2 \, d\theta \quad . \quad . \quad . \quad (2)$$

$$i_1 + i_2 = i \quad . \quad . \quad . \quad . \quad (3)$$

From these equations, by elimination,

$$\frac{di_1}{d\theta} + \frac{x_c i_1(R + r)}{rR} = \frac{ex_c}{rR},$$

which has for a solution

$$i_1 = Ae^{-\frac{(R+r)x_c}{rR}\theta} + \frac{e}{R + r} \quad . \quad . \quad . \quad (4)$$

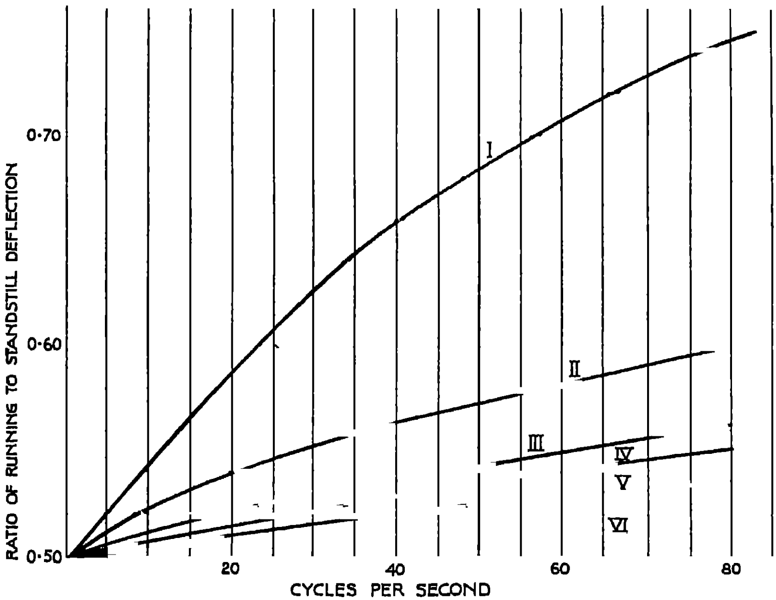


FIG. 366.—Ratio of running to standstill deflection for various speeds of a series commutator.

No. 1.—Leads for 10^6 ohm resistance. 100 feet of twisted flexible (= 0·005 mf.); leads for commutator 15 feet ditto (= 0·0007 mf.), brush narrower than insulation between segments.

No. 2.—Leads for 10 ohm resistance, short separated wires. Leads for commutator the same as No. 1, brush wider than insulation between segments.

No. 3.—Same as No. 1 except that the leads for the 10 ohm resistance were separated.

No. 4.—Same as No. 2 except that brush was narrower than insulation between segments.

No. 5.—Same as No. 3 except that the commutator leads were separated.

No. 6.—Same as No. 4 except that the commutator leads were separated.

From equations (2) and (4)

$$e_c = AR\epsilon^{-\frac{(R+r)x_c}{rR}\theta} + \frac{ER}{R+r} \quad . \quad . \quad . \quad (5)$$

and to evaluate (5) put $\theta = 0$ for the closing, and $\theta = \pi$ for the opening of the circuit, whence

$$e_c(\theta = 0) = AR + \frac{E}{R+r}$$

and
$$e_c(\theta = \pi) = AR\epsilon^{-\frac{(R+r)x_c}{Rr}\pi} + \frac{ER}{R+r}.$$

During the period when the circuit is open (it being remembered that a series commutator is being considered), the condenser discharges through R and the equations are

$$\frac{de'_c}{d\theta} + \frac{x_c e'_c}{R} = 0,$$

or
$$e'_c = A'\epsilon^{-\frac{x_c\theta}{R}}.$$

Hence as above

$$e'_c(\theta = 0) = A'$$

and
$$e'_c(\theta = 2\pi) = A'\epsilon^{-\frac{2\pi x_c}{R}}$$

and if commutation is to be sparkless the further condition is given that

$$e_c(\theta = 0) = e'_c\,(\theta = 2\pi)$$

and
$$e_c(\theta = 2\pi) = e'_c\,(\theta = 0)$$

respectively.

Eliminating the constants A and A' the current values are found to be

$$i_1 = \frac{E}{R+r}\left\{1 - \frac{1 - \epsilon^{-\frac{2\pi x_c}{R}}}{1 - \epsilon^{-2\pi x_c\left(\frac{2}{R}+\frac{1}{r}\right)}}\epsilon^{-\pi x_c\left(\frac{1}{R}+\frac{1}{r}\right)}\right\}$$

and

$$i = \frac{E}{R+r}\left\{1 + \frac{R}{r}\,\frac{1 - \epsilon^{-\frac{2\pi x_c}{R}}}{1 - \epsilon^{-2\pi x_c\left(\frac{2}{R}+\frac{1}{r}\right)}}\epsilon^{-\pi x_c\left(\frac{1}{R}+\frac{1}{r}\right)}\right\}$$

If I_M is the continuous current through the galvanometer

$$I_M = \frac{E}{R + r}$$

and the ratio i/I_M should correspond with the values given in the curves in Fig. 366. If this equation is plotted it will be found to be in close agreement with experiment.

The above analysis has been included because it is not generally realised what effect capacity has on the results of such a measurement, and the fact that most of the commutators in use are of the shunt type is based on direct experimental evidence.

Measurements of High Frequency Currents and Voltage by Means of a Thermionic Valve.—A sensitive direct reading

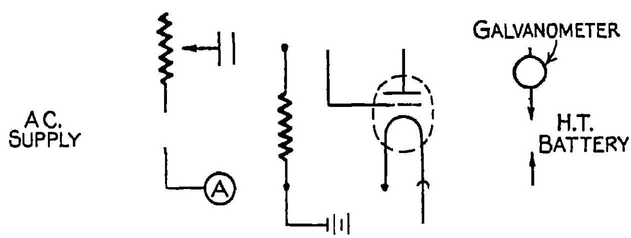


FIG. 367.—Moullin voltmeter.

voltmeter has been designed by E. B. Moullin, and is available for measurements of high frequency currents and voltages.

The circuit employed consists of a three-electrode valve connected as shown in Fig. 367.

As regards the constancy of the circuit arrangements it will be noted that a variation in any one of the following quantities may effect the reading of the galvanometer for a given variation of the supply :—

(1) High tension voltage,
(2) Low tension voltage,
(3) Grid condenser capacity,
(4) Grid leak value,

but these factors do not have as great an effect as would be expected. For instance, with an ordinary R type valve :—

(1) At about 75 volts high tension a variation of plus and minus 3·5 per cent. volts has no appreciable effect on the calibration.

At lower voltages a certain measure of inaccuracy is introduced, and it is therefore advisable to keep the anode potential at about that figure.

(2) A change of from 3·7 to 4·0 volts on the filament only produces an alteration in the calibration of 1·5 per cent.

(3) The grid condenser does not vary in capacity, so that its effect need not be considered.

(4) Grid leak. Changing the grid leak from 1·8 to 4 megohms affects the accuracy by 4 per cent.

It will be apparent that a somewhat complicated arrangement with various external supplies results in a measuring instrument, the accuracy of which can be controlled within definite limits.

The calibration can be effected by inserting a meter of known accuracy, in the supply circuit, and obtaining a curve by varying the rheostat connecting the amperes in the supply with the galvanometer readings. A family of curves will thus be obtained with different anode or filament potentials. These curves will form the calibration of the instrument.

Altogether six different methods of connection have been employed for the measurement of radio frequency voltages. A full description is given by Medlam in "Experimental Wireless," which should be consulted for the detailed descriptions. The methods affect the accuracy of the determination as well as the voltage range covered.

Peak Voltmeter.—An important and simple application of the rectifier as a piece of measuring apparatus, is the peak voltmeter. This instrument consists of a thermionic rectifier, a condenser of high insulation, whose capacity is large compared with that between the anode and filament of the rectifier, and an electrostatic voltmeter also of high insulation. Voltage peaks are measured with respect to earth potential, and depending on whether the peak is positive or negative, the arrangement of Fig. 368 or Fig. 369 must be used.

The principle involved is simple—take the case shown in Fig. 368, current will flow into the condenser whenever the

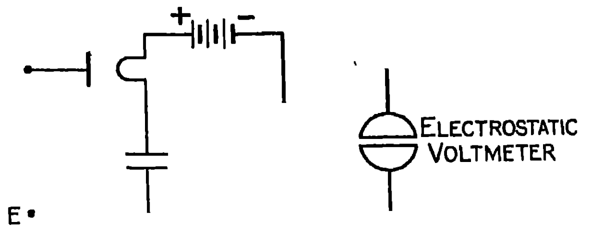


FIG. 368.—Peak voltmeter.

anode is at a higher potential, but no current can flow out of the condenser, since it is perfectly insulated, and further the

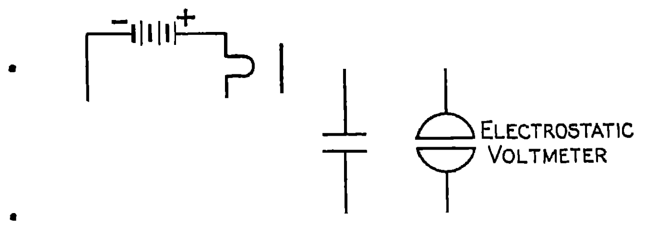


FIG. 369.—Peak voltmeter.

rectifier is a perfect insulator, as far as current flowing out of the condenser is concerned. The condenser thus rapidly charges up to the peak value of the voltage applied to its terminals.

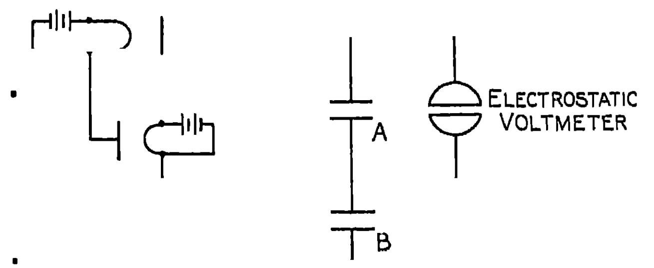


FIG. 370.—Ripple voltmeter.

It is necessary to ensure good insulation—the sharper the peaks the more important this factor will become. The rectifier must also be thoroughly evacuated so as to cause no trouble through

gas discharges. A small emission of a few milliamperes only is required.

By measuring peak voltages across non-inductive shunts, peak values of current can be obtained, while by measuring peak voltages across inductances a measure of the steepness of wave front can be obtained.

Such peak voltmeters are of value in studying the operation of three-electrode valves used as generators of oscillations; the complete voltage swing of grid and anode can be readily obtained.

A further use is in measuring the amount of ripple on a rectified supply, by using the circuit arrangement shown in

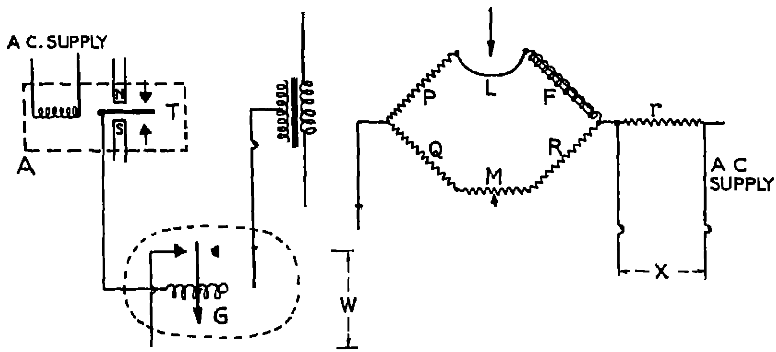


FIG. 371 —Temperature regulation of a furnace.

Fig. 370, which is merely a combination of Figs. 367 and 368; the voltmeter measures the difference between the maximum and the minimum value.

Application of Small Rectifiers to Voltage Control.—As an example of the special usages to which rectifiers may be put, the control of a furnace temperature between close limits is quoted. This particular control has been developed by H. S. Roberts and is shown diagrammatically in Fig. 371, a vibrating reed type of rectifier being used.

The furnace is shown at F and is placed in one of the arms of a Wheatstone's Bridge, the ratios of which are

$$\frac{P}{Q} \text{ and } \frac{F}{R}.$$

When the temperature rises, the resistance of F rises and the balance is upset, causing an alternating current to flow between the contacts L and M. Thus current will pass through the transformer to the vibrating reed rectifier A which is supplied from the main A.C. supply, and which vibrates once per cycle, owing to the polarising permanent magnet. A direct current flows to the galvanometer relay G and the needle gradually moves over to the right or left depending on the direction of the current, i.e. on whether L is at a greater potential than M or conversely. The galvanometer needle thus makes contact with

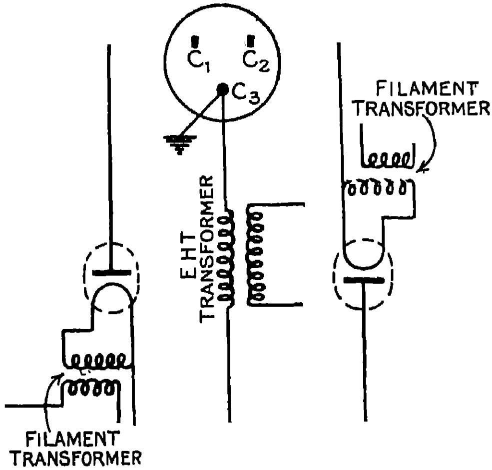


FIG. 373.—150 kilovolt direct current core C_1 to core C_2 of a three-core cable. 75 kilovolt cores C_1 and C_2 to earth.

one or other side of the circuit W, and on account of the inclusion of another relay and rectifier (not shown) the circuit X is opened or closed, thus regulating the furnace current by the short-circuiting of the resistance r. The details have been worked out carefully in the paper mentioned in the bibliography, but this case is mentioned to indicate what advantages a rectifier may bring if only it can be developed with sufficient care.

Application of Thermionic Rectifiers to Cable Testing and Dust Precipitation.—The high voltage thermionic rectifier is used in cable testing and dust precipitation as foreshadowed

on page 131. The particular plant for the former purpose is one developed by Messrs. Watson & Sons, and a portable form is illustrated in Fig. 372.

With the apparatus shown, it is possible to obtain a peak voltage for test purposes of 150 kilovolt by the use of a 75 kilovolt transformer. The method by which this voltage is obtained has already been described on page 336, but the actual connections in this case are indicated in Fig. 373 and 374.

The primary voltage can be suitably varied by means of an

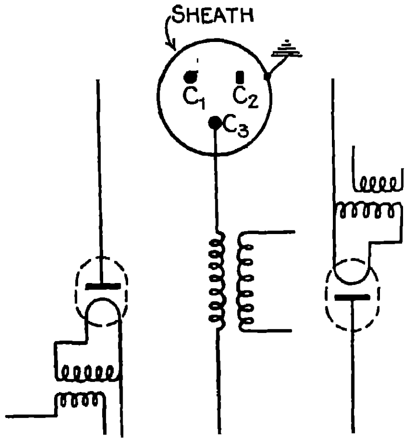


FIG. 374.—100 kilovolt cores C_1 to earth. 50 kilovolt cores C_3 to earth.

adjustable choke, and this, at the same time, will automatically limit the amount of current in the E.H.T. circuit in the event of a cable fault during testing operations.

Further banks of high resistance units are provided in the high tension circuit to protect the rectifiers against surges.

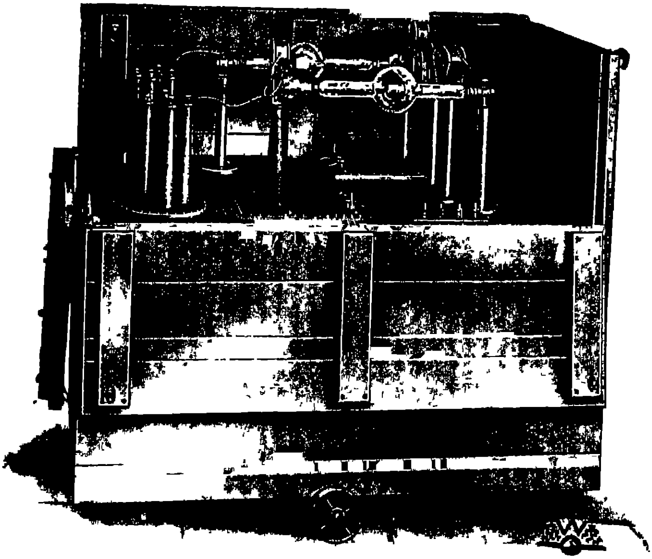


Fig. 372.—Cable-testing plant using thermionic rectifiers.

[To face page 510.

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B 1927 266	Gorce	S.F.B.	1926, p. 1182.
B 1926 399	Medlam	Exp. W.	1926, pp. 589, 664.

APPENDIX I.

TABLES OF $e^{-\theta}$.

θ.	0	·001.	·002.	·003.	·004.	·005.	·006.	·007.	·008.	·009.
·00	1·000	·9990	·9980	·9970	·9960	·9950	9940	·9980	·9920	·9910
·01	·9900	·9891	·9881	·9871	·9861	·9851	·9841	·9881	·9822	·9812
·02	·9802	·9792	·9782	·9778	·9768	·9758	·9748	·9784	·9724	·9714
·03	·9704	·9695	·9685	·9675	·9666	·9656	·9646	·9687	·9627	·9618
·04	·9608	·9598	·9589	·9579	·9570	·9560	·9550	·9541	·9581	·9522
·05	·9512	·9502	·9493	·9484	·9474	·9465	·9455	·9446	·9436	·9427
·06	·9418	·9408	·9399	·9389	·9380	·9871	·9361	·9352	·9848	·9388
·07	·9824	·9815	·9805	·9296	·9287	·9277	·9268	·9259	·9250	·9240
·08	·9231	·9222	·9213	·9204	·9194	·9185	·9176	·9167	·9158	·9148
·09	·9139	·9130	·9121	·9112	·9103	·9094	·9085	·9076	·9066	·9057

θ.	0.	·01.	·02.	·03.	·04.	·05.	·06.	·07.	·08	·09.
·1	·9048	·8958	·8869	·8781	·8694	·8607	·8521	·8437	·8353	·8270
·2	·8187	·8106	·8025	·7945	·7866	·7788	·7711	·7634	·7558	·7483
·3	·7408	·7334	·7261	·7189	·7118	·7047	·6977	·6907	·6839	·6771
·4	·6703	·6637	·6570	·6505	·6440	·6376	·6313	·6250	·6188	·6126
·5	·6065	·6005	·5945	·5886	·5827	·5769	·5712	·5655	·5599	·5543
·6	·5488	·5434	·5379	·5326	·5273	·5220	·5169	·5117	·5066	·5016
·7	·4966	·4916	·4868	·4819	·4771	·4724	·4677	·4630	·4584	·4538
·8	·4493	·4449	·4404	·4360	·4317	·4274	·4232	·4190	·4148	·4107
·9	·4066	·4025	·3985	·3946	·3906	·3867	·3829	·3791	·3753	·3716
1 0	·3679	·3642	·3606	·3570	·3535	·3499	·3465	·3430	·3396	·3362
1·1	·3329	·3296	·3263	·3230	·3198	·3166	·3135	·3104	·3073	·3042
1·2	·3012	·2982	·2952	·2923	·2894	·2865	·2837	·2808	·2780	·2753
1·3	·2725	·2698	·2671	·2645	·2618	·2592	·2567	·2541	·2516	·2491
1·4	·2466	·2441	·2417	·2393	·2369	·2346	·2322	·2299	·2276	·2254
1·5	·2231	·2209	·2187	·2165	·2144	·2122	·2101	·2080	·2060	·2039
1·6	·2019	·1999	·1979	·1959	·1940	·1920	·1901	·1882	·1864	·1845
1·7	·1827	·1809	·1791	·1773	·1755	·1738	·1720	·1703	·1686	·1670
1·8	·1653	·1637	·1620	·1604	·1588	·1572	·1557	·1541	·1526	·1511
1·9	·1496	·1481	·1466	·1451	·1437	·1423	·1409	·1395	·1381	·1367
2·0	·1353	·1340	·1327	·1313	·1300	·1287	·1275	·1262	·1249	·1237
2·1	·1225	·1212	·1200	·1188	·1177	·1165	·1153	·1142	·1130	·1119
2·2	·1108	·1097	·1086	·1075	·1065	·1054	·1044	·1033	·1023	·1013
2·3	·1003	·0993	·0983	·0973	·0963	·0954	·0944	·0935	·0926	·0916
2·4	·0907	·0898	·0889	·0880	·0872	·0863	·0854	·0846	·0837	·0829
2·5	·0821	·0813	·0805	·0797	·0789	·0781	·0773	·0765	·0758	·0750
2·6	·0743	·0735	·0728	·0721	·0714	·0707	·0699	·0693	·0686	·0679
2·7	·0672	·0665	·0659	·0652	·0646	·0639	·0633	·0627	·0620	·0614
2·8	·0608	·0602	·0596	·0590	·0584	·0578	·0573	·0567	·0561	·0556
2·9	·0550	·0545	·0539	·0534	·0529	·0523	·0518	·0513	·0508	·0503

TABLES OF $\epsilon^{-\theta}$ (*cont.*).

θ.	0.	·1.	2.	·3.	·4.	5	·6.	·7.	·8.	·9.
3	·0498	·0450	·0408	·0368	·0334	·0302	·0273	·0247	·0224	·0202
4	·0183	·0166	·0150	·0136	·0123	·0111	·0101	·0091	·0082	·0074
5	·0067	·0061	·0055	·0050	·0045	·0041	·0037	·0033	·0030	·0027
6	·0025	·0022	·0020	·0018	·0017	·0015	·0014	·0012	·0011	·0010
7	·0009	·0008	·0007	·0007	·0006	·0006	·0005	·0005	·0004	·0004
8	·0003	·0003	·0003	·0002	·0002	·0002	·0002	·0002	·0002	·0001

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